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FORMAL LOGIC
(1847)

AUGUSTUS DE MORGAN

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A. E. TAYLOR, F.B.A.
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FORMAL LOGIC
(1847)

BY

AUGUSTUS DE MORGAN

EDITED BY

A. E. TAYLOR, F.B.A.

PROFESSOR OF MORAL PHILOSOPHY AT
THE UNIVERSITY OF EDINBURGH

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EDITOR'S NOTE.

The object of the present reprint is simply to provide students of logic and the history of logical doctrine with an accurate text of a classical work which has become almost inaccessible to those who are unable to avail themselves of large libraries. In this volume I have not regarded it as any part of my task to supply biographical information about De Morgan or to indicate opinions of my own upon the correctness or incorrectness of his views, his historical place in the series of English writers on logic, his influence upon the subsequent developments of logical symbolism and other kindred topics. I have aimed at little beyond the presentation of a thoroughly trustworthy text ; and have sought simply to make the reprint in every possible way an adequate substitute for the almost unprocurable original edition. I have accordingly made it a rule for myself that no departure from the original, however minute, shall be permitted which can possibly affect the writer's sense. The orthography of the first edition has been strictly adhered to, even when it is not in conformity with current usage. Spellings like "premise," "synonyme," and the like have an historical interest, and were in some cases deliberately adopted by De Morgan for reasons of his own. It has therefore been made a rule of decent piety towards an eminent writer to preserve them. For weightier reasons the author's original punctuation and his use of italics, capitals and the like have been rigidly followed. De Morgan's stopping is heavier and more elaborate than that commonly employed in either his day or ours, but there is clear internal evidence that it was deliberately and carefully thought out. The mere insertion or omission of a

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comma may involve an editorial choice between slightly different interpretations of an ambiguous sentence, and I at any rate feel strongly that the making of such choices falls outside my province. If there are ambiguities in De Morgan's text, I feel it my duty to present them as such to my readers. For the same reason, I have considered it unlawful to run the risk of modifying the emphasis intended to be laid on a word or a clause by any departure from the author's employment of such devices as italics and capitals. The use of the long s has not been preserved in the body of the work, nor has it been considered necessary to retain the original pagination; but the page-numbers of the original have been inserted throughout for convenience of reference.

In regard to the few uncorrected errata of the first edition, the plan followed has been to correct them where they occur, but to record the erroneous readings in an editorial footnote, in order that the reader may be exactly informed on the text of the original work. De Morgan's own "Table of Contents," which has been exactly reprinted, contains a number of "corrections and additions." Where the correction is merely that of an error of the press, the change has been made silently; in other cases, the text has been printed as it stood, and the alterations or additions requested by De Morgan indicated in a footnote. All editorial notes are enclosed in square brackets, numbered consecutively, and carefully distinguished from De Morgan's annotations by the signature "Ed."

I desire to express my obligation to Mr. J. M. Child of the University of Manchester for the care he has lavished on the proof-sheets of this reprint: the detection of more than one oversight in De Morgan's Text is due to Mr. Child, who was also good enough to re-cast and greatly improve the note on an interesting problem in probabilities at p. 260.

A. E. TAYLOR.

Edinburgh,
September, 1925.

NOTE.

THE matters collected in De Morgan's *Syllabus of a Proposed System of Logic*, London, Walton & Maberly, 1860; 72 pp. [Preface dated Nov. 12, 1859], is to be found in I. *Paper on the Syllogism, C.P.T.* 8, part 3, 1847; II. This *Formal Logic*; III. Paper in *C.P.T.*, 9, part 1, 1850; IV. Paper in *C.P.T.*, 10, part 1, 1858. Of these the formulæ and notation of I. are entirely superseded; the notation only of II. may be advantageously replaced by that of III., IV. and the *Syllabus*. "There is very little in the first 3 writings on which my opinion has varied; but of all 3 it is to be said that they are entirely based on what I now call the *arithmetical* view of the proposition and syllogism (§§ 8, 173, 174), extending this term not merely to the *numerically* definite syllogism, but to the ordinary form, to my own extension of it, and to Sir W. Hamilton's departure from it" (*Syllabus*, p. 3).

"The relations of my work on *Formal Logic* to the present syllabus are as follows. Chapter I. *First Notions*, may afford previous knowledge to the student who has hitherto paid no attention to the subject. Chapter III, on the *Abstract Form of the Proposition*, may be consulted at § 93 of this syllabus, and are partially superseded. Chapter XIV, on the *Verbal Description of the Syllogism*, is entirely superseded. All the rest of the work may be read as the titles of the chapters suggest" (*Syllabus*, p. 4).

P. 14. "21. Let X, totally spoken of, be X) or (X: let X, partially spoken of, be)X or X(. Let a negative proposition be denoted by one dot; an affirmative proposition by two dots or more, at pleasure."

P. 15. Note to § 24, De Morgan compares notation of *Formal Logic* with that used in the second and third papers in *C.P.T.*, and in the *Syllabus*.

<table>
<tr><td>“ For X))Y,</td><td>X)Y and $A_{\prime}$</td><td>For X()Y,</td><td>XY and $I_{\prime}$</td></tr>
<tr><td>For X((Y,</td><td>X(Y and A'</td><td>For X)(Y,</td><td>xy and I'</td></tr>
<tr><td>For X).(Y,</td><td>X.Y and $E_{\prime}$</td><td>For X(.(Y,</td><td>X:Y and $O_{\prime}$</td></tr>
<tr><td>For X(.)Y,</td><td>x.y. and E'</td><td>For X).)Y,</td><td>Y:X and O'</td></tr>
<tr><td>For X)o)Y,</td><td>$D_{\prime}$ and $A_{\prime} + O'$</td><td>For X)o(Y,</td><td>$C_{\prime}$ and $E_{\prime} + I'$</td></tr>
<tr><td>For X||Y,</td><td>D and $A_{\prime} + A'$</td><td>For X|.|Y,</td><td>C and $E_{\prime} + E'$</td></tr>
<tr><td>For X(o(Y,</td><td>D' and $A' + O'$</td><td>For X(o)Y,</td><td>C' and $E' + I_{\prime}$</td></tr>
</table>

FORMAL LOGIC:

OR,

The Calculus of Inference,

Neceſſary and Probable.

BY

AUGUSTUS DE MORGAN
Of Trinity College Cambridge,
Fellow of the Cambridge Philoſophical Society, Secretary of the Royal
Aſtronomical Society, Profeſſor of Mathematics in
Univerſity College London.

Καλὸς ὁ νόμος ἐάν τις αὐτῷ νομίμως χρῆται.

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PREFACE.

THE system given in this work extends beyond that commonly received, in several directions. A brief statement of what is now submitted for adoption into the theory of inference will be the matter of this preface.

In the form of the proposition, the copula is made as abstract as the terms: or is considered as obeying only those conditions which are necessary to inference.

Every name is treated in connection with its *contrary* or *contradictory* name; the distinction between these words not being made, and others supplied in consequence. Eight really separable forms of predication are thus obtained, between any two names: the eight of the common system amounting only to six, when, as throughout my work, the two forms of a convertible proposition are considered as identical.

The complex proposition is introduced, consisting in the coexistence of two simple ones. The theory of the syllogism of complex propositions is made to precede that of the simple or ordinary syllogism; which last is deduced from it. I have only used the word *complex*, because *simple* was already appropriated (see page 85).

[iv] By the introduction of contraries, the number of valid syllogistic forms is increased to thirty-two, connected together by many rules of relation, but all shewn to contain, each with reference to its own disposition of names and contraries, only one form of inference.

The distinction of figure is avoided from the beginning by introducing into every proposition an order of reference to its terms.

A simple notation, which includes the common one, gives the means of representing every syllogism by three letters, each accented above or below. By inspection of one of these symbols it is seen immediately, 1. What syllogism is represented, 2. Whether it be valid or invalid, 3. How it is at once to be written down, 4. What axiom the inference contains, or what is the act of the mind when it makes that inference (chapter XIV).

A subordinate notation is used (page 60) [1] in abbreviation of the proposition at length.

Compound names are considered, both when the composition is conjunctive, and when it is disjunctive. Distinct notation and rules of transformation are given, and the compound syllogisms are treated as reducible to ordinary ones, by invention of compound names.

The theory of the numerical syllogism is investigated, in which, upon the hypothesis of numerical quantity in both terms of every proposition, a numerical inference is made.

But, when the numerical relations of the several terms are fully known, all that is unusual in the quantity of the predicate is shown to be either superfluous, or else, as I have called it, spurious.

[v] The old doctrine of modals is made to give place to the numerical theory of probability. Many will object to this theory as extralogical. But I cannot see on what definition, founded on real distinction, the exclusion of it can be maintained. When I am told that logic considers the validity of the inference, independently of the truth or falsehood of the matter, or supplies the conditions under which the hypothetical truth of the matter of the premises gives hypothetical truth to the matter of the conclusion, I see a real definition, which propounds for consideration the forms and laws of inferential thought. But when it is further added that the only hypothetical truth shall be absolute truth, certain knowledge, I begin to see arbitrary distinction, wanting the reality of that which preceded.

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[1] [I.e. p. 70 of the present edition.—Ed.]

Without pretending that logic can take cognizance of the probability of any given matter, I cannot understand why the study of the effect which partial belief of the premises produces with respect to the conclusion, should be separated from that of the consequences of supposing the former to be absolutely true. Not however to dispute upon names, I mean that I should maintain, against those who would exclude the theory of probability from logic, that, call it by what name they like, it should accompany logic as a study.

I have, of course, been obliged to express, in my own manner, my own convictions on points of mental philosophy. But any one will see that, in all which I have proposed for adoption, it matters nothing whether my views of the phenomena of thought, or others, be made the basis of the explanation. So far therefore, as I am **[vi]** considered as proposing forms of syllogism, &c. to the logician, and not giving instruction to the student of the science, the reader has nothing to do with my choice of the terms in which mental operations are spoken of.

In the appendix will be found some remarks on the personal controversy between Sir W. Hamilton of Edinburgh and myself, of which I suppose the celebrity of my opponent, and the appearance of part of it in a journal so widely circulated as the *Athenæum*, has caused many students of logic to hear or read something.

At the end of the contents of some chapters in the following table, are a few additions and corrections, to which I request the reader's attention.[2]

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON,
October 14, 1847.

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[2] [In the present edition mere errors of impression have been silently corrected. The author's other additions and corrections have been appended as notes to the passages indicated in the Table of Contents.—ED.]

[vii] TABLE OF CONTENTS.

⁂ The articles entered in *Italic*, are those, the contents of which belong to the peculiar system presented in this work.

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⁂ This chapter may be omitted by those who have some knowledge of the ordinary definitions and phraseology of logic. It is strictly confined to the Aristotelian forms and syllogisms, and is the reprint of a tract published in 1839, under the title of 'First Notions of Logic (preparatory to the study of Geometry)': the only alterations are;—the change of phraseology, as altering 'some X is Y' into 'some Xs are Ys,' &c.; the correction of a faulty demonstration; and a few omissions, particularly of some insufficient remarks on the probability of arguments.

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[3] [Throughout this table the numbers refer to the original pagination, given by clarendon numbers inserted in the text.—Ed.]

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Additions and corrections. Page 56, *line* 7, *insert* except only one which consists of four simple propositions. *Page* 62, *line* 23; Say X and Y are not *complements* (instead of *contraries*) that is, do not together either fill, or more than fill, the universe. *Page* 72, *lines* 4 *and* 3, *from the bottom*; The oppositions are incorrect. It ought to be *cannot do without* and *cannot fail with*: *must precede*, and *must follow*. The reader may easily identify the eight forms of predication as having X for subject, Y for predicate, with the copulæ, cannot be without, can be without, cannot be with, can be with, cannot fail without, can fail without, cannot fail with, can fail with.

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Additions and corrections.—Page 79, in the first diagram, for $\mathrm{D_{\prime}D_{\prime}D}$, read $\mathrm{D_{\prime}D_{\prime}D_{\prime}}$; *page* 88, *line* 23, *instead of* has the other two for its opponents, *read* has its opponents in the set ; *page* 90, *line* 4, *from the bottom, for* premiss *read* premise : the first spelling has been common enough, but it seems strange that the cognate words promise, surmise, demise, &c. should not have dictated the second. *Page* 96 ; **[xi]** The inverted forms of the strengthened syllogisms are omitted : of these, four are their own inversions, namely, $\mathrm{A_{\prime}A'I'}$, $\mathrm{A'AI_{\prime}}$, $\mathrm{E'E'I_{\prime}}$, and $\mathrm{E_{\prime}E_{\prime}I'}$: of the remainder, $\mathrm{A_{\prime}E'O'}$ and $\mathrm{E'A'O_{\prime}}$ are inversions ; and also $\mathrm{A'E_{\prime}O_{\prime}}$ and $\mathrm{E_{\prime}A_{\prime}O'}$. *Page* 100, *line* 12, *from the bottom* ; for —011 *read* —011), the first time it occurs. *Page* 101 : Read the symbols of the strengthened syllogisms so as to begin from the middle in both premises : thus, Xyz| is y)X+y)z=Xz. *Page* 101. I might have said a word or two on the case in which a complex particular is combined with a universal ; to form the results will be an easy exercise for the reader. *Page* 102, *line* 7, *from the bottom,* for $\mathrm{I_{\prime}A'I_{\prime}}$ *read* $\mathrm{I_{\prime}A_{\prime}I_{\prime}}$.

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Additions and corrections. Page 121, *line* 8, *from the bottom.* For [x,y][p,q])u *read* [X,Y][p,q])u.

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[4] [The technical names Baroco, Bocardo should strictly be spelt with c, not with k, since the c is significant. Its meaning is that the O premise of the syllogism in question is the one which must be contradicted in order to obtain an "indirect reduction" to Barbara in Fig. 1.—Ed.]

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[xiii] *Additions and corrections. Page* 143, *line* 12: Supply the propositions X)M,P and Y)N,Q, as deducible from the numbers of instances in the several names. *Page* 148, *line* 10, *from the bottom*: *for* propositions *read* prepositions. *Page* 152, *line* 4: *for* m *read m*. *Page* 153, *line* 22: *for* will presently show us, *read* have shown us in page 145. *Page* 154, *line* 2, *from the bottom*, *for* ys *read* zs. *Page* 155, *line* 6 *from the bottom*, *for* mXY *read* *m*XY. *Page* 162, *line* 2, *after the table*: *for* last chapter *read* chapter V. *Page* 166, *line* 17, *for m'xy read m'*xy. *Page* 167, *line* 24: *for* 62 *read* 92.

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Additions and corrections. Page 199, *line* 4, *from the bottom: for* $(1-\lambda)$ *read* $(1-\lambda)^m$. *Page* 201, *line* 14, *from the bottom: for* ^{7} *read* $\frac{7}{10}$.

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Additions and corrections. Page 230, *lines* 16 *and* 15, *from the bottom; transpose the words* former *and* latter. *Page* 234, *line* 2 *from bottom, for* after *read* before. *Page* 237, *note;* I find that etymologists are decidedly of opinion that ῥῆσις, speech, and ʽϱέω, flow, have different roots, and that the former is *speech* in its primitive meaning. The reader must make the alteration, which however does not affect my suggestion.

[xv] CHAPTER XIII.—*On Fallacies* (pages 237—286).

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Additions and corrections. Page 250, *lines* 3 *and* 5 *; for* millenium *read* millennium, *and for* Newtonion *read* Newtonian.

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[1]

ELEMENTS OF LOGIC.

CHAPTER I.

FIRST NOTIONS.

THE first notion which a reader can form of Logic is by viewing it as the examination of that part of reasoning which depends upon the manner in which inferences are formed, and the investigation of general maxims and rules for constructing arguments, so that the conclusion may contain no inaccuracy which was not previously asserted in the premises. It has so far nothing to do with the truth of the facts, opinions, or presumptions, from which an inference is derived; but simply takes care that the inference shall certainly be true, if the premises be true. Thus, when we say that all men will die, and that all men are rational beings, and thence infer that some rational beings will die, the *logical* truth of this sentence is the same whether it be true or false that men are mortal and rational. This logical truth depends upon the *structure of the sentence*, and not upon the particular matters spoken of. Thus,

Instead of	Write,
All men will die.	Every Y is X.
All men are rational beings.	Every Y is Z.
Therefore some rational beings will die.	Therefore some Zs are Xs.

The second of these is the same proposition, logically considered, as the first; the consequence in both is virtually contained in, and rightly inferred from, the premises. Whether the premises be true or false, is not a question of logic, but of morals, philosophy, history, or any other knowledge to which their subject-[2]matter belongs : the question of logic is, does the conclusion certainly follow if the premises be true ?

Every act of reasoning must mainly consist in comparing together different things, and either finding out, or recalling from previous knowledge, the points in which they resemble or differ from each other. That particular part of reasoning which is called *inference*, consists in the comparison of several and different things with one and the same other thing ; and ascertaining the resemblances, or differences, of the several things, by means of the points in which they resemble, or differ from, the thing with which all are compared.

There must then be some propositions already obtained before any inference can be drawn. All propositions are either assertions or denials, and are thus divided into *affirmative* and *negative*. Thus, X is Y, and X is not Y, are the two forms to which all propositions may be reduced. These are, for our present purpose, the most simple forms ; though it will frequently happen that much circumlocution is needed to reduce propositions to them. Thus, suppose the following assertion, " If he should come to-morrow, he will probably stay till Monday ; " how is this to be reduced to the form X is Y ? There is evidently something spoken of, something said of it, and an affirmative connection between them. Something, if it happen, that is, the happening of something, makes the happening of another something probable ; or *is* one of the things which render the happening of the second thing probable.

X	is	Y
The happening of his arrival to-morrow	is	an event from which it may be inferred as probable that he will stay till Monday.

The forms of language will allow the manner of asserting to be varied in a great number of ways ; but the reduction to the preceding form is always possible. Thus, " so he said " is an affirmation, reducible as follows:

What you have just said (or whatever else "so" refers to)	is	the thing which he said.

[3] By changing " is " into " is not," we make a negative proposition; but care must always be taken to ascertain whether a proposition which appears negative be really so. The principal danger is that of confounding a proposition which is negative with another which is affirmative of something requiring a negative to describe it. Thus, " he resembles the man who was not in the room," is affirmative, and must not be confounded with " he does not resemble the man who was in the room." Again, " if he should come to-morrow, it is probable he will not stay till Monday," does not mean the simple denial of the preceding proposition, but the affirmation of a directly opposite proposition. It is,

X	is	Y
The happening of his arrival to-morrow,	is	an event from which it may be inferred to be *im*probable that he will stay till Monday:

whereas the following,

The happening of his arrival to-morrow,	is *not*	an event from which it may be inferred as probable that he will stay till Monday,

would be expressed thus: " If he should come to-morrow, that is no reason why he should stay till Monday."

Moreover, the negative words not, no, &c., have two kinds of meaning which must be carefully distinguished. Sometimes they deny, and nothing more: sometimes they are used to affirm the direct contrary. In cases which offer but two alternatives, one of which is necessary, these amount to the same thing, since the denial of one, and the affirmation of the other, are obviously equivalent propositions. In many idioms of conversation, the negative implies affirmation of the contrary in cases which offer not only alternatives, but degrees of alternatives. Thus, to the question, "Is he tall?" the simple answer, "No," most frequently means that he is the contrary of tall, or considerably under the average. But it must be remembered, that, in all logical reasoning, the negation is simply negation, and nothing more, never implying affirmation of the contrary.

The common proposition that two negatives make an affirmative, is true only upon the supposition that there are but two [4] possible things, one of which is denied. Grant that a man must be either able or unable to do a particular thing, and then *not unable* and able are the same things. But if we suppose various degrees of performance, and therefore degrees of ability, it is false, in the common sense of the words, that two negatives make an affirmative. Thus, it would be erroneous to say, "John is able to translate Virgil, and Thomas is not unable; therefore, what John can do Thomas can do," for it is evident that the premises mean that John is so near to the best sort of translation that an affirmation of his ability may be made, while Thomas is considerably lower than John, but not so near to absolute deficiency that his ability may be altogether denied. It will generally be found that two negatives imply an affirmative of a weaker degree than the positive affirmation.

Each of the propositions, "X is Y," and "X is not Y," may be subdivided into two species: the *universal*, in which every possible case is included; and the *particular*, in which it is not meant to be asserted that the affirmation or negation is universal. The four species of proposition are then as follows, each being marked with the letter by which writers on logic have always distinguished it.

A	*Universal Affirmative*	Every	X is	Y
E	*Universal Negative*	No	X is	Y
I	*Particular Affirmative*	Some	Xs are	Ys
O	*Particular Negative*	Some	Xs are not	Ys

In common conversation the affirmation of a part is meant to imply the denial of the remainder. Thus, by "some of the apples are ripe," it is always intended to signify that some are not ripe. This is not the case in logical language, but every proposition is intended to make its amount of affirmation or denial, and no more. When we say, "Some X is Y," or, more grammatically, "Some Xs are Ys," we do not mean to imply that some are not: this may or may not be. Again, the word some means, "one or more, possibly all." The following table will shew the bearing of each proposition on the rest.

Every X is Y	affirms *Some Xs are Ys* and denies	*No X is Y* *Some Xs are not Ys*	
[5] *No X is Y*	affirms *Some Xs are not Ys* and denies	*Every X is Y* *Some Xs are Ys*	
Some Xs are Ys	does not contradict	*Every X is Y* *Some Xs are not Ys*	but denies *No X is Y*
Some Xs are not Ys	does not contradict	*No X is Y* *Some Xs are Ys*	but denies *Every X is Y*

Contradictory propositions are those in which one denies *any thing* that the other affirms; *contrary* propositions are those in which one denies *every thing* which the other affirms, or affirms every thing which the other denies. The following pair are contraries,

Every X is Y and No X is Y

and the following are contradictories,

Every X is Y to Some Xs are not Ys
No X is Y to Some Xs are Ys

A contrary, therefore, is a complete and total contradictory; and a little consideration will make it appear that the decisive distinction between contraries and contradictories lies in this, that contraries may both be false, but of contradictories, one must be true and the other false. We may say, "Either P is true, or *something* in contradiction of it is true;" but we cannot say, "Either P is true, or *every thing* in contradiction of it is true." It is a very common mistake to imagine that the *denial* of a proposition gives a right to *affirm* the contrary; whereas it should be, that the *affirmation* of a proposition gives a right to *deny* the contrary. Thus, if we deny that Every X is Y, we do not affirm that No X is Y, but only that Some Xs are not Ys; while, if we affirm that Every X is Y, we deny No X is Y, and also Some Xs are not Ys.

But, as to contradictories, affirmation of one is denial of the other, and denial of one is affirmation of the other. Thus, either Every X is Y, or Some Xs are not Ys: affirmation of either is denial of the other, and *vice versâ.*

Let the student now endeavour to satisfy himself of the following. Taking the four preceding propositions, A, E, I, O, let the simple letter signify the affirmation, the same letter in parentheses the denial, and the absence of the letter, that there is neither affirmation nor denial.

[6] From A follow (E), I, (O)
From E ......(A), (I), O
From I ......(E)
From O ......(A)

From (A) follow O
From (E) I
From (I) ......(A), E, O
From (O) ......A, (E), I

These may be thus summed up: The affirmation of a universal proposition, and the denial of a particular one, enable us to affirm or deny all the other three; but the denial of a universal proposition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others.

In such propositions as "Every X is Y," "Some Xs are not Ys," &c., X is called the *subject,* and Y the *predicate,* while the verb "is" or "is not," is called the *copula.* It is obvious that the words of the proposition point out whether the subject is spoken of universally or partially, but not so of the predicate, which it is therefore important to examine. Logical writers generally give the name of *distributed* subjects or predicates to those which are spoken of universally; but as this word is rather technical, I shall say that a subject or predicate enters wholly or partially, according as it is universally or particularly spoken of.

1. In A, or "Every X is Y," the subject enters wholly, but the predicate only partially. For it obviously says, "Among the Ys are all the Xs," "Every X is part of the collection of Ys, so that all the Xs make a part of the Ys, the whole it *may* be." Thus, "Every horse is an animal," does not speak of all animals, but states that all the horses make up a portion of the animals.

2. In E, or "No X is Y," both subject and predicate enter wholly. "No X whatsoever is any one out of all the Ys;" "search the whole collection of Ys, and *every* Y shall be found to be something which is not X."

3. In I, or "Some Xs are Ys," both subject and predicate enter partially. "Some of the Xs are found among the Ys, or make up a part (the whole possibly, but not known from the preceding) of the Ys."

4. In O, or "Some Xs are not Ys," the subject enters partially, and the predicate wholly. "Some Xs are none of them any whatsoever of the Ys; every Y will be found to be no one out of a certain portion of the Xs."

It appears then that,

In affirmatives, the predicate enters partially.

[7] In negatives, the predicate enters wholly.

In contradictory propositions, both subject and predicate enter differently in the two.

The *converse* of a proposition is that which is made by interchanging the subject and predicate, as follows:

	The proposition.	Its converse.
A	Every X is Y	Every Y is X
E	No X is Y	No Y is X
I	Some Xs are Ys	Some Ys are Xs
O	Some Xs are not Ys	Some Ys are not Xs

Now, it is a fundamental and self-evident proposition, that no consequence must be allowed to assert more widely than its premises; so that, for instance, an assertion which is only of some Ys can never lead to a result which is true of all Ys. But if a proposition assert agreement or disagreement, any other proposition which asserts the same, to the same extent and no further, must be a legitimate consequence; or, if you please, must amount to the whole, or part, of the original assertion in another form. Thus, the converse of A is not true: for, in "Every X is Y," the predicate enters partially; while in "Every Y is X," the subject enters wholly. "All the Xs make up a part of the Ys, then a part of the Ys are among the Xs, or some Ys are Xs." Hence, the only *legitimate* converse of "Every X is Y" is, "Some Ys are Xs." But in "No X is Y," both subject and predicate enter wholly, and "No Y is X" is, in fact, the same proposition as "No X is Y." And "Some Xs are Ys" is also the same as its converse "Some Ys are Xs:" here both terms enter partially. But "Some Xs are not Ys" admits of no converse whatever; it is perfectly consistent with all assertions upon Y and X in which Y is the subject. Thus neither of the four following lines is inconsistent with itself.

Some Xs are not Ys and Every Y is X
Some Xs are not Ys and No Y is X
Some Xs are not Ys and Some Ys are Xs
Some Xs are not Ys and Some Ys are not Xs.

Having thus discussed the principal points connected with the simple assertion, I pass to the manner of making two assertions [8] give a third. Every instance of this is called a *syllogism*, the two assertions which form the basis of the third are called *premises*, and the third itself the *conclusion*.

If two things both agree with a third in any particular, they agree with each other in the same; as, if X be of the same colour as Y, and Z of the same colour as Y, then X is of the same colour as Z. Again, if X differ from Y in any particular in which Z agrees with Y, then X and Z differ in that particular. If X be not of the same colour as Y, and Z be of the same colour as Y, then X is not of the colour of Z. But if X and Z both differ from Y in any particular, nothing can be inferred; they may either differ in the same way and to the same extent, or not. Thus, if X and Z be both of different colours from Y, it neither follows that they agree, nor differ, in their own colours.

The paragraph preceding contains the essential parts of all inference, which consists in comparing two things with a third, and finding from their agreement or difference with that third, their agreement or difference with one another. Thus, Every X is Y, every Z is Y, allows us to infer that X and Z have all those qualities in common which are necessary to Y. Again, from every X is Y, and "No Z is Y," we infer that X and Z differ from one another in all particulars which are essential to Y. The preceding forms, however, though they represent common reasoning better than the ordinary syllogism, to which we are now coming, do not constitute the ultimate forms of inference. Simple *identity* or *non-identity* is the ultimate state to which every assertion may be reduced; and we shall, therefore, first ask, from what identities, &c., can other identities, &c., be produced? Again, since we name objects in species, each species consisting of a number of individuals, and since our assertion may include all or only part of a species, it is further necessary to ask, in every instance, to what extent the conclusion drawn is true, whether of all, or only of part?

Let us take the simple assertion, "Every living man respires;" or every living man is one of the things (however varied they may be) which respire. If we were to enclose all living men in a large triangle, and all respiring objects in a large circle, the preceding assertion, if true, would require that the whole of the triangle should be contained in the circle. And in the same way we **[9]** may reduce any assertion to the expression of a coincidence, total or partial, between two figures. Thus, a point in a circle may represent an individual of one species, and a point in a triangle an individual of another species: and we may express that the whole of one species is asserted to be contained or not contained in the other by such forms as, "All the △ is in the ○"; "None of the △ is in the ○".

Any two assertions about X and Z, each expressing agreement or disagreement, total or partial, with or from Y, and leading to a conclusion with respect to X or Z, is called a syllogism, of which Y is called the *middle term.* The plainest syllogism is the following:—

Every X is Y	All the △ is in the ○
Every Y is Z	All the ○ is in the □
Therefore Every X is Z	Therefore All the △ is in the □

In order to find all the possible forms of syllogism, we must make a table of all the elements of which they can consist; namely—

X and Y		Z and Y
Every X is Y	A	Every Z is Y
No X is Y	E	No Z is Y
Some Xs are Ys	I	Some Zs are Ys
Some Xs are not Ys	O	Some Zs are not Ys
Every Y is X	A	Every Y is Z
Some Ys are not Xs	O	Some Ys are not Zs

Or their synonymes,

△ and ○		□ and ○
All the △ is in the ○	A	All the □ is in the ○
None of the △ is in the ○	E	None of the □ is in the ○
Some of the △ is in the ○	I	Some of the □ is in the ○
Some of the △ is not in the ○	O	Some of the □ is not in the ○
All the ○ is in the △	A	All the ○ is in the □
Some of the ○ is not in the △	O	Some of the ○ is not in the □

Now, taking any one of the six relations between X and Y, and combining it with either of those between Z and Y, we have six pairs of premises, and the same number repeated for every different relation of X to Y. We have then thirty-six **[10]** forms to consider: but, thirty of these (namely, all but (A, A) (E, E), &c.,) are half of them repetitions of the other half. Thus, "Every X is Y, no Z is Y," and "Every Z is Y, no X is Y," are of the same form, and only differ by changing X into Z and Z into X. There are then only 15+6, or 21 distinct forms, some of which give a necessary conclusion, while others do not. We shall select the former of these, classifying them by their conclusions; that is, according as the inference is of the form A, E, I, or O.

I. In what manner can a universal affirmative conclusion be drawn; namely, that one figure is entirely contained in the other? This we can only assert when we know that one figure is entirely contained in the circle, which itself is entirely contained in the other figure. Thus,

Every X is Y	All the △ is in the ○	A
Every Y is Z	All the ○ is in the □	A
Every X is Z	All the △ is in the □	A

is the only way in which a universal affirmative conclusion can be drawn.

II. In what manner can a universal negative conclusion be drawn; namely, that one figure is entirely exterior to the other? Only when we are able to assert that one figure is entirely within, and the other entirely without, the circle. Thus,

Every X is Y	All the △ is in the ○	A
No Z is Y	None of the □ is in the ○	E
No X is Z	None of the △ is in the □	E

is the only way in which a universal negative conclusion can be drawn.

III. In what manner can a particular affirmative conclusion be drawn; namely, that part or all of one figure is contained in the other? Only when we are able to assert that the whole circle is part of one of the figures, and that the whole, or part of the circle, is part of the other figure. We have then two forms.

Every Y is X	All the ○ is in the △	A
Every Y is Z	All the ○ is in the □	A
Some Xs are Zs	Some of the △ is in the □	I

[11]

Every Y is X	All the ○ is in the △	A
Some Ys are Zs	Some of the ○ is in the □	I
Some Xs are Zs	Some of the △ is in the □	I

The second of these contains all that is strictly necessary to the conclusion, and the first may be omitted. That which follows when an assertion can be made as to some, must follow when the same assertion can be made of all.

IV. How can a particular negative proposition be inferred; namely, that part, or all of one figure, is not contained in the other? It would seem at first sight, whenever we are able to assert that part or all of one figure is in the circle, and that part or all of the other figure is not. The weakest syllogism from which such an inference can be drawn would then seem to be as follows.

Some Xs are Ys	Some of the △ is in the ○
Some Zs are not Ys	Some of the □ is not in the ○
∴ Some Zs are not Xs	∴ Some of the △ is not in the □

But here it will appear, on a little consideration, that the conclusion is only thus far true; that those Xs which are Ys cannot be *those* Zs which are not Ys; but they may be *other* Zs, about which nothing is asserted when we say that *some* Zs are not Ys. And further consideration will make it evident, that a conclusion of this form can only be arrived at when one of the figures is entirely within the circle, and the whole, or part of the other without; or else when the whole of one of the figures is without the circle, and the whole or part of the other within; or lastly, when the circle lies entirely within one of the figures, and not entirely within the other. That is, the following are the distinct forms which allow of a particular negative conclusion, in which it should be remembered that a particular proposition in the premises may always be changed into a universal one, without affecting the conclusion. For that which necessarily follows from "some," follows from "all."

Every X is Y	All the △ is in the ○	A
Some Zs are not Ys	Some of the □ is not in the ○	O
∴ Some Zs are not Xs	Some of the □ is not in the △	O

[12]

No X is Y	None of the △ is in the ○	E
Some Zs are Ys	Some of the □ is in the ○	I
∴ Some Zs are not Xs	Some of the □ is not in the △	O

Every Y is X	All the ○ is in the △	A
Some Ys are not Zs	Some of the ○ is not in the □	O
∴ Some Xs are not Zs	Some of the △ is not in the □	O

It appears, then, that there are but six distinct syllogisms. All others are made from them by strengthening one of the premises, or converting one or both of the premises, where such conversion is allowable; or else by first making the conversion, and then strengthening one of the premises. And the following arrangement will show that two of them are universal, three of the others being derived from them by weakening one of the premises in a manner which does not destroy, but only weakens, the conclusion.

1. Every X is Y
Every Y is Z
Every X is Z

3. Every X is Y
No Z is Y
No X is Z

....

2. Some Xs are Ys
Every Y is Z
Some Xs are Zs

4. Some Xs are Ys
No Z is Y
Some Xs are not Zs

5. Every X is Y
Some Zs are not Ys
Some Zs are not Xs

6. Every Y is X
Some Ys are not Zs
Some Xs are not Zs

We may see how it arises that one of the partial syllogisms is not immediately derived, like the others, from a universal one. In the preceding, A E E may be considered as derived from A A A, by changing the term in which Y enters universally into a universal negative. If this be done with the other term instead, we have

No X is Y. } from which universal premises we cannot deduce a
Every Y is Z } universal conclusion, but only some Zs are not Xs.

If we weaken one and the other of these premises, as they stand, we obtain

Some Xs are not Ys		No X is Y
Every Y is Z	and	Some Ys are Zs
No conclusion		Some Zs are not Xs

[13] equivalent to the fourth of the preceding: but if we convert the first premise, and proceed in the same manner,

From	No Y is X	we obtain	Some Ys are not Xs
	Every Y is Z		Every Y is Z
	Some Zs are not Xs		Some Zs are not Xs

which is legitimate, and is the same as the last of the preceding list, with X and Z interchanged.

Before proceeding to show that all the usual forms are contained in the preceding, let the reader remark the following rules, which may be proved either by collecting them from the preceding cases, or by independent reasoning.

1. The middle term must enter universally into one or the other premise. If it were not so, then one premise might speak of one part of the middle term, and the other of another; so that there would, in fact, be no middle term. Thus, "Every X is Y, Every Z is Y," gives no conclusion: it may be thus stated;

All the Xs make up *a part* of the Ys
All the Zs make up *a part* of the Ys

And, before we can know that there is any common term of comparison at all, we must have some means of showing that the two parts are to some extent the same; or the preceding premises by themselves are inconclusive.

2. No term must enter the conclusion more generally than it is found in the premises; thus, if X be spoken of partially in the premises, it must enter partially into the conclusion. This is obvious, since the conclusion must assert no more than the premises imply.

3. From premises both negative no conclusion can be drawn. For it is obvious, that the mere assertion of disagreement between each of two things and a third, can be

no reason for inferring either agreement or disagreement between these two things. It will not be difficult to reduce any case which falls under this rule to a breach of the first rule: thus, No X is Y, No Z is Y, gives

Every X is (something which is not Y)
Every Z is (something which is not Y)

[14] in which the middle term is not spoken of universally in either. Again, "No Y is X, some Ys are not Zs," may be converted into

Every X is (a thing which is not Y)
Some (things which are not Zs) are Ys

in which there is no middle term.

4. From premises both particular no conclusion can be drawn. This is sufficiently obvious when the first or second rule is broken, as in "Some Xs are Ys, Some Zs are Ys." But it is not immediately obvious when the middle term enters one of the premises universally. The following reasoning will serve for exercise in the preceding results. Since both premises are particular in form, the middle term can only enter one of them universally by being the predicate of a negative proposition; consequently (Rule 3) the other premise must be affirmative, and, being particular, neither of its terms is universal. Consequently both the terms as to which the conclusion is to be drawn enter partially, and the conclusion (Rule 2) can only be a particular *affirmative* proposition. But if one of the premises be negative, the conclusion must be *negative* (as we shall immediately see). This contradiction shows that the supposition of particular premises producing a legitimate result is inadmissible.

5. If one premise be negative, the conclusion, if any, must be negative. If one term agree with a second and disagree with a third, no agreement can be inferred between the second and third.

6. If one premise be particular, the conclusion must be particular. This may be shown as follows. If two pro-

positions P and Q, together prove a third, R, it is plain that P and the denial of R, prove the denial of Q. For P and Q cannot be true together without R. Now if possible, let P (a particular) and Q (a universal) prove R (a universal). Then P (particular) and the denial of R (particular) prove the denial of Q. But two particulars can prove nothing.

In the preceding set of syllogisms we observe one form only which produces A, or E, or I, but three which produce O.

Let an assertion be said to be weakened when it is reduced from universal to particular, and strengthened in the contrary case. Thus, "Every X is Z" is called stronger than "Some Xs are Zs."

[15] Every usual form of syllogism which can give a legitimate result is either one of the preceding six, or another formed from one of the six, either by changing one of the assertions into its converse, if that be allowable, or by strengthening one of the premises, without altering the conclusion, or both. Thus,

$$\left.\begin{array}{l}\text{Some Xs are Ys}\\ \text{Every Y is Z}\end{array}\right\}\text{ may be written }\left\{\begin{array}{l}\text{Some Ys are Xs}\\ \text{Every Y is Z}\end{array}\right.$$

$$\text{What follows will still follow from }\left\{\begin{array}{l}\textit{Every}\text{ Y is X}\\ \text{Every Y is Z}\end{array}\right.$$

for all which is true when "Some Ys are Xs," is not less true when "Every Y is X."

It would be possible also to form a legitimate syllogism by weakening the conclusion, when it is universal, since that which is true of all is true of some. Thus, "Every X is Y, Every Y is Z," which yields "Every X is Z," also yields "Some Xs are Zs." But writers on logic have always considered these syllogisms as useless, conceiving it better to draw from any premises their strongest conclusion. In this they were undoubtedly right; and the only question is, whether it would not have been advisable to make the premises as weak as possible, and not to admit any syllogisms in which more appeared than was absolutely necessary to the conclusion. If such had been the practice, then

Every Y is X, Every Y is Z, therefore Some Xs are Zs

would have been considered as formed by a spurious and unnecessary excess of assertion. The minimum of assertion would be contained in either of the following,

Every Y is X, Some Ys are Zs, therefore Some Xs are Zs
Some Ys are Xs, Every Y is Z, therefore Some Xs are Zs

In this chapter, syllogisms have been divided into two classes: first, those which prove a universal conclusion; secondly, those which prove a partial conclusion, and which are (all but one) derived from the first by weakening one of the premises, in such manner as to produce a legitimate but weakened conclusion. Those of the first class are placed in the first column, and of the other in the second.

[16]	Universal.		Particular.	
	A Every X is Y		Some Xs are Ys	I
	A Every Y is Z	———	Every Y is Z	A
	A Every X is Z		Some Xs are Zs	I
			Some Xs are Ys	I
			No Y is Z	E
	A Every X is Y		Some Xs are not Zs	O
	E No Y is Z	———	Every X is Y	A
	E No X is Z		Some Zs are not Ys	O
			Some Zs are not Xs	O
			Every Y is X	A
	....		Some Ys are not Zs	O
			Some Xs are not Zs	O

In all works on logic, it is customary to write that premise first which contains the predicate of the conclusion. Thus,

Every Y is Z		Every X is Y
Every X is Y	would be written, and not	Every Y is Z
Every X is Z		Every X is Z

The premises thus arranged are called major and minor; the predicate of the conclusion being called the major term, and its subject the minor. Again, in the preceding case we see the various subjects coming in the order Y, Z; X, Y; X, Z: and the number of different orders which can appear is four, namely—

$$\begin{array}{ccccccc} \text{Y Z} & & \text{Z Y} & & \text{Y Z} & & \text{Z Y} \\ \text{X Y} & & \text{X Y} & & \text{Y X} & & \text{Y X} \\ \cline{1-1} \cline{3-3} \cline{5-5} \cline{7-7} \text{X Z} & & \text{X Z} & & \text{X Z} & & \text{X Z} \end{array}$$

which are called the four *figures*, and every kind of syllogism in each figure is called a *mood*. I now put down the various moods of each figure, the letters of which will be a guide to find out those of the preceding list from which they are derived. Co means that a premise of the preceding list has been converted; + that it has been strengthened; Co +, that both changes have taken place. Thus,

$$\begin{array}{llclll} [17] & \text{A} & \text{Every Y is Z} & & \text{A} & \text{Every Y is Z} \\ & \text{I} & \text{Some Xs are Ys} & \text{becomes} & \text{A} & \text{Every Y is X : (Co+)} \\ \cline{3-3} \cline{6-6} & \text{I} & \text{Some Xs are Zs} & & \text{I} & \text{Some Xs are Zs} \end{array}$$

And Co + points out the following: If some Xs be Ys, then some Ys are Xs (Co); and all that is true when Some Ys are Xs, is true when Every Y is X (+); therefore the second syllogism is legitimate, if the first be so.

First Figure.

$$\begin{array}{llcll} \text{A} & \text{Every Y is Z} & & \text{A} & \text{Every Y is Z} \\ \text{A} & \text{Every X is Y} & & \text{I} & \text{Some Xs are Ys} \\ \cline{2-2} \cline{5-5} \text{A} & \text{Every X is Z} & & \text{I} & \text{Some Xs are Zs} \end{array}$$

$$\begin{array}{llcll} \text{E} & \text{No Y is Z} & & \text{E} & \text{No Y is Z} \\ \text{A} & \text{Every X is Y} & & \text{I} & \text{Some Xs are Ys} \\ \cline{2-2} \cline{5-5} \text{E} & \text{No X is Z} & & \text{O} & \text{Some Xs are not Zs} \end{array}$$

Second Figure.

E No Z is Y (Co)
A Every X is Y
E No X is Z

A Every Z is Y
E No X is Y (Co)
E No X is Z

E No Z is Y (Co)
I Some Xs are Ys
O Some Xs are not Zs

A Every Z is Y
O Some Xs are not Ys
O Some Xs are not Zs

Third Figure.

A Every Y is Z
A Every Y is X (Co+)
I Some Xs are Zs

I Some Ys are Zs (Co)
A Every Y is X
I Some Xs are Zs

A Every Y is Z
I Some Ys are Xs (Co)
I Some Xs are Zs

E No Y is Z
A Every Y is X (Co+)
O Some Xs are not Zs

O Some Ys are not Zs
A Every Y is X
O Some Xs are not Zs

E No Y is Z
I Some Ys are Xs (Co)
O Some Xs are not Zs

[18]

Fourth Figure.

A Every Z is Y (+)
A Every Y is X
I Some Xs are Zs

A Every Z is Y
E No Y is X
E No X is Z

I Some Zs are Ys
A Every Y is X
I Some Zs are Xs

E No Z is Y (Co)
A Every Y is X (Co+)
O Some Xs are not Zs

E No Z is Y (Co)
I Some Ys are Xs (Co)
O Some Xs are not Zs

The above is the ancient method of dividing syllogisms; but, for the present purpose, it will be sufficient to consider the six from which the rest can be obtained. And since some of the six have X in the predicate of the conclusion, and not Z, I shall join to them the six other syllogisms which are found by transposing Z and X. The complete list, therefore, of syllogisms with the weakest premises and the strongest conclusions, in which a comparison of X and Z is obtained by comparison of both with Y, is as follows:

Every X is Y	Every Z is Y	Some Xs are Ys	Some Zs are Ys
Every Y is Z	Every Y is X	No Y is Z	No Y is X
Every X is Z	Every Z is X	Some Xs are not Zs	Some Zs are not Xs
Every X is Y	Every Z is Y	Every X is Y	Every Z is Y
No Y is Z	No Y is X	Some Zs are not Ys	Some Xs are not Ys
No X is Z	No Z is X	Some Zs are not Xs	Some Xs are not Zs
Some Xs are Ys	Some Zs are Ys	Every Y is X	Every Y is Z
Every Y is Z	Every Y is X	Some Ys are not Zs	Some Ys are not Xs
Some Xs are Zs	Some Zs are Xs	Some Xs are not Zs	Some Zs are not Xs

In the list of page 12,[5] there was nothing but recapitulation of forms, each form admitting a variation by interchanging X and Z. This interchange having been made, and the results collected as above, if we take every case in which Z is the predicate, or can be made the predicate by allowable conversion, we [19] have a collection of all possible *weakest* forms in which the result is one of the four "Every X is Z," "No X is Z," "Some Xs are Zs," "Some Xs are not Zs;" as follows. The premises are written in what appeared the most natural order, without distinction of major or minor.

Every X is Y
Every Y is Z
Every X is Z

Some Xs are Ys	Some Zs are Ys
Every Y is Z	Every Y is X
Some Xs are Zs	Some Xs are Zs

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[5] [Here, as wherever De Morgan refers to the pages of *Formal Logic*, the figures give the page of the original edition.—Ed.]

Every X is Y No Z is Y No X is Z	Every Z is Y No X is Y No X is Z

Some Xs are Ys No Z is Y Some Xs are not Zs	Every Z is Y Some Xs are not Ys Some Xs are not Zs	Every Y is X Some Ys are not Zs Some Xs are not Zs

Every assertion which can be made upon two things by comparison with any third, that is, every simple inference, can be reduced to one of the preceding forms. Generally speaking, one of the premises is omitted, as obvious from the conclusion; that is, one premise being named and the conclusion, that premise is implied which is necessary to make the conclusion good. Thus, if I say, "That race must have possessed some of the arts of life, for they came from Asia," it is obviously meant to be asserted, that all races coming from Asia must have possessed some of the arts of life. The preceding is then a syllogism, as follows:

That race is "a race of Asiatic origin:"
Every "race of Asiatic origin" is "a race which must have possessed some of the arts of life:"
Therefore, That race *is* a race which must have possessed some of the arts of life.

A person who makes the preceding assertion either means to imply, antecedently to the conclusion, that all Asiatic races must have possessed arts, or he talks nonsense if he assert the conclu-**[20]**sion positively. "X must be Z, for it is Y," can only be an inference when "Every Y is Z." This latter proposition may be called the suppressed premise; and it is in such suppressed propositions that the greatest danger of error lies. It is also in such propositions that men convey opinions which they would not willingly express. Thus, the honest witness [6] who said, "I always thought him a respectable man—he kept his gig," would probably not have admitted in direct terms, "Every man who keeps a gig must be respectable."

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[6] [The allusion is to the answer of a witness in the trial of John Thurtell (executed at Hertford, January 9th, 1824), for the murder of William Weare. The same incident is responsible for Carlyle's endless jests about "Gigmanity."—Ed.]

I shall now give a few detached illustrations of what precedes.

"His imbecility of character might have been inferred from his proneness to favourites; for all weak princes have this failing." The preceding would stand very well in a history, and many would pass it over as containing very good inference. Written, however, in the form of a syllogism, it is,

All weak princes are prone to favourites
He was prone to favourites
Therefore He was a weak prince

which is palpably wrong. (Rule 1.) The writer of such a sentence as the preceding might have meant to say, "for all who have this failing are weak princes;" in which case he would have inferred rightly. Every one should be aware that there is much false form of inference arising out of badness of style, which is just as injurious to the habits of the untrained reader as if the errors were mistakes of logic in the mind of the writer.

"X is less than Y; Y is less than Z: therefore X is less than Z." This, at first sight, appears to be a syllogism; but, on reducing it to the usual form, we find it to be,

X is (a magnitude less than Y)
Y is (a magnitude less than Z)
Therefore X is (a magnitude less than Z)

which is not a syllogism, since there is no middle term. Evident as the preceding is, the following additional proposition must be formed before it can be made explicitly logical. "If Y be a magnitude less than Z, then every magnitude less than Y is also less than Z." There is, then, before the preceding can be reduced to a syllogistic form, the necessity of a deduction from the second [21] premise, and the substitution of the result instead of that premise. Thus,

X is less than Y
Less than Y is less than Z : following from Y is less than Z.
Therefore X is less than Z

But, if the additional argument be examined—namely, if Y be less than Z, then that which is less than Y is less than Z—it will be found to require precisely the same considerations repeated; for the original inference was nothing more. In fact, it may easily be seen as follows, that the proposition before us involves more than any simple syllogism can express. When we say that X is less than Y, we say that if X were applied to Y, every part of X would match a part of Y, and there would be parts of Y remaining over. But when we say, "Every X is Y," meaning the premise of a common syllogism, we say that every instance of X is an instance of Y, without saying any thing as to whether there are or are not instances of Y still left, after those which are also X are taken away. If, then, we wish to write an ordinary syllogism in a manner which shall correspond with "X is less than Y, Y is less than Z, therefore X is less than Z," we must introduce a more definite amount of assertion than was made in the preceding forms. Thus,

Every X is Y, and there are Ys which are not Xs
Every Y is Z, and there are Zs which are not Ys
Therefore Every X is Z, and there are Zs which are not Xs

Or thus:

The Ys contain all the Xs, and more
The Zs contain all the Ys, and more
The Zs contain all the Xs, and more

The most technical form, however, is,

From Every X is Y; [Some Ys are not Xs]
Every Y is Z; [Some Zs are not Ys]
Follows Every X is Z; [Some Zs are not Xs]

This sort of argument is called *à fortiori* argument, because the premises are more than sufficient to prove the conclusions and the extent of the conclusion is thereby greater than its mere form would indicate. Thus, "X is less than Y, Y is less than Z, **[22]** therefore, *à fortiori*, X is less than Z," mean, that the extent to which X is less than Z must be greater than that to which X is less than Y, or Y than Z. In the syllogism last written, either of the bracketted premises might be struck out without destroying the conclusion; which last would, however, be weakened. As it stands, then, the part of the conclusion, "Some Zs are not Xs," follows *à fortiori*.

The argument *à fortiori* may then be defined as a universally affirmative syllogism, in which both of the premises are shewn to be less than the whole truth, or greater. Thus, in "Every X is Y, Every Y is Z, therefore Every X is Z," we do not certainly imply that there are more Ys than Xs, or more Zs than Ys, so that we do not know that there are more Zs than Xs. But if we be at liberty to state the syllogism as follows,

All the Xs make up part (and part only) of the Ys
Every Y is Z;

then we are certain that

All the Xs make up part (and part only) of the Zs.

But if we be at liberty further to say that

All the Xs make up part (and part only) of the Ys
All the Ys make up part (and part only) of the Zs

then we conclude that

All the Xs make up *part of part* (only) of the Zs

and the words in Italics mark that quality of the conclusion from which the argument is called *à fortiori*.

Most syllogisms which give an affirmative conclusion are generally meant to imply *à fortiori* arguments, except only in mathematics. It is seldom, except in the exact sciences, that we meet with a proposition, "Every X is Z," which we cannot immediately couple with "some Zs are not Xs."

When an argument is completely established, with the exception of one assertion only, so that the inference may be drawn as soon as that one assertion is established, the result is stated in a form which bears the name of an *hypothetical* syllogism. The word hypothesis means nothing but supposition; and the species of syllogism just mentioned first lays down the assertion that a consequence will be true if a certain condition be fulfilled, and **[23]** then either asserts the fulfilment of the condition, and thence the consequence, or else denies the consequence, and thence denies the fulfilment of the condition. Thus, if we know that

When X is Z, it follows that P is Q;

then, as soon as we can ascertain that X is Z, we can conclude that P is Q; or, if we can shew that P is not Q, we know that X is not Z. But if we find that X is not Z, we can infer nothing; for the preceding does not assert that P is Q *only* when X is Z. And if we find out that P is Q we can infer nothing. This conditional syllogism may be converted into an ordinary syllogism, as follows. Let K be any "case in which X is Z," and V, a "case in which P is Q;" then the preceding assertion amounts to "Every K is V." Let L be a particular instance, the X of which may or may not be Z. If X be Z in the instance under discussion, or if X be not Z, we have, in the one case and the other,

Every K is V	Every K is V
L is a K	L is not a K
Therefore L is a V	No conclusion

Similarly, according as a particular case (M) is or is not V, we have

Every K is V	Every K is V
M is a V	M is not a V
No conclusion	M is not a K

That is to say: the assertion of an hypothesis is the assertion of its necessary consequence, and the denial of the necessary consequence is the denial of the hypothesis: but the assertion of the necessary consequence gives no right to assert the hypothesis, nor does the denial of the hypothesis give any right to deny the truth of that which would (were the hypothesis true) be its necessary consequence.

Demonstration is of two kinds: which arises from this, that every proposition has a contradictory; and of these two, one must be true and the other must be false. We may then either prove a proposition to be true, or its contradictory to be false. "It is true that every X is Z," and "it is false that there are some Xs which are not Zs," are the same proposition; and the proof of either is called the indirect proof of the other.

[24] But how is any proposition to be proved false, except by proving a contradiction to be true? By proving a necessary consequence of the proposition to be false. But this is not a complete answer, since it involves the necessity of doing the same thing; or, so far as this answer goes, one proposition cannot be proved false unless by proving another to be false. But it may happen, that a necessary consequence can be obtained which is obviously and self-evidently false, in which case no further proof of the falsehood of the hypothesis is necessary. Thus the proof which Euclid gives that all equiangular[7] triangles are equilateral is of the following structure, logically considered.

(1.) If there be an equiangular triangle not equilateral, it follows that a whole can be found which is not greater than its part.*

(2.) It is false that there can be any whole which is not greater than its part (self evident).

(3.) Therefore it is false that there is any equiangular

* This is the proposition in proof of which nearly the whole of the demonstration of Euclid is spent.

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[7] [De Morgan's reference is to the demonstration of Euclid I. 6.—Ed.]

triangle which is not equilateral; or all equiangular triangles are equilateral.

When a proposition is established by proving the truth of the matters it contains, the demonstration is called *direct;* when by proving the falsehood of every contradictory proposition, it is called *indirect.* The latter species of demonstration is as logical as the former, but not of so simple a kind; whence it is desirable to use the former whenever it can be obtained.

The use of indirect demonstration in the Elements of Euclid is almost entirely confined to those propositions in which the converses of simple propositions are proved. It frequently happens that an established assertion of the form

Every X is Z..................(1)

may be easily made the means of deducing,

Every (thing not X) is not Z..(2)

which last gives

Every Z is X..................(3)

[25] The conversion of the second proposition into the third is usually made by an indirect demonstration, in the following manner: If possible, let there be one Z, which is not X, (2) being true. Then there is one thing which is not X and is Z; but every thing not X is not Z; therefore there is one thing which is Z and is not Z: which is absurd. It is then absurd that there should be one single Z which is not X; or, Every Z is X.

The following proposition contains a method which is of frequent use.

HYPOTHESIS.—Let there be any number of propositions or assertions,—three for instance, X, Y, and Z,—of which it is the property that one or the other must be true, *and one only.* Let there be three other propositions, P, Q, and R, of which it is also the property that one, and one only, must be true. Let it also be a connexion of those assertions, that

When X is true, P is true
When Y is true, Q is true
When Z is true, R is true

CONSEQUENCE: then it follows that

When P is true, X is true
When Q is true, Y is true
When R is true, Z is true

For, when P is true, then Q and R must be false; consequently, neither Y nor Z can be true, for then Q or R would be true. But either X, Y, or Z must be true, therefore X must be true; or, when P is true, X is true. In a similar way the remaining assertions may be proved.

Case 1. If When P is Q, X is Z
When P is not Q, X is not Z
It follows that When X is Z, P is Q
When X is not Z, P is not Q

Case 2. If
When X is greater than Z, P is greater than Q
When X is equal to Z, P is equal to Q
When X is less than Z, P is less than Q

It follows that
When P is greater than Q, X is greater than Z
When P is equal to Q, X is equal to Z
When P is less than Q, X is less than Z

[26]

CHAPTER II.

ON OBJECTS, IDEAS, AND NAMES.

Logic is derived from a Greek word (λόγος) which signifies communication of thought, usually by speech. It is the name which is generally given to the branch of inquiry (be it called science or art), in which the act of the mind in reasoning is considered, particularly with reference to the connection of thought and language. But no definition yet given in few words has been found satisfactory to any considerable number of thinking persons.

All existing things upon this earth, which have knowledge of their own existence, possess, some in one degree and some in another, the power of *thought*, accompanied by *perception*, which is the awakening of thought by the effect of external objects upon the senses. By thought I here mean, all mental action, not only that comparatively high state of it which is peculiar to man, but also that lower degree of the same thing which appears to be possessed by brutes.

With respect to the mind, considered as a complicated apparatus which is to be studied, we are not even so well off as those would be who had to examine and decide upon the mechanism of a watch, merely by observation of the functions of the hands, without being allowed to see the inside. A mechanician, to whom a watch was presented for the first time, would be able to give a good guess as to its structure, from his knowledge of other pieces of contrivance. As soon as he had examined the law of the motion of the hands, he might conceivably invent an instrument with similar properties, in fifty different ways. But in the case of the mind, we have manifestations only, without the smallest power of reference to other similar things, or the least knowledge of structure or process, other than what may be derived from those manifestations. It is the problem of the watch to those who have never seen any mechanism at all.

[27] We have nothing more to do with the science of mind, usually called *metaphysics*,* than to draw a very few necessary distinctions, which, whatever names we use to denote them, are matters of fact connected with our subject. Some modes of expressing them favour one system of metaphysics, and some another; but still they are matters of observed fact. Our words must be very imperfect symbols, drawn from comparison of the manifestations of thought with those of things in corporeal existence. For instance, I just now spoke of the mind as an apparatus, or piece of mechanism. It is a structure of some sort, which has the means of fulfilling various purposes: and so far it resembles the hand, which by the disposition of bone and muscle, can be made to perform an immense variety of different motions and grasps. Where the resemblance begins to be imperfect, and why, is what we cannot know. In all probability we should need new modes of perception, other senses besides sight, hearing, and touch, in order to know thought as we know colour, size, or motion. But the

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* All systems make an assumption of the uniformity of process in all minds, carried to an extent the propriety of which ought to be a matter of special discussion. There are no writers who give us so much *must* with so little *why*, as the metaphysicians. If persons who had only seen the outside of the timepiece, were to invent machines to answer its purpose, they might arrive at their object in very different ways. One might use the pendulum and weight, another the springs and the balance: one might discover the combination of toothed wheels, another a more complicated action of lever upon lever. Are we *sure* that there are not differences in our minds, such as the preceding instance may suggest by analogy; if so, *how* are we sure? Again, if our minds be as tables with many legs, do we know that a weight put upon different tables will be supported in the same manner in all. May not the same leg support much or all of a certain weight in one mind, and little or nothing in another? I have seen striking instances of something like this, among those who have examined for themselves the grounds of the mathematical sciences.

I would not dissuade a student from metaphysical inquiry; on the contrary, I would rather endeavour to promote the desire of entering upon such subjects: but I would warn him, when he tries to look down his own throat with a candle in his hand, to take care that he does not set his head on fire.

purpose of the present treatise is only the examination of some of the manifestations of thinking power in their relation to the language in which they are expressed. Knowledge of thought and knowledge of the results of thought, **[28]** are very different things. The watch abovementioned might have the functions of its hands discovered, might be used in finding longitude (and even latitude) all over the world, without the parties using it having the smallest idea of its interior structure.

That our minds, souls, or thinking powers (use what name we may) exist, is the thing of all others of which we are most certain, each for himself. Next to this, nothing can be more certain to us, each for himself, than that other things also exist; other minds, our own bodies, the whole world of matter. But between the character of these two certainties there is a vast difference. Any one who should deny his own existence would, if serious, be held beneath argument: he does not know the meaning of his words, or he is false or mad. But if the same man should deny that any thing exists except himself, that is, if he should affirm the whole creation to be a dream of his own mind, he would be absolutely unanswerable. If I (who *know* he is wrong, for *I* am certain of *my own* existence) argue with him, and reduce him to silence, it is no more than might * happen in his dream. A celebrated metaphysician, Berkeley, maintained that with regard to *matter*, the above is the state of the case: that our impressions of matter are only impressions, communicated by the Creator without any intervening cause of communication.

Our most convincing communicable proof of the existence of other things, is, not the appearance of objects, but the necessity of admitting that there are *other minds* besides our own. The external inanimate objects might be creations of our own thought, or thinking and perceptive function: they are so sometimes, as in the case of insanity, in which the mind has frequently the appearance of making the whole or part of its own external world. But when we see other beings, performing similar functions to those which we ourselves perform, we come so irresistibly to the conclusion that there must be other sentients like ourselves, that we should rather compare a person who doubted it to one who denied his own existence, than to one who simply denied the real external existence of the *material* world.

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* It is not impossible that in a real dream of sleep, some one may have created an antagonist who beat him in an argument to prove that he was awake.

[29] When once we have admitted different and independent minds, the reality of external objects (external to all those minds) follows as of course. For different minds receive impressions at the same time, which their power of communication enables them to know are similar, so far as any impressions, one in each of two different minds, can be known to be similar. There must be a *somewhat* independent of those minds, which thus acts upon them all at once, and without any choice of their own. This *somewhat* is what we call an external object: and whether it arise in Berkeley's mode, or in any other, matters nothing to us here.

We shall then, take it for granted that external *objects* actually exist, independently of the mind which perceives them. And this brings us to an important distinction, which we must carry with us throughout the whole of this work. Besides the actual external object, there is also the mind which perceives it, and what (for want of better words or rather for want of knowing whether they be good words or not) we must call the *image of that object in the mind*, or the *idea* which it communicates. The term *subject* is applied by metaphysicians to the perceiving mind: and thus it is said that a thing may be considered *subjectively* (with reference to what it is in the mind) or *objectively* (with reference to what it is independently of any particular mind). But logicians use the word subject in another sense. In a proposition such as "bread is wholesome," the thing spoken of, "bread", is called the subject of the proposition: and in fact the word *subject* is in common language so frequently confounded with *object*, that it is almost hopeless to speak clearly to beginners about themselves as *subjects*. I shall therefore adopt the words *ideal* and *objective*, *idea* and *object*, as being, under explanations, as good as any others: and better than *subject* and *object* for a work on logic.

The word *idea*, as here used, does not enter in that vague sense in which it is generally used, as if it were an opinion that might be right or wrong. It is that which the object gives to the mind, or the state of the mind produced by the object. Thus the idea of a horse is *the horse in the mind*: and we know no other horse. We admit that there is an external *object*, a horse, which may give a *horse in the mind* to twenty different persons: but no one of these twenty knows the object; each one only knows his *idea*. **[30]** There is an object, because each of the twenty persons receives an idea without communicating with the others: so that there is something external to give it them. But when they talk about it, under the name of a horse, they talk about their ideas. They all refer to the object, as being the thing they are talking about, until the moment they begin to differ: and then they begin to speak, not of external horses, but of impressions on their minds; at least this is the case with those who know what knowledge is; the positive and the unthinking part of them still talk of the *horse*. And the latter have a great advantage * over the former with those who are like themselves.

Why then do we introduce the term *object* at all, since all our knowledge lies in ideas? For the same reason as we introduce the term *matter* into natural philosophy, when all we know is form, size, colour, weight, &c., no one of which is matter, nor even all together. It is convenient to have a word for that external source from which *sensible* ideas are produced: and it is just as convenient to have a word for the external source, material or not, from which *any* idea is produced. Again, why do we speak of our power of considering things either ideally or objectively, when as we can know nothing but ideas, we can have no right to speak of any thing else? The answer is that, just as in other things, when we speak of an object, we speak of the *idea of an object.* We learn to speak of the external world, because there are others like ourselves who evidently draw ideas from the same sources as ourselves: hence we come to have the idea of those sources, the idea of external objects, as we call them. But we do not know those sources; we know only our ideas of them.

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* One man asserts *a fact* on his own knowledge, another asserts his full *conviction* of the contrary fact. Both use the evidence of their senses: but the second knows that *full conviction* is all that man can have. The first will carry it hollow in a court of justice, in which persons are constantly compelled to swear, not only that they have an impression, but that the impression is correct; that is to say, is the impression which mankind in general would have, and must have, and ought to have.

We can even use the terms ideal and objective in what may appear a metaphorical sense. When we speak of ourselves in the manner of this chapter, we put ourselves, as it were, in the position of spectators of our own minds: speak and think of our [31] own minds objectively. And it must be remembered that by the word object, we do not mean *material* object only. The mind of another, any one of its thoughts or feelings, any relation of minds to one another, a treaty of peace, a battle, a discussion upon a controverted question, the right of conveying a freehold, —are all objects, independently of the persons or things engaged in them. They are things external to our minds, of which we have ideas.

An object communicates an idea: but it does not follow that every idea is communicated by an object. The mind can create ideas in various ways; or at least can derive, by combinations which are not found in external existence, new collections of ideas. We have a perfectly distinct idea of a unicorn, or a flying dragon: when we say there are no such things, we speak objectively only: ideally they have as much existence as a horse or a sheep; to a herald, more. Add to this, that the mind can separate ideas into parts, in such manner that the parts alone are not ideas of any existing separate material objects, any more than the letters of a word are constituent parts of the meaning of the whole. Hence we get what are called *qualities* and *relations*. A ball may be hard and round, or may have hardness and roundness: but we can not say that hardness and roundness are separate external material objects, though they are objects the ideas of which necessarily accompany our perception of certain objects. These ideas are called *abstract* as being removed or abstracted from the complex idea which gives them: the abstraction is made by comparison or observation of resemblances. If a person had never seen any thing round except an apple, he would perhaps never think of roundness as a distinct object of thought. When he saw another round body, which was evidently not an apple, he would immediately, by perception of the resemblance, acquire a separate idea of the thing in which they resemble one another.

Abstraction is not performed upon the ideas of material objects only. For instance, from conduct of one kind, running through a number of actions, performed by a number of persons, we get the ideas of goodness, wickedness, talent, courage. But we must not imagine that we can make ideally external representation of these words. They are *objects*, that is to say, the mind considers them as external to itself: but they are not material objects. [32] Some people deny their existence, and look upon them as only abstract words, or words under which we speak of minds or bodies without specifying any more than one of the ideas produced by these minds or bodies. For instance, they assert that when we say "knowledge gives power" it is really that persons with knowledge are therefore able, or have power, to produce, or to do, what persons without it cannot. This is a question which it does not concern me here to discuss.

Seeing that the mind possesses a power of originating new combinations of ideas, and also of abstracting from complex ideas the more simple ones of which, it seems natural to say, they are composed, it has long been a question among metaphysicians whether the mind has any ideas of its own which it possesses independently of all suggestion from external objects. It is not necessary that I should attempt to lead the student to any conclusion * on this subject: for our purpose, the distinction between ideas and objects, though it were false, is of more importance than that between innate and acquired ideas, though it be true. But one of these two things must be true: either we have ideas which we do *not* acquire from or by means of communication with the external world (experience, trial of our senses) or there is a power in the mind of acquiring a certainty and a generality which experience alone could not properly give. For instance, we are satisfied as of our own existence that seven and three collected are the same as five and five, *whatever the objects may be* [33] *which are counted:* the thing is true of fingers, pebbles, counters, sheep, trees, &c. &c. &c. We cannot have assured ourselves of this by experience: for example, we *know* it to be true of pebbles at the North Pole, though we have never been there; we are as sure of it as of our own existence. I do not mean that we have a rational conviction only, fit to act upon, that it is so at the North Pole, because it is so in every place in which it has been tried: if we had nothing else, we should have this; but we feel that this lesser conviction is swallowed up by a greater. We have the lesser conviction that the pebbles at the pole fall to the ground when they are let go: we are very sure of this, without asserting that it cannot be otherwise: we see no *impossibility* in those pebbles being such as always to remain in air wherever they are placed.* But that seven and three are no other than five and five is a matter which we are prepared to affirm as positively of the pebbles at the North Pole as of our own fingers, both that it is so, and that it *must* be so. Whence arises this actual difference in point of fact, between our mode of viewing and knowing [34] different species of assertions? the truth of the last named assertion is not born with us, for children are without it, and learn it by experience, as we know. The *must be so* cannot be acquired from experience in the common way, for that same experience on which we rely tells us that however often a thing may have been found true, whatever rule may have been established by repeated instances, an exception may at last occur. There seems then to be in the mind a power of developing, from the ideas which experience gives, a real and true distinction of necessary and not necessary, possible and impossible. The things which are without us always confirm our *necessary* propositions: but how we derive that complete assurance that they *will* do so as faithfully as hitherto they *have* done so, is not within our power to say.

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* It has always appeared to me much such a question as the following. There are hooks which certainly catch fish if put into the water; and most certainly they have been put into the water. There are then fish upon them. But these fish might have been on some of them when they were put into the water. It is to no purpose to inquire whether it was so or not, unless there be some distinction between the fishes which may make it a question whether some of them could have been bred in the river into which the hooks were put. The mind has certainly a power of acquiring and retaining ideas, which power, when put into communication with the external world, it must exercise. There is no mind to experiment upon, except those which have had such communication. Are there found any ideas which we have reason to think *could* not have been acquired by this communication? any fishes which *could* not have come out of the river? Metaphysicians seem to admit that if any ideas be innate, they are those of space, time, and of cause and effect: they seem also to admit, that if there be any ideas, which, not being innate, are sure to be acquired, they are these very ones.

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* Metaphysicians, in their systems, have often taken this distinction to be one of system only, treating it as a thing to be accepted or rejected with the system, instead of an actual and indisputable phenomenon which requires explanation under any system. Dr. Whewell, of all English writers on natural science I know, is the one who has made the fact, as a fact, pervade his writings, sometimes attached to a system, sometimes not. The following remarks on the general subject are worth consideration: "It is indeed, extremely difficult to find, in speaking of this subject, expressions which are satisfactory. The reality of the objects which we perceive is a profound, apparently an insoluble problem. We cannot but suppose that existence is something different from our knowledge of existence:—that what exists, does not exist merely in our knowing that it does:—truth is truth whether we know it or not. Yet how can we conceive truth, otherwise than as something known? How can we conceive things as existing, without conceiving them as objects of perception? Ideas and Things are constantly opposed, yet necessarily coexistent. How they are thus opposite and yet identical, is the ultimate problem of all philosophy. The successive phases of philosophy have consisted in separating and again uniting these two opposite elements; in dwelling sometimes upon the one and sometimes upon the other, as the principal or original or only element; and then in discovering that such an account of the state of the case was insufficient. Knowledge requires ideas. Reality requires things. Ideas and things coexist. Truth *is*, and is known. But the complete explanation of these points appears to be beyond our reach."

Connected with ideas are the *names* we give them; the spoken or written sounds by which we think of them, and communicate with others about them. To have an idea, and to make it the subject of thought as an idea, are two perfectly distinct things: the *idea of an idea* is not the idea itself. I doubt whether we could have made thought itself the subject of thought without language. As it is, we give names to our ideas, meaning by a name not merely a single word, but any collection of words which conveys to one mind the idea in another. Thus a-man-in-a-black-coat-riding-along-the-high-road-on-a-bay-horse is as much the name of an idea as man, black, or horse. We can coin words at pleasure; and, were it worth while, might invent a single word to stand for the preceding phrase.

Names are used indifferently, both for the objects which produce ideas, and for the ideas produced by them. This is a disadvantage, and it will frequently be necessary to specify whether we speak ideally or objectively. In common conversation we speak ideally and think we speak objectively: we take for granted that our own ideas are fit to pass to others, and will convey to them the same ideas as the objects themselves would have done. That this may be the case, it is necessary first, that the object should really give us the same ideas as to others; secondly, that our words should carry from us to our correspondents the same ideas as those which we intended to express by them. How, and in what cases, the first or the second condition is not ful-[35]filled, it is impossible to know or to enumerate. But we have nothing to do here except to observe that we are only incidentally concerned with this question in a work of logic. We presume fixed and, if objective, objectively true ideas, with certain names attached: so that it is never in doubt whether a name be or be not properly attached to any idea. This method must be followed in all works of science: a conceivably attainable end is first presumed to be attained, and the consequences of its attainment are studied. Then, afterwards, comes the question whether this end is always attained, and if not, why. The way to mend bad roads must come at the end, not at the beginning, of a treatise on the art of making good ones.

Every name has a reference to every idea, either affirmative or negative. The term *horse* applies to every thing, either positively or negatively. This (no matter what I am speaking of) either *is* or *is not* a horse. If there be any doubt about it, either the idea is not precise, or the term *horse* is ill understood. A name ought to be like a boundary, which clearly and undeniably either shuts in, or shuts out, every idea that can be suggested. It is the imperfection of our minds, our language, and our knowledge of external things, that this clear and undeniable inclusion or exclusion is seldom attainable, except as to ideas which are *well within the boundary*: at and near the boundary itself all is vague. There are decided greens and decided blues: but between the two colours there are shades of which it must be unsettled by universal agreement to which of the two colours they belong. To the eye, green passes into blue by imperceptible gradations: our senses will suggest no place on which all agree, at which one is to end and the other to begin.

But the advance of knowledge has a tendency to supply means of precise definition. Thus, in the instance above cited, Wollaston and Fraunhofer have discovered the black lines which always exist in the spectrum of solar colours given by a glass prism, in the same relative places. There are definite places in the spectrum, by the help of which the place of any shade of colour therein existing may * be ascertained, and means of definition given.

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* It is quite within the possibilities of the application of science to the arts that the time should come when the spectrum, and the lines in it, will be used for matching colours in every linen-draper's shop.

When a name is complex, it frequently admits of definition, [36] nominal or real. A name may be said to be *defined nominally* when we can of right substitute for it other terms. In such a case, a person may be made to know the meaning of the word without access to the object of which it is to give the idea. Thus, an *island* is completely defined in "land surrounded by water." In definition, we do not mean that we are necessarily to have very precise terms in which to explain the name defined: but, as the terms of the definition so is the name which is defined; according as the first are precise or vague, clear or obscure, so is the second. Thus there may be a question as to the meaning of *land*: is a marsh sticking up out of the water an island? Some will say that, as opposed to water, a marsh is land, others may consider marsh as intermediate between what is commonly called [dry] land and water. If there be any vagueness, the term island must partake of it: for island is but short for "land surrounded by water," whether this phrase be vague or precise. This sort of definition is *nominal*, being the substitution of names for names. It is complete, for it gives all that the name is to mean. An island, as such, can have nothing necessarily belonging to it except what necessarily belongs to "land surrounded by water." By *real* definition, I mean such an explanation of the word, be it the whole of the meaning or only part, as will be sufficient to separate the things contained under that word from all others. Thus the following, I believe, is a complete definition of an *elephant*; "an animal which naturally drinks by drawing the water into its nose, and then spirting it into its mouth." As it happens, the animal which does this is the elephant only, of all which are known upon the earth: so long as this is the case, so long the above definition answers every purpose; but it is far from involving all the ideas which arise from the word. Neither sagacity, nor utility, nor the production of ivory, are necessarily connected with drinking by help of the nose. And this definition is purely objective; we do not mean that every idea we could form of an animal so drinking is to be called an elephant. If a new animal were to be discovered, having the same mode of drinking, it would be a matter of pure choice whether it should be called elephant or not. It [37] must then be settled whether it shall be called an elephant, and that race of animals shall be divided into two species, with distinctive definitions; or whether it shall have another name, and the definition above given shall be incomplete, as not serving to draw an entire distinction between the elephant and all other things.

It will be observed that the nominal definition includes the real, as soon as the terms of substitution are really defined: while the real definition may fall short of the nominal.

When a name is clearly understood, by which we mean when of every object of thought we can distinctly say, this name does or does not, contain that object—we have said that the name applies to everything, in one way or the other. The word man has an application both to Alexander and Bucephalus: the first *was* a man, the second *was not*. In the formation of language, a great many names are, as to their original signification, of a purely negative character: thus, parallels are only lines which do *not* meet, aliens are men who are *not* Britons (that is, in our country). If language were as perfect and as copious as we could imagine it to be, we should have, for every name which has a positive signification, another which merely implies all other things: thus, as we have a name for a tree, we should have another to signify every thing that is not a tree. As it is, we have sometimes a name for the positive, and none for the negative, as in *tree:* sometimes for the negative and none for the positive, as in *parallels:* sometimes for both, as in a frequent use of *person* and *thing*. In logic, it is desirable to consider names of inclusion with the corresponding names of exclusion: and this I intend to do to a much greater extent than is usual: inventing names of exclusion by the prefix not, as in tree and not-tree, man and not-man. Let these be called *contrary*,* or *contradictory*, names.

Let us take a pair of contrary names, as man and not-man. It is plain that between them they represent everything imaginable or real, in the universe. But the contraries of common language usually embrace, not the whole universe, but some one general idea. Thus, of men, Briton and alien are contraries: every man must be one of the two, no man can be both. Not-Briton and alien are identical names, and so are not-alien and Briton. [38] The same may be said of integer and fraction among numbers, peer and commoner among subjects of the realm, male and female among animals, and so on. In order to express this, let us say that the whole idea under consideration is *the universe* (meaning merely the whole of which we are considering parts) and let names which have nothing in common, but which between them contain the whole idea under consideration, be called contraries *in, or with respect to, that universe*. Thus, the universe being mankind, Briton and alien are contraries, as are soldier and civilian, male and female, &c.: the universe being animal, man and brute are contraries, &c.

Names may be represented by the letters of the alphabet: thus A, B, &c., may stand for any names we are considering, simple or complex. The contraries may be represented by not-A, not-B, &c., but I shall usually prefer to denote them by the small letters *a*, *b*, &c. Thus, everything in the universe (whatever that universe may embrace) is either A or not-A, either A or *a*, either B or *b*, &c. Nothing can be both B and *b*; every not-B is *b*, and every not-*b* is B: and so on.

No language, as may well be supposed, has been constructed beforehand with any intention of providing for the wants of any metaphysical system. In most, it is seen that the necessity of providing for the formation of contrary terms has been obeyed. Our own language has borrowed from the Latin as well as from its parent: thus we have *imperfect, disagreeable,* as well as *unformed* and *witless.* There is a choice of contraries without very well settled modes of appropriation: standing for different degrees of contrariety. Thus we have *not perfect* which is not so strong a term as *imperfect;* and *not imperfect,* the contrary of a contrary, which is not so strong as *perfect.* The wants of common conversation have sometimes retained a term and allowed the contrary to sink into disuse; sometimes retained the contrary and neglected the original term; sometimes have even introduced the contrary without introducing any term for the original notion, and allowed no means of expressing the original notion except as the contrary of a contrary. If we could imagine a perfect language, we should suppose it would contain a mode of signifying the contrary of every name: this indeed our own language may be said to have, though sometimes in an awkward and [39] unidiomatic manner. One inflexion, or one additional word, may serve to signify a contrary of any kind: thus *not man* is effective to denote all that is other than man. But there is a wider want, which can only be partially supplied, for its complete satisfaction would require words almost beyond the power of arithmetic to count: and all that has been done to make it less consists, in our language and in every other, mostly in the formation of compound terms, be they substantive and adjective, double substantives, or any others. A class of objects has a sub-class contained within it, the individuals of which are distinguished from all others of the class by something common to them and them only. If the distinguishing characteristic have been separated, and a word formed to signify the abstract idea, that word, or an adjective formed from it (if it be not an adjective) is joined with the general name of the class. Thus we have strong men, white horses, &c. Or it may happen that the individuals of the sub-class take, in right of the distinguishing characteristic, a perfectly new name, and by the most varied rules. A corn-grinding man is called from the implement he uses, a *miller* ; a meat-killing man from the organ which he supplies, a *butcher*, (if the first idea of the etymology of this word be correct).[8] Other men use mills and other trades feed the mouth : still custom has settled these terms, though the first is only connected with its origin by the spelling, and the second by a derivation which must be sought in another language. But again, it will more often happen that a distinctive characteristic, belonging to some only, gives no distinctive name to those *some*, which still remain an unnamed *some* out of the whole, to be separated by the description of their characteristic when wanted, instead of being the *all* of a name invented to express them, and them alone of their class. In such a predicament, for instance, are men who have never seen the sea, as distinguished from those who have seen it. Hence it appears that particular propositions are not so distinct from universal ones in real character as they are generally made to be. If I say " some As are Bs " the reader may well suppose that it is not often necessary to advert to this fact : had it been so, a name would have been invented specially to signify "As which are Bs." If this name hadbeen C, the proposition would have been " *every* C is B."

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* I intend to draw no distinction between these words.

[40] The same convenience which dictates the formation of a name for one sub-class and not for another, rules in the formation of contrary terms, as already noted. And these caprices of language—for logically considered they are nothing else, though their formation is far from lawless—make it desirable to include in a formal treatise the most complete consideration of all propositions, with reference not only to their terms, but also to the contraries of those terms. Every negative proposition is affirmative, and every affirmative is negative. Whatever completely does one of the two, include or exclude, also does the other. If I say that " no A is B," then, *b* being the name of every thing not B in the universe of the proposition, I say that " every A is *b* : " and if I say that " every A is B," I say that " no A is *b*." Whether a language will happen to possess the name B, or *b*, or both, depends on circumstances of which logical preference is never one, except in treatises of science. The English may possess a term for B, the French only for *b* : so that the same idea must be presented in an affirmative form to an Englishman, as in " every A is B," and in a negative one to a Frenchman, as " no A is *b*." From all this it follows that it is an accident of language whether a proposition is universal or particular, positive or negative. We, having the names A and B, may be able to say " every A is B : " another language, which only names the contrary of B, must say " no A is *b*." A third language, in which As have not a separate name, but are only individuals of the class C, must say " some Cs are Bs ; " while a fourth, which is in the further predicament of naming only *b*, must have it " some Cs are not *bs*." When we come to consider the syllogism, we shall have full confirmation of the correctness and completeness of this view.

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[8] [The etymology suggested is an (incorrect) derivation of *butcher* from *bouche*.—Ed.]

It may be objected that the introduction of terms which are merely negations of the positive ideas contained in other terms is a species of fiction. I answer, that, first, the fiction, if it be a fiction, exists in language, and produces its effects : nor will it easily be proved more fictitious than the invention of sounds to stand for things. But, secondly, there is a much more effective answer, which will require a little development.

When writers on logic, up to the present time, use such contraries as man and not-man, they mean by the alternative, man and everything else. There can be little effective meaning, and [41] no use, in a classification which, because they are not men, includes in one word, *not-man*, a planet and a pin, a rock and a featherbed, bodies and ideas, wishes and things wished for. But if we remember that in many, perhaps most, propositions, the range of thought is much less extensive than the whole universe, commonly so called, we begin to find that the whole extent of a subject of discussion is, for the purpose of discussion, what I have called a *universe,* that is to say, a range of ideas which is either expressed or understood as containing the whole matter under consideration. In such universes, contraries are very common : that is, terms each of which excludes every case of the other, while both together contain the whole. And, it must be observed that the contraries of a limited universe, though it be a sufficient real definition of either that it is not the other, are frequently both of them the objects from which positive ideas are obtained. Thus, in the universe of property, personal and real are contraries, and a definition of either is a definition of the other. But though each be a negative term as compared with the other, no one will say that the idea conveyed by either is that of a mere negation. Money is *not* land, but it *is* something. And even when the contrary term is originally invented merely as a negation, it may and does acquire positive properties. Thus alien is strictly non-Briton : but suppose a man taken in arms against the crown on some spot within its dominions, and claiming to be a prisoner of war. The answer that he *is* a British subject is a negation : to establish his positive claim he first must prove himself an alien, and moreover that he is in another positive predicament, namely, that he is the subject of a power at war with Great Britain. Accordingly, of two contraries, neither must be considered as *only* the negation of the other : except when the universe in question is so wide, and the positive term so limited, that the things contained under the contrary name have nothing but the negative quality in common.

Perception of agreements and disagreements is the foundation of all *assertion :* the acquirement of such perception with respect to any two ideas by the comparison of both with a third, is the process of all *inference.* To infer, by comparison of abstract ideas, is the peculiar privilege of man ; to need inference is his imperfection. To what point man would carry inference if he wanted **[42]** language, how much further the lower animals could carry what they have of it if they had language, are questions on which it is vain to speculate. The words *is* and *is not,* which imply the agreement or disagreement of two ideas, must exist, explicitly or implicitly, in every assertion. And what we call agreement or disagreement, may be reduced to identity or non-identity. When we say John is a man, we have the first and most objective form of assertion. Looked at in the most objective point of view it is only this, *John* is one of the individual objects who are called *man.* Looked at ideally, the proposition is more general. The idea of man, gathered from instances, presents itself as a collective mass of ideas, of which we can figure to ourselves an instance without necessarily calling up the idea of any man that ever existed. In the ideal conception of man, Achilles is a man as much as the Duke of Wellington, whether the former ever existed objectively or no: of all the ideas of man which the mind can imagine, the former is one as well as the latter.

The separation of ideas, or formation of abstract ideas, and assertion by means of them, presents nothing, for our purpose, which differs from the former case. If we say "this picture is beautiful," the mere phrase is incomplete, for "beautiful" is only an attribute, a purely ideal reference to a classification which the mind makes, dictated by its own judgment. The picture being a material object, cannot *be* anything but an object, cannot belong to any class of notions, unless that class contain objects. What the proposition may mean is to a certain extent dependent upon the implied substantive to which beautiful belongs: that is, to the class of objects which the proposition implies the mind to have separated into beautiful and not beautiful. It may be that the picture is a beautiful picture: or a beautiful work of art, taking its place in that division by which not only pictures, but statues, buildings, reliefs, &c. are separated into beautiful and otherwise: or a beautiful creation of human thought, placed among works of art, imagination, or science, &c. in the subdivision beautiful: or finally, it may be a beautiful *thing*, placed with all objects of perception in a similar subdivision.

In all assertions, however, it is to be noted, once for all, that *formal logic*, the object of this treatise, deals with *names* and not with either the *ideas* or *things* to which these names belong. We [43] are concerned with the properties of "A is B" and "A is not B" so far as they present an idea independently of any specification of what A and B mean: with such ideas upon propositions as are presented by their *forms*, and are common to all forms of the same kind. The reality of logic is the examination of the use of *is* and *is not*: the tracing of the consequences of the application of these words. The argument "when the sun shines it is day: but it is not day, therefore the sun does not shine," contains a theory and two facts, the latter of which is made to follow from the former by the theory. That inference is made is seen in the word *therefore*: and the sentence is capable of being put upon its trial for truth or falsehood by logical examination. But this examination rejects the meaning of sun and day, the truth of the theory and of the facts; and only inquires into the right which the sentence, of its own structure, gives us to introduce the word *therefore*. It merely enters upon "when A *is*, B *is*; but B *is not*; therefore A *is not*:" and decides that this is a correct junction of precedents and consequent, an exhibition of necessary connexion between what goes before and after *therefore*, and a development, in the latter, of what is virtually, though not actually, expressed in the former. What A and B may mean is of no consequence to the *inference*, or right to *bring in* "A is *not*."

Thus A and B, divested of all specific meaning, are really names as names, independently of things: or at least may be so considered. For the truth of the proposition, under all meanings, gives us a right to suppose, if we like, that *names are the meanings*—that is to say, that we may put it thus, "When the name A is, the name B is: but the name B is not; therefore the name A is not."

It is not therefore the object of logic to determine whether conclusions be true or false; but whether what are asserted to be conclusions are *conclusions*. By a *conclusion* is meant that which is and must be *shut in with* certain other preceding things put in first: it is that which must have been put into a sentence because certain other things were put in. To *infer a conclusion* is to *bring in*, as it were, the direct statement of that which has been virtually stated already—has been *shut in*. When we say "A is B, B is C; we *conclude* A is C"; it would be more correct to say "A is B, B is C; we *have concluded* A is C". We should never [44] think of saying "we have put into a box a man's upper dress of the colour of the trees; therefore we *must put in* a green coat";—we should say "*we have put in*." To infer the conclusion then is to bring in a statement that we have *concluded*.

Inference does not give us more than there was before: but it may make us *see* more than we *saw* before: ideally speaking, then, it does give us (in the mind) more than there was before. But the homely truth that no more can come out than was in, though accepted as to all material objects even by metaphysicians—who are generally well pleased to find the key of a box which contains what they want, though sure that it will put in no more than was there already—has been applied to logic, and even to mathematics, in depreciation of their rank as branches of knowledge. Those who have made this strangest of human errors must have assumed an ideal omniscience, and looked at human imperfection objectively. Omniscience need neither compare ideas, nor draw inferences: the conclusion which we deduce *from* premises, is always present *with* them; truths are *concomitants*, not *consequences*. When we say that one assertion *follows* from another, we speak purely ideally, and describe an imperfection of our own minds: it is not that the consequence follows from the premises, but that our *perception* of the consequence follows our *perception* of the premises: the consequence, objectively speaking, is in, and with, and of, the premises. We speak wrongly if we speak ideally, when we say that "A is C," *is* in "A is B and B is C': in fact, it is only by giving an objective view to the argument, that we can even assert that it *will be* seen. To uncultivated minds, this simple conclusion is never concomitant with the premises, and only with some difficulty a consequence.

From the certainty that a consequence may be made to come out, which is an allegorical use of the word *out*, we assume a right to declare, by the same sort of allegory,* that it was *in*. The premises therefore contain the conclusion: and hence some have spoken as if in studying how to draw the conclusion, we were studying to know what we knew before. All the propositions of pure geometry, which multiply so fast that it is only a small [45] and isolated class even among mathematicians who know all that has been done in that science, are certainly contained in, that is necessarily deducible from, a very few simple notions. But *to be known from* these premises is very different from being *known with* them.

Another form of the assertion is that consequences are *virtually* contained in the premises, or (I suppose) *as good as* contained in the premises. Persons not spoiled by sophistry will smile when they are told that knowing two straight lines cannot enclose a space, the whole is greater than its part, &c.—they as good as knew that the three intersections of opposite sides of a hexagon inscribed in a circle must be in the same straight line. Many of my readers will learn this now for the first time: it will comfort them much to be assured, on many high authorities, that they virtually knew it ever since their childhood. They can now ponder upon the distinction, as to the state of their own minds, between virtual knowledge and absolute ignorance.

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* I am of opinion that it is more consistent with analogy to say that the hypothesis *is contained in* its necessary consequence, than to say that the former contains the latter. My reason will appear in the course of the work.

There must always be some contention as to the relative value of their knowledge between the students of the things which we can see must have been, and of the things which, for what we can see, might have been otherwise. How much of the distinction is due to our ignorance, no one can tell. In the mean time, it is of more use to point out the advantage, as things are, of studying both kinds of knowledge, than to attempt to institute a rivalry between them. Those who have undervalued the study of necessary consequences, have allowed themselves, in illustrating their argument, phrases * which taken literally, mean more perhaps than they intended.

[46] The study of logic, then, considered relatively to human knowledge, stands in as low a place as that of the humble rules of arithmetic, with reference to the vast extent of mathematics and their physical applications. Neither is the less important for its lowliness: but it is not every one who can see that. Writers on the subject frequently take a scope which entitles them to claim for logic one of the highest places: they do not confine themselves to the connexion of premises and conclusion, but enter upon the *periculum et commodum* of the formation of the premises themselves. In the hands of Mr. Mill, for example (and to some extent in those of Dr. Whateley) logic is the science of distinguishing truth from falsehood, so as both to judge

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* We might sometimes take them to mean that the study of necessary connexion in logic, mathematics, &c., is at least useless, if not pernicious. Now we should suppose, if this be what they mean, that close connexion, short of absolute necessity, must partake somewhat of the same character. If the absolute mathematical necessity that three angles of a triangle are equal to two right angles is therefore to be avoided, the study of physics, in which there are the necessities which we express by the term *laws of nature*, must do some harm. History, in which we may so often count upon the actions which motives will produce, cannot be quite faultless: and there are laws of formation in language which might as well be kept out of sight, for they act almost with the uniformity of laws of nature. True knowledge must consist in the study of the actions of madmen: that a certain man imagined himself to be Cæsar, when he might just as well have been Newton or Nebuchadnezzar, must be a real bit of knowledge, not virtually contained in anything else, wholly or partially.

the premises and draw the conclusion, to compare name with name, not only as to identity or difference, but in all the varied associations of thought which arise out of this comparison.

CHAPTER III.

ON THE ABSTRACT FORM OF THE PROPOSITION.

IN the preceding chapter, I have endeavoured to put together such notions on the actual sources of our knowledge as may give the reader the means of thinking upon points which any system of logic, however restricted, must necessarily suggest. We cannot attempt to connect our use of words with our notions of things, without the occurrence of a great many difficulties, a great many sources of adverse theories, and of never-ending disputes. We cannot even represent phænomena, as phænomena, except in the language of some system, and it may be of a wrong one. The confidence which the favourers of these several theories place in their correctness is a sufficient reason for keeping the account of the process of the understanding, so far as it can be made an exact science, as distinct as possible from all of them: for they differ widely, and if they agree in anything which can [47] be distinctly apprehended, it is only in having names of great authority enrolled among the partisans of every one.

In order to examine the laws of inference, of the way of distinctly perceiving the right to say "therefore," "so that," "whence it must be," &c., &c., in a manner which may be admitted, so far as we go, by all, we must make this separation very complete. All admit propositions, as "man is animal," "no man is faultless;" all are, after a little thought, agreed upon the modes of inference: but upon the import of a simple proposition, there is every kind of difference. How much we mean, when we say "man is animal," and how we arrive at our meaning, is matter for volumes on different sides of unsettled questions.

In order properly to examine the laws of inference, or of any thing else, we must first endeavour to arrive at a distinct abstraction of so much of the idea we are concerned with, as is itself the precedent reason, if it be right so to speak, of the law in question. This is an easy process upon familiar things. We do not give the carriers of goods much credit for profundity, in seeing that, on a given road, there is only the difference of weight by which they are concerned to know how one parcel differs from another; and further that, as long as they have to carry a pound, it matters nothing whether it be of sugar or iron. It is this process which we want to perform to the utmost, upon the simple proposition. Writers on logic, from Aristotle downwards, have made a large and important step in substituting for specific names, with all their suggestions about them, the mere letters of the alphabet, A, B, C, &c. These letters are *symbols*, and *general* symbols: each of them stands for any one we please of its class. But what are they symbols of, names, ideas,* or the objects which give those ideas? The answer is, that this is precisely one of those considerations which we may leave behind, in abstracting what is necessary to an examination of the laws of inference. The only condition is, that we are to confine ourselves to one or the other. When we say man is animal, it may be that the name man is contained in the name animal, that the idea of man is contained in that of animal, or that the object man is in the object animal. Or if there were twenty more different appropriations of the [48] symbols, the same thing might be said of each. This, I believe, the first use of the general symbol in order of time; the algebraical use of letter or other symbol, to designate number, being both subsequent and derived.

When therefore we say "Every X is Y," we understand that X is a symbol which represents an instance of a name, idea, object, &c., as the case may be. There may be more or fewer of such instances; they may be numerable or innumerable. And the same of Y. The language of logicians has generally been unfavourable to the distinct perception of their terms being distributively applicable to classes of instances. They have rather been *quantitative* than *quantuplicitative:* expressing themselves as if, in saying that animal is a larger or wider term than man, they would rather draw their language from the idea of two areas, one of which is larger than the other, than from two collections of indivisible units, one of which is in number more than the other. They have even carried this so far as to make it doubtful, except from context, whether their distinction between universal and particular is that of *all* and *some*, or of *the whole* and *part*. If their instances had been *white squares*, their "all A is B" and "some A is B" might have applied as well to "All the square is white" and "Some of the square is white" as to "All the squares are white" and "Some of the squares are white." I shall take particular care to use numerical language, as distinguished from magnitudinal, throughout this work, introducing of course, the plurals Xs, Ys, Zs, &c.

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* Meaning of course (page 30) ideas of ideas, and ideas of objects.

I may mention here another mode of speaking, which will, I think, appear objectionable to all who are much used to consideration of quantity. When a compound idea contains two or more simpler ones, some logicians have spoken as if the combination were legitimately represented by *arithmetical addition*. Thus the combination of the ideas of *animal* and *rational* must give the idea of *man:* for the two notions co-exist in nothing else that we know of. Accordingly, some write *animal* + *rational* = *man*. If this be intended as an abstraction of the notation of arithmetic, for the purpose of fitting to it entirely different meaning, there is of course no objection which I need consider here: but it seems to me that more is meant, and that those who have used this notation imagine a great resemblance between *combining* [49] ideas, and *cumulating* them. What the difference is, I cannot pretend to say, any more than I can pretend to say what the difference is between chemically combining volumes of oxygen and hydrogen, so as to produce water, and simple cumulation of them in the same vessel, so as to produce a mixed gas: every beginner knows that the electric spark, or some other inexplicable agency, is necessary to turn the mixed gas into a new chemical combination. But that the difference exists in the former case also, seems to me as clear as any thing I can imagine. Even in chemistry the cumulative notation, which was once thought an all-sufficient mode of expressing the results of the atomic theory, has failed with the progress of knowledge. To a considerable extent, the introduction of modes of cumulation as yet answers the purpose: but there still remain *isomeric* compounds, differing in properties, but of the same composition. For example, the tartaric and racemic acids: of which Professor Graham says (*Elements of Chemistry*, p. 158), "A nearer approach to identity could scarcely be conceived than is exhibited by these bodies, which are, indeed, the same both in form and composition. . . . . . . But by no treatment can the one acid be transmuted into the other." If the above mode of confounding cumulation and combination be admissible, I suppose we might easily give ourselves a right to say that

$$2 + 2 + \text{addition} = 4$$

an equation at which the mathematician would stare.

So much for the characteristics of the *terms* of a proposition, as wanted for the abstract forms of inference. It remains to consider those of the connecting copulæ *is* and *is not*.

The complete attempt to deal with the term *is* would go to the form and matter of every thing in *existence*, at least, if not to the possible form and matter of all that does not exist, but might. As far as it could be done, it would give the grand Cyclopædia, and its yearly supplement would be the history of the human race for the time. That logic exists as a treated science, arises from the characteristics of the word, requisite to be abstracted in studying inference, being few and easily apprehended. It may be used in many senses, all having a common property. Names, ideas, and objects, require it in three different senses. Speak of *names*, and say "man *is* animal": the *is* is here an *is* of applicability; to **[50]** whatsoever (idea, object, &c.) man is a name to be applied, to that same (idea, object, &c.) animal is a name to be applied. As to ideas, the *is* is an *is* of possession of all essential characteristics; *man* is an idea which possesses, contains, presents, all that is constitutive of the idea *animal*. As to absolute external objects, the *is* is an *is* of identity, the most common and positive use of the word. Every man *is* one of the animals; touch him, you touch an animal, destroy him, you destroy an animal.

These senses are not all interchangeable. Take the *is* of identity, and the name *man* is not, as a name, the name *animal*: the idea man is not, as an idea, the idea animal. Now we must ask, what common property is possessed by each of these three notions of *is*, on which the common laws of inference depend. Common laws of inference there certainly are. If the applicability of the name A be always accompanied by that of B, and that of B by that of C, then that of A is always accompanied by that of C. If the idea A contain all that is essential to the idea B, and B all that is essential to C, then A contains all that is essential to C. If the object A be actually the object B, and if B be actually C, then A is actually C.

The following are the characteristics of the word *is* which, existing in any proposed meaning of it, make that meaning satisfy the requirements of logicians when they lay down the proposition "A is B." To make the statement distinct, let the proposition be doubly singular, or refer to one instance of each, one A and one B: let it be "this one A is this one B."

First, the double singular proposition above mentioned, and every such double-singular, must be indifferent to conversion: the "A is B," and the "B is A" must have the same meaning, and be both true or both false.

Secondly, the connexion *is*, existing between one term and each of two others, must therefore exist between those two others; so "A is B" and "A is C" must give "B is C."

Thirdly, the essential distinction of the term *is not* is merely that *is* and *is not* are *contradictory alternatives*, one must, both cannot, be true.

Every connexion which can be invented and signified by the terms *is* and *is not*, so as to satisfy these three conditions, makes all the rules of logic true. No doubt absolute identity was the sug-[51]gesting connexion from which all the others arose: just as arithmetic was the medium in which the forms and laws of algebra were suggested. But, as now we *invent algebras* by abstracting the forms and laws of operation, and fitting new meanings to them, so we have power to invent new meanings for all the forms of inference, in every way in which we have power to make meanings of *is* and *is not* which satisfy the above conditions. For instance, let X, Y, Z, each be the symbol attached to every instance of a class of *material* objects, let *is* placed between two, as in "X is Y" mean that the two are tied together, say by a cord, and let X be considered as tied to Z when it is tied to Y which is tied to Z, &c. There is no syllogism but what remains true under these meanings. Thus

The syllogism	Is true in the sense
Every X is Y	Every X is tied to a Y
Some Zs are not Ys	Some Zs are not tied to Ys
∴Some Zs are not Xs	∴Some Zs are not tied to Xs

This last instance might be considered as a material representation of attachment together of ideas in the mind.

We must distinctly observe that it is not every case of inference which demands all the characteristics to be satisfied. Thus in the most common case of all, "Every A is B, every B is C, therefore every A is C," of all the three conditions only the second is wanted to secure the validity of this case. Though it be seldom thought worth while to make this observation, yet it is universal practice to act upon it, and so as to introduce into formal logic apparent contradictions of its own rules. For example, the following are allowed to pass for syllogisms, in the ordinary definition of that word.

" Every A is greater than some one B ; every B is greater than some one C, therefore every A is greater than some one C." And the same when instead of *greater than* is read *equal to* or *less than.* The form which most commonly appears is the pair of doubly singular propositions, " A (one thing) is greater than B ; B is greater than C ; therefore A is greater than C." Here " greater than greater " is " greater," the second rule is satisfied, and no other is wanted. But this meaning for *is* (or this substitute for it, if the reader like it better) will not satisfy all the con-**[52]**ditions, and therefore will not apply to all the forms of inference.

But *is* in the sense " is equal to " *does* satisfy all the conditions. This sense of *is*, namely agreement in magnitude, is the copula of the mathematician's syllogism, when he is reasoning on quantity only.

It will probably be affirmed that the generalization thus made, or shown to be possible, in the conception of the word *is* for purposes of inference, amounts only to a very frequent, if not most usual, use of the word, namely, as signifying a certain mode, not of identity, but of agreement in quality. As when we say " these two things are the same—in colour " or " the one thing *is* the other—in colour : " that the name man *is* the name animal, in a certain respect, namely, in what the latter can be applied to : that the idea man is animal, in both possessing certain characteristics : that every object man is an object animal, in actual substance : that A is B in magnitude,—when we say A equals B ; and so on. But I admit only the converse, namely, that all these uses satisfy the conditions. It would hardly be for any one to say, that every possible use of *is* which satisfies three such simple requirements, has been or can be exhausted. Even the material example which was just now given, cannot be identified with any common use, or easily imaginable one, of the common verb. But if no invented meaning, proper to satisfy the conditions, can be found, other than already exists in more or less of use, still, these conditions are the laws to which the word must submit in its logical acceptation.

There are common uses of the word which are not admitted in logic : and among them, one of the most common, connection of an object with its quality, and of an idea with one of its constituent or associated ideas. As when we say, the rose *is* red, prudence *is* desirable. Here the logical conditions are not satisfied. For example, " red is the rose," though a poetical inversion of the first assertion, is not logically true. It is usual to consider such propositions, in logic, as elliptical ; thus " the rose is red " is considered as " the rose is a red object, or an object of red colour ; " in which the *is* now takes one of the senses which allows of conversion. Similarly, in all other cases, the subject and predicate are made to take the same character ; both names, both ideas, or both objects. This reduction renders unnecessary both the study **[53]** of the varieties of meaning of the word *is* (meaning varieties out of the pale of the conditions above enumerated), and also that of the transitions of meaning within the circle of which the inference remains good.

The most common uses of the verb are ;—first absolute identity, as in " the thing he sold you *is* the one I sold him : " secondly, agreement in a certain particular or particulars understood, as in " He is a negro " said of a European in reference to his colour : thirdly, possession of a quality, as in " the rose is red : " fourthly, reference of a species to its genus, as in " man is an animal." All these uses are independent of the use of the verb alone, denoting existence, as in " man is [i.e. exists]." In all these senses, and in all which might be added consistently with the conditions in page 50, some propositions sometimes admit of having the sense of *is* shifted, and some do not. Thus, in negative propositions, the *is* of agreement in particulars may be lawfully converted into that of identity: if "No A is B in colour," then absolutely "No A is B." But "Every A is B" in colour, does not prove "Every A is B." But the first pair might be connected by a syllogism.

The *is* of agreement in particulars may always be reduced to the *is* of identity, by alteration of the predicate; thus "Every A is B in colour" is "Every A is a thing having the colour of one of the Bs." When a syllogism has a negative conclusion, and the middle term is, or can be made, the predicate of both premises, then the whole syllogism can be transformed from one in which there is only the *is* of agreement to one in which there is no *is* but that of identity. For example, suppose the premises to be "No X is Y (in colour); every Z is Y (in colour)," not meaning necessarily that all the Ys are of one colour, but reading it as "No X is of the colour of any one of the Ys; every Z is of the colour of one of the Ys." The conclusion is that "no Z is X (in colour)," or "no Z is of the colour of any one of the Xs." But from this it follows that no Z is X, for if any one Z were absolutely X, it would have * the colour of that X. This **[54]** last conclusion can be brought directly from altered premises: thus, *is* being that of identity, we have "No X is [a thing having the colour of one of the Ys]; every Z is [a thing having the colour of one of the Ys]; therefore no Z is X." But suppose we take the following premises, "Some Ys are not Xs (in colour); every Y is Z

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* The reader must not paint any of the letters during the process. The sense in which we say a door *is* the same door as before, after it has been painted of a different colour, is not the sense of logical identity: it is the same in all but colour and colouring matter; and the *is* is one of agreement. Except as a joke in sufficient answer to a captious objection or a trap, no change whatever must take place in the terms of conclusion, during inference. The American calculating boy, Zerah Colburn, was asked how many black beans it would take to make ten white ones; to which he very properly answered, "Ten, if you skin 'em;" but the ten skinned beans would not be the *same beans* as before: except, indeed, to those to whom black is white.

(in colour)." From this it follows that some Zs are not Xs (in colour), and thence that some Zs *are not* Xs. But we cannot now alter the premises, so as to produce the last conclusion from X, Z, and a middle term.

CHAPTER IV.

ON PROPOSITIONS.

A NAME is a symbol which is attached to one or more objects of thought, on account of some resemblance, or community of properties. Or else it is a symbol attached to some one or more objects of thought, to distinguish them from others having the same properties. Objects of the same name are, so far as that name is concerned, undistinguishable. And one object may have many names, as being one in each of many classes of objects of thought.

Names, as explained in chapter II, are exclusively the objects of formal logic. The identity and difference of things is described by asserting the right to assert, or the right to deny, the application of names. And names whether simple or complex, will be represented by letters of the alphabet, as X, Y, Z.

A proposition is the assertion of agreement, more or less, or disagreement, more or less, between two names. It expresses that of the objects of thought called Xs, there are some which are, or are not, found among the objects of thought called Ys: [55] that there are objects which have both names, or which have one but not the other, or which have neither.

For the most part, the objects of thought which enter into a proposition are supposed to be taken, not from the whole universe of possible objects, but from some more definite collection of them. Thus when we say " All animals require air," or that the name *requiring air* belongs to every thing to which the name *animal* belongs, we should understand that we are speaking of things on this earth: the planets, &c., of which we know nothing, not being included. By the *universe* of a proposition, I mean the whole range of names in which it is expressed or understood that the names in the proposition are found. If there be no such expression nor understanding, then the universe of the proposition is the whole range of possible names. If, the universe being the name U, we have a right to say "every X is Y," then we can only extend the universe so as to make it include all possible names, by saying "Every X which is U is one of the Ys which are Us," or something equivalent.

Contrary names, with reference to any one universe, are those which cannot both apply at once, but one or other of which always applies. Thus, the universe being *man*, *Briton* and *alien* are contraries; the universe being *property*, *real* and *personal* are contraries. Names which are contraries in one universe, are not necessarily so in a larger one. Thus in geometry, when the universe is *one plane*, pairs of straight lines are either parallels or intersectors, and never both: parallels and intersectors are then contraries. But when the student comes to solid geometry, in which *all space* is the universe, there are lines which are neither parallels nor intersectors; and these words are then not contraries. But names which are contraries in the larger and containing universe, are necessarily contraries in the smaller and contained, unless the smaller universe absolutely exclude one name, and then the other name is *the* universe.

In future, I always understand some one universe as being that in which all names used are wholly contained: and also (which it is very important to bear in mind) that no one name mentioned in a proposition fills this universe, or applies to everything in it. Nothing is more easy than to treat the supposition of a name being the universe as an extreme case. And I shall denote con-[56]traries by large and small letters: thus, X being a name, x is the contrary name. And everything (in the universe understood) is either X or x: and nothing is both.

A proposition may be either *simple* and *incomplete*, or *complex* and *complete*. The simple proposition only asserts that Xs are Ys, or are not Ys: the complex proposition, which always consists of two simple ones,[9] disposes in one manner or the other of every X and every Y. Thus "Every X is Y" is a simple proposition: but it forms a part of two complex propositions. It may belong either to "every X is Y and every Y is X," or to "Every X is Y and some Ys are not Xs."

The propositions advanced in common life are usually complex, with one simple proposition expressed and one understood: but books of logic have hitherto considered only the simple proposition. And this last should be considered before the complex form.

The simple proposition must be considered with respect to *sign*, *relative quantity*, and *order*.

Simple propositions are of two *signs*: *affirmative* and *negative*. It is either "Xs are Ys," or "Xs are not Ys." The phrases *are* and *are not*, or *is* and *is not*, which mark the distinction, are called *copulæ*.

The *relative quantity* of a proposition has reference to the numbers of instances of the different names which enter it. The distinctions of quantity usually recognized are *all* and *some* *: leading to the distinction of *universal* and *particular*. Thus "Every X is Y" and "Every X is not Y" are the universal affirmative and negative propositions: the latter is usually stated as "No X is Y." And "some Xs are Ys" and "some Xs are not Ys" are the particular affirmative and negative propositions. And when the

* *Some*, in logic, means *one or more, it may be all*. He who says that *some are*, is not to be held to mean that *the rest are not*. "Some men breathe," "some horses are distinguishable by shape from their riders" would be held false in common language. The reason is, as above noted, that common language usually adopts the complex particular proposition, and implies that some are not in saying that some are. The student cannot be too careful to remember this distinction. A particular proposition is only a "may be particular."

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[9] ["Except only one which consists of four simple propositions," De Morgan, "Table of Contents."—ED.]

propositions are reduced strictly to these four forms, [57] the first named, X, is called the *subject*, and the second named, Y, the *predicate*.

It has been proposed to consider the universal propositions as *definite* with respect to *quantity*: but this is not quite correct. The phrase "all Xs are Ys" does not tell us how many Xs there are, but that, be the unknown number of Xs in *existence* what it may, the unknown number mentioned in the proposition is the same. That which is definite is the *ratio* of the number of Xs of the proposition to the Xs of the universe. So understood, however, the "definite quantity," as an abbreviation, may be said to belong to universals. And the indefiniteness of the particular proposition is only hypothetical. It is in our power to suppose the *some* to be one half of the whole, or two-thirds, or any other fraction.

The quantity of the subject is expressed; that of the predicate, though not expressed, is necessarily implied by the meaning of language. The predicate of an *affirmative* is *particular*: the predicate of a *negative* is *universal*. If I say "Xs are Ys," even though I speak of all the Xs, I only really speak of so many Ys as are compared with Xs and found to agree: and these need not be all the Ys. "Every horse is an animal," declares that so many horses as there are to speak of, so many animals are spoken of: and leaves it wholly unsettled whether there be or be not more animals left. But if I should say "Xs are not Ys," though it should be only one X, as in "this X is not a Y," yet I speak of every Y which exists. The assertion is "this X is not any one whatsoever of all the Ys in existence." A person who should wish to verify by actual inspection, "these 20 Xs are Ys," might, perchance, be enabled to affirm the result upon the examination of only 20 Ys, if he came first upon the right ones. But he could not verify "this one X is not a Y" until he had examined every Y in existence. This is the common doctrine, but though admitting of course that the affirmative proposition only enables us to infer of some instances of the predicate, yet I think it more correct to say that the predicate itself is spoken of universally, but *indivisibly*, and that in the negative proposition the predicate is spoken of universally *and divisibly*. " Some Xs are Ys " tells us that each X mentioned is *either* the first Y, *or* the second Y, *or* the third Y, &c., no Y being excluded from comparison. But [58] " Some Xs are not Ys " tells us that each X mentioned is absolutely *not* the first Y, *nor* the second, *nor* the third, &c. ; is not, in fact, any one of all the Ys. Still, however, the predicate of an affirmative yields no more than it would do if the Ys finally accepted as Xs were specially separated, and considered as the only Ys spoken of.

The relation of the universal quantity to the whole quantity of instances in existence is *definite*, being that whole quantity itself. But the particular quantity is wholly *indefinite* : " Some Xs are Ys " gives no clue to the fraction of all the Xs spoken of, nor to the fraction which they make of all the Ys. Common language makes a certain conventional approach to definiteness, which has been thrown away in works of logic. " Some," usually means a rather small fraction of the whole ; a larger fraction would be expressed by " a good many " ; and somewhat more than half by " most " ; while a still larger proportion would be " a great majority " or " nearly all." A perfectly *definite particular*, as to quantity, would express how many Xs are in existence, how many Ys, and how many of the Xs are or are not Ys : as in " 70 out of the 100 Xs are among the 200 Ys." In this chapter I shall treat only the *indefinite particular*, leaving the *definite particular* for future consideration.

The *order* of a proposition has relation to the choice of subject and predicate. Thus " Every X is Y " and " every Y is X," though both establish a universal affirmative relation between X and Y, yet are in fact two different propositions. They are called *converse* forms. When the subject and predicate are of the same sort of quantity, both universal or both particular, the converse forms give the same proposition. Thus " No X is Y " and " No Y is X " are the same; neither has any meaning, except perhaps of emphasis, which the other has not. And " Some Xs are Ys " is the same as " Some Ys are Xs." The universal negative, then, in which both terms are universal, and the particular affirmative, in which both are particular —are *necessarily convertible propositions.* But the universal affirmative, in which the subject is universal and the predicate particular, and the particular negative, in which the subject is particular, and the predicate universal—are not necessarily convertible, and are generally called *inconvertible.* They *may* be convertible, in one case, and inconvertible in an-**[59]**other. But the term *inconvertible* is not incorrect, for the following reason.

The agreements and disagreements which are treated in logic are of this character; there can only be agreement with one, but there may be disagreement with all. If " this X be a Y " it is one Y only: it is " this X is *either* the first Y, *or* the second Y, *or* the third Y, &c." If there be 100 Ys, there is, to those who can know it, 99 times as much negation as affirmation in the proposition: and yet most assuredly it is properly called *affirmative.* But if it be " this X is not a Y," we have " this X is not the first Y, *and* it is not the second Y, *and* it is not the third Y, &c." The affirmation is what is commonly called disjunctive, the negation conjunctive. A disjunctive negation would be no proposition at all, except that one and the same thing cannot be two different things: any X is either not the first Y *or* not the second Y. And in like manner a conjunctive affirmation would be an impossibility: it would state that one thing is two or more different things.

We must be prepared, then, to consider cases of opposition in which on the one side there is fixed necessity, and on the other side possibility of alternatives: and we must be prepared to denote these by opposite terms, which, looking to etymology only, denote fixed necessities of opposite characters. This happens in the case above: *convertible* means absolutely and necessarily convertible, inconvertible means *convertible or inconvertible as the case may be.* Taking the four forms of one order, we find that each of the universals cannot exist with either proposition of opposite form. Thus " Every X is Y " cannot be true if either " No X is Y " or " Some Xs are not Ys : " while " No X is Y " cannot be true if either " Every X is Y " or " Some Xs are Ys." But each of the particulars is necessarily inconsistent with nothing but the universal of opposite form. That " Some Xs are Ys " cannot be true if " No X is Y," but it may be true if " Some Xs are not Ys." And " Some Xs are not Ys " cannot be true if " Every X is Y," but it may be true though " Some Xs are Ys."

The pair " Every X is Y " and " some Xs are not Ys " are called *contradictory :* and so are the pair " No X is Y " and " Some Xs are Ys." Of each pair of contradictories, one must be true and [60] one must be false : so that the affirmation of either is the denial of the other, and the denial of either is the affirmation of the other. The pair " Every X is Y " and " No X is Y " are usually called *contraries ;* contrariety implying the utmost extreme of contradiction. Contraries may both be false, but cannot both be true. The pair " Some Xs are Ys," and " Some Xs are not Ys," which may both be true, but cannot both be false, are usually called *subcontraries.* But, for reasons hereafter to be given, I intend to abandon the distinction between the words *contrary* and *contradictory,* and to treat them as synonymous. And the propositions usually called *contraries,* " Every X is Y " and " No X is Y " I shall call *subcontraries :* while those usually called *subcontraries,* " Some Xs are Ys " and " Some Xs are not Ys " I shall call *supercontraries.*

I shall now proceed to an enlarged view of the proposition, and to the structure of a notation proper to represent its different cases.

As usual, let the universal affirmative be denoted by A, the particular affirmative by I, the universal negative by E, and the particular negative by O. This is the extent of the common symbolic expression of propositions: I propose to make the following additions for this work. Let one particular choice of order, as to subject and predicate, be supposed established as a standard of reference. As to the letters X, Y, Z, let the order be always that of the alphabet, XY, YZ, XZ. Let x, y, z, be the contrary names of X, Y, Z; and let the same order be adopted in the standard of reference. Let the four forms when choice is made out of X, Y, Z, be denoted by $A_,$, $E_,$, $I_,$, $O_,$; but when the choice is made from the contraries, let them be denoted by A', E', I', O'. Thus, with reference to Y and Z, "Every Y is Z" is the $A_,$ of that pair and order: while "Every y is z" is the A'. I should recommend $A_,$ and A' to be called the *sub*-A and the *super*-A of the pair and order in question: the helps which this will give the memory will presently be very apparent. And the same of $I_,$ and I', &c.

Let the following abbreviations be employed;—

X)Y means "Every X is Y"
X:Y — "Some Xs are not Ys"
X.Y means "No X is Y"
XY — "Some Xs are Ys"

[61] There are eight distinct modes, independent of contraries, in which a simple proposition may be made by means of X and Y. These eight modes are X)Y and Y)X, X:Y and Y:X, X.Y and Y.X, and XY and YX. But the eight are equivalent only to six: for X.Y and Y.X are the same, and so are XY and YX. Again, there are six simple propositions between x and y, six between X and y, six between x and Y. Taking in contraries, there are then twenty-four apparent modes of forming a simple proposition from X and Y: but these are not all distinct. Eight of them contain all the rest: these eight being the $A_,$, $E_,$, $I_,$, $O_,$, A', E', I', O', above described. This is seen in the following table, the study of which should be carefully made,

$A_,$	X)Y = X.y = y)x	A'	x)y = x.Y = Y)X
$O_,$	X:Y = Xy = y:x	O'	x:y = xY = Y:X
$E_,$	X.Y = X)y = Y)x	E'	x.y = x)Y = y)X
$I_,$	XY = X:y = Y:x	I'	xy = x:Y = y:X

I suppose most readers will readily see the truth of the identities here affirmed: if not, the following mode of illustration (which will be very useful when I come to treat of the syllogism) may be tried. Let U be the name which is the universe of the proposition: and write down in a line as many Us as there are distinct objects to which this name applies. A dozen will do as well for illustration as a million. Under every U which is an X write down X: and x of course, under all the rest. Follow the same plan with Y. The occurrence of letters in the same column shows that they are names of the same object. The following are specimens of the eight standard varieties of assertion, to which all the rest may be referred.

A_1	UUUUUUUUUUUU	A'	UUUUUUUUUUUU
	XXXXXxxxxxxx		XXXXXXXXxxxx
	YYYYYYYYyyyy		YYYYYyyyyyyy
O_1	UUUUUUUUUUUU	O'	UUUUUUUUUUUU
	XXXXXXXxxxxx		XXXXXxxxxxxx
I_1	yyyyYYYYYYyy	I'	YYyyyyyyYYYY
E_1	UUUUUUUUUUUU	E'	UUUUUUUUUUUU
	XXXXxxxxxxxx		XXXXXXXXxxxx
	yyyyyyyYYYYY		yyyyyYYYYYYY

[62] In the first scheme, A_1, there exist twelve Us, the first five of which are both Xs and Ys, the next three Ys but not Xs, the last four neither Xs nor Ys. This case, so constructed that X)Y is true, shows X.y and y)x.

The propositions A_1 and A', X)Y and x)y, may be called *contranominal,* as having each names contrary of those in the other. It appears, then, that as to inconvertibles, contranominal and converse are terms of the same meaning, for X)Y and y)x are the same, and x:y and Y:X. And since it is more natural to speak of direct names than of their contraries, it will be best to attach to A' and O' the ideas of Y)X and Y:X; but not so as to forget their derivation from x)y and x:y. Observe also that each universal proposition has converted contranominals for its *affirmative* forms. Thus X)Y = y)x: and though X.Y is not y.x, yet if we make X.Y take the affirmative form X)y, it is equivalent to Y)x. In particular propositions, the negative forms have the same property. The contranominals of the convertible propositions E_1 and I_1 are of totally different meaning. They have never till now been introduced into logic, and a few words of explanation are wanted.

First as to I' or xy. We here express that some not-Xs are not-Ys, or that there are things in the universe which are neither Xs nor Ys. That is, X and Y are not contraries.[10] Next as to E' or x.y. We here express that no not-X is not-Y, or that everything in the universe is either X or Y, *or both*. These last words are important: by omitting them, we should imagine that x.y signifies that X and Y are contraries; which is not necessarily true.

Accordingly, the eight standard forms of expression, with reference to the order XY, and exhibited in the form in which it will be most convenient to think and speak of them, are as follows,

A_1 or X)Y	Every X is Y	A' or Y)X	Every Y is X
O_1 or X:Y	Some Xs are not Ys	O' or Y:X	Some Ys are not Xs
E_1 or X.Y	No X is Y	E' or x.y	Everything is either X or Y
I_1 or XY	Some Xs are Ys	I' or xy	Some things are neither Xs nor Ys

Returning to the table, we now see the following general laws.

1. Each triad of equivalents contains two inconvertibles and one convertible. 2. Of the four, X, Y, x, y, each of the eight forms [63] speaks universally of two, and particularly of two. 3. A proposition speaks in different ways of each name and its contrary; universally of one and particularly of the other. 4. The propositions called *contradictory*, from the common meaning of this word, may be so called in another sense: for they speak in the same manner of contraries. Thus X)Y speaks universally of X, and par-

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[10] [" Say X and Y are not *complements* (instead of *contraries*) that is, do not together either fill, or more than fill, the universe."—De Morgan, " Table of Contents."—Ed.]

ticularly of Y: its denial, X:Y or y:x, speaks universally of x, and particularly of y.

Any two of the eight forms being taken, it is clear either that they cannot exist together, or that one must exist when the other exists, or that one may exist either with or without the other. The alternatives of each case are presented in the following table.

	Denies	Contains	Is indifferent to		Denies	Is contained in	Is indifferent to
$A_,$	$O_,E_,E'$	$I_,I'$	$A'O'$	$O_,$	$A_,$	$E_,E'$	$A'O'I_,I'$
A'	$O'E'E_,$	$I'I_,$	$A_,O_,$	O'	A'	$E'E_,$	$A_,O_,I'I_,$
$E_,$	$I_,A_,A'$	$O_,O'$	$E'I'$	$I_,$	$E_,$	$A_,A'$	$E'I'O_,O'$
E'	$I'A'A_,$	$O'O_,$	$E_,I_,$	I'	E'	$A'A_,$	$E_,I_,O'O_,$

Let the *concomitants* of a proposition be those to which it is wholly indifferent. Then it appears that each universal has for concomitants its contranominal and the contradictory of the last: but each particular has all for concomitants except only its own contradictory. Each universal denies, besides its own contradictory, the two universals of opposite name; and contains the two particulars of the same name. The two concomitants of a universal may be described as its universal and its particular concomitant.

There is a certain sort of repetition in our choice of the four forms, combined with the four selections XY, Xy, xy, xY. If *any one* of the four forms $A_,$ $E_,$ A' E' be applied to all the above, it will give the four forms derived from XY. Thus the $A_,$ of XY, Xy, xy, xY, are severally the $A_,$, $E_,$, A' and E' of XY; and the E' of XY, Xy, xy, and xY are severally the E', A', $E_,$ and $A_,$ of XY: and so on. It will serve for exercise to verify the above, and still more the cases contained in the following.

There are four things in a proposition, each of which may be changed into its contrary: subject, predicate, order, and copula. Let S be the direction to change the subject into its contrary: P [64] the same for the predicate: let T be the direction to transform the order: and F the direction to change the form, from affirmative to negative, or from negative to affirmative. When T enters, let it be done last, to avoid confusion. Thus SPT performed upon X)Y gives x)Y from S, x)y, from P, and y)x from T; which is X)Y, so that in this case alteration of subject, predicate, and order, is no alteration at all. Let L be the representation of no alteration at all. To investigate equivalent alterations, observe, first, that F and P, singly, are identical: thus F performed on X.Y gives X)Y, and P on X.Y gives X.y. And X)Y = X.y. This perfect identity of F and P in effect, remains in all combinations into which T does not enter. But when T enters, it is S and F which are identical. Thus ST performed on Y)X gives X)y or X.Y: and FT performed on Y)X gives X.Y. The reason is, that T interchanges subject and predicate; so that F, after T, makes a change which is counterbalanced by a change in what was the subject. Accordingly, remembering that each operation performed twice is no operation at all (thus PP is L, and TT is L), we have in all cases

$$\mathrm{P} = \mathrm{F},\ \mathrm{SP} = \mathrm{SF},\ \mathrm{PF} = \mathrm{L},\ \mathrm{SPF} = \mathrm{S}$$
$$\mathrm{ST} = \mathrm{FT},\ \mathrm{SPT} = \mathrm{FPT},\ \mathrm{SFT} = \mathrm{T},\ \mathrm{SPFT} = \mathrm{PT}$$

all which should be tried for exercise. Again, in a *convertible* proposition, transformation is no alteration or T = L: in an *inconvertible* one, transformation changes it into its contranominal; or T = SP. Now set out as follows;— L, in *convertible* propositions is T; which in *inconvertibles*, is SP; which, in *convertibles* again, is SPT; which, in *inconvertibles* again, is TT, or L. Put these down as follows, writing under them the operations which are always equivalent to them, as shewn above,

$$\begin{array}{c||c|c||c|c} \mathrm{L} & \mathrm{T} & \mathrm{SP} & \mathrm{SPT} & \mathrm{L} \\ \mathrm{PF} & \mathrm{SFT} & \mathrm{SF} & \mathrm{PFT} & \mathrm{PF} \end{array}$$

The combinations written under one another are always the same in effect: those separated by double lines have the same effect on convertibles: those separated by single lines, have the same effect on inconvertibles. Again P, for convertibles, is the same as PT; which, for inconvertibles is the same as PSP, or S; which, for convertibles again, is the same as ST; which, for inconvertibles, is SSP or P. These treated as before, give the table

[65]

P	PT	S	ST	P
F	SPFT	SPF	FT	F

In these two cycles there are L and all the fifteen selections which can be made out of S, P, F, T. And every possible case of equivalent changes is contained in these two tables. Thus PT is in all cases equivalent to SPFT; in convertible cases, to P and to F; in inconvertible ones, to S and to SPF. And no other combination is in any case equivalent to PT. In verification of these tables, observe that the operation F always occurs in the lower line, and never in the upper; and that this operation changes convertibles into inconvertibles, and *vice versâ*. We ought then to expect, that the equivalences which, containing F, apply to inconvertibles, will be those which when F is struck out, apply to convertibles; and *vice versâ*. And so we shall find it: for instance, SPFT and SPF are equivalent when performed on inconvertibles; strike out F and we have SPT and SP, which are equivalent when performed on convertibles.

It appears, then, that any change which can be made on a proposition, amounts in effect to L, P, S, or PS. This is another verification of the preceding table: for all our forms may be derived from applying those which relate to XY in the cases of Xy, xY, and xy.

We have seen that $A_{,}$ and A' both contain $I_{,}$ and I'; and that $E_{,}$ and E' both contain $O_{,}$ and O'. Hence each of the universals may be said to be the *strengthened form* of either of its particulars of the same sign: and each of the particulars the *weakened form* of its universals of the same sign. The only distinction which appears between the two forms of the convertible particulars, XY and YX, xy and yx, is that the strengthened forms derived from extending the subjects are different. Thus xy gives x)y or Y)X; but yx gives y)x or X)Y.

A *complex proposition* is one which involves within itself the assertion or denial of each and all of the eight simple propositions. If these eight propositions were all concomitants, or if any number of them might be true, and the rest false, there would be 256 possible cases of the complex proposition. As it is, owing to the connexion established in the table of page 63, there are but *seven*.

[66] First, let the names X and Y be so related that neither of the four universals are true. Then all the four particulars are true: and this is the first case. Let it be called a *complex particular*, and denoted by P. Then, denoting coexistence of simple propositions by writing + between their several letters, we have

$$P = O' + O_{\prime} + I' + I_{\prime}$$

This case is of the least frequent mention in the theory of the syllogism.

Next, let one of the universal propositions be true. Then five of the other propositions are settled, either by affirmation or denial. There remain the two concomitants, which are contradictory; so that only one is true. Accordingly, with the exception of the complex particular just described, every complex proposition must consist of the coexistence of a universal and one of its concomitants. But there are not therefore eight more such propositions: for $A' + A_{\prime}$ and $A_{\prime} + A'$ are the same, and so are $E_{\prime} + E'$ and $E' + E_{\prime}$. The remaining number is then reduced to six, which are

$$A_{\prime} + O', \qquad A_{\prime} + A', \qquad A' + O_{\prime}$$
$$E_{\prime} + I', \qquad E_{\prime} + E', \qquad E' + I_{\prime},$$

These must be separately examined.

First, take $A_{\prime} + A'$ (the order XY always understood). We have then X)Y and Y)X. That is, there is no object whatsoever which has one of these names, but what also has the other. The names X and Y are then *identical*, not as names, but as subjects of application.[11] Where either can be applied, there can the other also. Thus, in geometry (the universe being plane rectilinear figure[12]) equilateral and equiangular are identical names. Not that they agree in etymology nor in meaning: more than this, a few words would explain the first to many who could not comprehend the second without difficulty. But they agree in that what figure soever has a right to either name, it has the same right to the other. It will tend to uniformity of language, if we call X, in this case, *an identical* of Y, and Y *an identical* of X. Let the symbol of an identical be D: then we have

$$D = A_{\prime} + A'$$

[67] Next, take $A_{\prime} + O'$. We have then X)Y and Y:X. Every X is Y, and so far there is a character of identity. But some Ys are not Xs; there are more Ys than Xs, and X stops short of a complete claim of identity with Y. Let X be called a *subidentical* of Y (thus *man* is a subidentical of *animal*), and let $D_{\prime}$ denote this case. Then

$$D_{\prime} = A_{\prime} + O'$$

Let $A' + O_{\prime}$ exist. We have then Y)X and X:Y. Every Y is X, and so far there is identity. But some Xs are not Ys, there are more Xs than Ys, or X goes beyond a claim of identity with Y. Let X be now called a *superidentical* of Y, and let it be denoted by D'. Then

$$D' = A' + O_{\prime}$$

The terms superidentical and subidentical are obviously correlative. If X be either of Y, Y is the other of X. Now let us consider $E_{\prime} + E'$. We have then X.Y and x.y. There is nothing which is both X and Y, there is nothing which is neither. Consequently X and Y are *contraries*, or just fill up the universe. Let C be the mark of this relation. Then

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[11] [There is apparently an ambiguity in the use of the word "name" in this paragraph. Where it first occurs it appears to mean "sounds" or "vocables"; on its second appearance it obviously means "subjects of application."—Ed.]

[12] [De Morgan apparently means "triangles," though he writes by an oversight "plane rectilinear figure." Manifestly all rectangles are equiangular, though not necessarily equilateral, and the rhombus is equilateral but not equiangular.—Ed.]

$$C = E_{\prime} + E^{\prime}$$

Next, take $E_{\prime} + I^{\prime}$. We have then X.Y and xy. Nothing is both X and Y, but there are things which are neither. X and Y are clear of one another, but do not amount to contraries, for they do not fill up the universe. Let them be called *subcontraries* (thus in the universe *metal, gold* and *silver* are subcontraries), and let $C_{\prime}$ denote the relation. Then

$$C_{\prime} = E_{\prime} + I^{\prime}$$

Lastly, take $E^{\prime} + I_{\prime}$. We have x.y and XY. The names fill the universe; for there is nothing but what is either X or Y. But they *overfill* it; for some things are both Xs and Ys. There is then all the completeness of a contrary and more. Let X and Y be called *supercontraries*,* and let $C^{\prime}$ denote the relation. Then we have

$$C^{\prime} = E^{\prime} + I_{\prime}$$

[68] To complete our language, let $A_{\prime}$ or X)Y, with reference to the order XY, be called *sub-affirmative*; and $A^{\prime}$ or Y)X, *superaffirmative*. Let $E_{\prime}$ or X.Y be called *subnegative*; and E or x.y, *supernegative*. Let the particulars $I_{\prime}$, $I^{\prime}$, and $O_{\prime}$, $O^{\prime}$, have also these several names. This extension of our language will require a little explanation.

When I say that X is a subidentical of Y, I mean that the etymological suggestions are actually satisfied. The whole name X, and more, is contained in Y. But when I say that X is a universal sub*affirmative* of Y, or X)Y, I mean no more than that we have the proposition whose form is not superaffirmative, according to the etymology of that word. An algebraist would well understand the distinction at a glance. He has often to distinguish the case in which a is less than b from that in which a is less than or equal to b: the case in which the extreme limit of the assertion is not included from that in which it is included.

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* The *supercontrary* relation, though essential to a complete system of syllogism, is not frequently met with. The other extreme of the supercontrary, or the subidentical, is so much the easiest of all our complex relations, that the latter rarely allows the former to appear. The first instance that suggested itself to me was *man* and *irrational* (as descriptive of the quality of the individual and not of the species) in the universe *animal*. These more than fill that universe, *idiot* being common to both. But it is more natural to say that *rational* (in this sense) is subidentical of *man*.

Again, the word negative had better be viewed as not so much presenting *exclusion* for its first idea, as *inclusion in the contrary*. Thus a subnegative, when universal, is to suggest complete inclusion in the contrary, meaning the extreme case, possibly; namely, that the subnegative names may be contraries. Again, supernegative is to suggest the idea of supercontrary, with the lowest extreme, the relation of contrary, possibly included.

For exercise in this language, and in the ideas which it is meant to present, I now state the following results.

Universal affirmation, though as a general term, it is to include super and sub affirmation, yet looked at as one of the three, and distinguished from the rest, it means identity. The same of negation and contrariety. Subidentity requires universal subaffirmation and particular supernegation. Identity is universal sub and super affirmation, both. Superidentity requires universal superaffirmation and particular subnegation. Subcontrariety requires universal subnegation and particular superaffirmation. Con-[69]trariety is universal sub and super negation, both. Supercontrariety requires universal supernegation and particular subaffirmation. Again, universal subaffirmation is either subidentity or identity: particular subaffirmation is a denial of contrariety and subcontrariety. Universal superaffirmation is either superidentity or identity: particular superaffirmation denies contrariety and supercontrariety. Universal subnegation is either subcontrariety or contrariety: particular subnegation denies subidentity and identity. Universal supernegation is either supercontrariety or contrariety: particular supernegation denies superidentity and identity. All this is expressed in the following table,

	$D_,$	affirms	$A_,$ *and* O'	$A_,$	affirms	$D_,$ *or* D
	D	—	$A_,$ and A'	A	—	$D_,$ or D or D'
	D'	—	A' and $O_,$	A'	—	D' or D
	$C_,$	—	$E_,$ and I'	$E_,$	—	$C_,$ or C
	C	—	$E_,$ and E'	E	—	$C_,$ or C or C'
	C'	—	E' and $I_,$	E'	—	C' or C
Denial of	$D_,$	—	A' *or* $O_,$	$O_,$	denies	$D_,$ *and* D
—	D	—	O' or $O_,$	O	—	$D_,$ and D, or D' and D
—	D'	—	$A_,$ or O'	O'	—	D' and D
—	$C_,$	—	E' or $I_,$	$I_,$	—	$C_,$ and C
—	C	—	I' or $I_,$	I	—	$C_,$ and C or C' and C
—	C'	—	$E_,$ or I'	I'	—	C' and C

Every subidentical of a name is the subcontrary of its contrary; every subcontrary is the subidentical of the contrary. Treat the word contrary as negative, the word identical as positive; and the two as of different signs. Then the algebraical rule " like signs give a positive, unlike signs a negative," holds in every case: including the variety of it so well known as " two negatives make an affirmative." When the modifying preposition comes first it must be retained; when it comes second, it must be changed. Thus the subcontrary of a contrary is a subidentical: but the contrary of a subcontrary is a superidentical. In putting two relations together, however, we have got into syllogism, as we shall presently see.

The following tables will show a connexion between the expressions, for different orders and selections, which it may be useful to verify.

[70]

XY	YX	xY	Yx	Xy	yX	xy	yx
$A_,O'D_,$	$A'O_,D'$	$E'I_,C'$	$E'I_,C'$	$E_,I'C_,$	$E_,I'C_,$	$A'O_,D'$	$A_,O'D_,$
$A'O_,D'$	$A_,O'D_,$	$E_,I'C_,$	$E_,I'C_,$	$E'I_,C'$	$E'I_,C'$	$A_,O'D_,$	$A'O_,D'$
$E_,I'C_,$	$E_,I'C_,$	$A'O_,D'$	$A_,O'D_,$	$A_,O'D_,$	$A'O_,D'$	$E'I_,C'$	$E'I_,C'$
$E'I_,C'$	$E'I_,C'$	$A_,O'D_,$	$A'O_,D'$	$A'O_,D'$	$A_,O'D_,$	$E_,I'C_,$	$E_,I'C_,$

This table only contains some of the rules already laid down in pp. 64, 65. It expresses that, for instance, the $A_{\prime}$, $O^{\prime}$, and $D_{\prime}$ of XY, are severally the same as the $E_{\prime}$, $I^{\prime}$, and $C_{\prime}$ of yX. This table may be exhibited thus, the identicals counting as inconvertibles, the contraries as convertibles.

Change of	In Convertibles, changes	In Inconvertibles, changes
Subject	Sign and Preposition	Sign and Preposition
Predicate	Sign	Sign
Subject and Predicate	Preposition	Preposition
Order	Neither	Preposition
Subject and Order	Sign	Sign and Preposition
Predicate and Order	Sign and Preposition	Sign
Subject, Predicate, and Order	Preposition	Neither

In all cases, change of subject is change both of sign and preposition ; change of predicate is change of sign ; change of subject and predicate is change of preposition. These three cases are of great importance in the syllogism : and the reader would do well to connect in his mind

Subject with *Sign and preposition*
Subject and Predicate — *Preposition*
Predicate — *Sign*

It is desirable to consider the several complex relations as to the continuous transition from one into another : *the growth of names* concerns not only the etymologist, but the logician also.

With the analogies and affinities by which the dominion of one name is extended to instance after instance, and class after class—and sometimes, in scientific language at least, deprived of a part of what it has held—I have here nothing to do. It is enough that the phenomena exist which may be described as the gradual transformation of one relation into another. The words *butt* and *bottle*, for example, are now subcontraries in the universe *receptacle :* but the etymology of the second word shows [71] that it *was* a subidentical of the first, being a diminutive. And if we were to take the whole class butt, buss, boot, bushel, box, boat, bottle, pottle, &c., which are all of one origin, the number of transitions would be found to be very large.

I assume that all the instances of a name are counted and arranged in its universe: a conceivable, though not attainable, supposition. Also, that the instances of the name are arranged contiguously, as in page 61. Whatever the reason may be which dictates the particular arrangement chosen, it will generally happen that the instances near to the boundary possess the characteristics of the name in a smaller degree than those nearer the middle. Let the contiguous arrangement be made of all the instances of the name Y, the universe being U. Let another name X begin to grow, commencing with one instance, that is, being applied to one of the objects in the universe U, be it a Y or not; then to another contiguous, and so on. We are to enumerate the ways in which such changes, whether of increase or diminution, may cause one name to change its relation to another. According as the change is made by accession or retrenchment, it may be denoted by $(+)$ or $(-)$.

Let the name X begin within the limits of the name Y: its initial relation to Y is then $D_{\prime}$. And the possibility of the following continuous changes is obvious:

$$\begin{array}{ll} D_{\prime}\ (+)\ D\ (+)\ D' & \qquad D_{\prime}\ (+)\ P\ (+)\ C' \\ D_{\prime}\ (+)\ P\ (+)\ D' & \qquad D_{\prime}\ (+)\ P\ (-)\ C_{\prime} \end{array}$$

Hence $D_{\prime}$ may become D' through either D or P, but $C_{\prime}$ or C' only through P. Next, let X begin without the limits of Y: the initial relation is $C_{\prime}$. We may have then

$$\begin{array}{ll} C_{\prime}\ (+)\ C\ (+)\ C' & \qquad C_{\prime}\ (+)\ P\ (+)\ D' \\ C_{\prime}\ (+)\ P\ (+)\ C' & \qquad C_{\prime}\ (+)\ P\ (-)\ D_{\prime} \end{array}$$

Let X begin both within and without Y: its initial relation is then P. And we have

$$P\ (+)\ D',\ P\ (+)\ C',\ P\ (-)\ D_{\prime},\ P\ (-)\ C_{\prime}$$

But when (−) follows $D_{,}$ or D, $C_{,}$ or C, we have nothing except

$$D_{,}(-)D_{,},\quad D(-)D_{,},\quad C_{,}(-)C_{,},\quad C(-)C_{,}$$

[72] If we begin at the other extreme, with the name U, we have

$$U(-)D' \qquad U(-)C'$$

Beginning from D' and C' we have

$$D'(-)D(-)D_{,} \qquad D'(-)P(-)C_{,}$$
$$D'(-)P(-)D_{,} \qquad D'(-)P(+)C'$$
$$C'(-)C(-)C_{,} \qquad C'(-)P(-)D_{,}$$
$$C_{,}(-)P(-)C_{,} \qquad C'(-)P(+)D'$$

But when (+) follows D' or D, C' or C, we have only

$$D'(+)D',\quad D(+)D',\quad C'(+)C',\quad C(+)C'$$

From the above list it appears that the transition which is accompanied by a change of preposition only can be made either through the letter without preposition or through P: and in all cases with one continued mode of alteration. But when the transition involves change of letter, it can only be made through P: with continuation of the mode of alteration when the prepositions are different, and change in the mode when they are the same. The following successions contain the arrangement of the results.

With one alteration (+)	With one alteration (−)	With two alterations (+ −)
$D_{,}DD'$	$D'DD_{,}$	$D_{,}PC_{,}$
$D_{,}PD'$	$D'PD_{,}$	$C_{,}PD_{,}$
$C_{,}CC'$	$C'CC_{,}$	— —
$C_{,}PC'$	$C'PC_{,}$	(− +)
———	———	$D'PC'$
$D_{,}PC'$	$D'PC_{,}$	$C'PD'$
$C_{,}PD'$	$C'PD_{,}$	

The following considerations will further serve to illustrate the want of the extension of the doctrine of propositions made in this chapter, and also the completeness of it.

Among our most fundamental distinctions is that of *necessity* and *sufficiency* ; of what we *cannot do without,* and what we *cannot fail with* ; of that which *must precede,* and that which *must follow.*[13] The contraries of these are *non-necessity* and *non-sufficiency.* In these four words, applied to both Y and y, we have the description of the eight re-[73]lations of X to Y. For instance $A_\prime$ or X)Y tells us that to have an X, we must take a Y, or to be X, it is *necessary* to be Y. Treating all in the same way, we have

$A_\prime$	X) Y	To take an X it is	*necessary*	to take a Y
A'	Y) X	. . . . X . .	*sufficient*	. . . Y
$E_\prime$	X . Y	. . . . X . .	*necessary*	. . . y
E'	x.y	. . . . X . .	*sufficient*	. . . y
$I_\prime$	X Y	. . . . X .	*not necessary*	. . . y
I'	x y	. . . . X .	*not sufficient*	. . . y
$O_\prime$	X : Y	. . . . X .	*not necessary*	. . . Y
O'	Y : X	. . . . X .	*not sufficient*	. . . Y

And the convertibility of the ordinary mode of description with this new one may be easily shown in any case. For example, what can we mean by saying that to take an X, it is not sufficient to take what is not Y ? Clearly that by taking not Y, or y, we may at the same time take an x, or that there are xs which are ys. And so on for the rest.

Of the four pairs XY, Xy, xy, xY, we know that each proposition may be expressed by three, and refuses to be expressed by one. If we now admit the two words *impossible* and *contingent,* meaning by the latter that which, as the case may be, is possible or impossible, we shall easily see the following table for the universals :

		XY	Xy	xy	xY
$A_\prime$	X) Y	N	I	S	C
$E_\prime$	X . Y	I	N	C	S
A'	Y) X	S	C	N	I
E'	x . y	C	S	I	N

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[13] [See Table of Contents, additions and corrections to ch. IV.—Ed.]

The letters N, I, S, C, are the initials of necessary, &c. And we read in the first line, that if X) Y, then to be X it is necessary to be Y ; to be X, it is impossible to be y ; to be x it is sufficient to be y ; and to be x, it is contingently possible or impossible to be Y. Again, if by *n* and *s* we mean *not necessary* and *not sufficient ;* by P, actually possible ; and by C, as before (C being its own contrary), we have the following table for the particulars :

[74]

		XY	Xy	xy	xY
$O_,$	X : Y	n	P	s	C
$I_,$	X Y	P	n	C	s
O'	Y : X	s	C	n	P
I'	xy	C	s	P	n

Of the four contrary pairs, n, P, s, C, are related to the particulars precisely as N, I, S, C, are to the universals. The interchange of Y and y is always accompanied by the interchange of N and I, S and C, n and P, s and C ; the interchange of X and x is that of N and C, S and I, n and C, s and P ; of both X and x, Y and y, is that of N and S, C and I, n and s, C and P.

The complex relations may be thus described. According as X is subidentical, identical, or superidentical of Y, to be X it is necessary and not sufficient, necessary and sufficient, or not necessary and sufficient, to be Y : according as X is subcontrary, contrary, or supercontrary of Y, to be X it is necessary and not sufficient, necessary and sufficient, or not necessary and sufficient, to be y. Or, as in the following table :

	XY	Xy	xy	xY
$D_,$	Ns	I	Sn	P
$C_,$	I	Ns	P	Sn
D'	Sn	P	Ns	I
C'	P	Sn	I	Ns
D	NS	I	NS	I
C	I	NS	I	NS
P	nsP	nsP	nsP	nsP

Instead of IC and PC, write I and C: for "impossible, and possible or impossible as the case may be" is "impossible," &c.

The names of the complex relations, subidentity, identity, &c. I suppose will be held tolerably satisfactory: those of the simple relations suggested in page 68, subaffirmative &c. have nothing in their favor except analogy with the former, and close connexion with the notation. A little practice in their use might render these last names available: but it will be advisable to con-[75]nect them with names more descriptive of the meaning, and to adopt these last, whether we reject or maintain their synonymes.

When X) Y, the relation of X to Y is well understood as that of the *species* to the *genus*. We may adopt these words, with the understanding that the word species includes the extreme case in which the species is as extensive as the genus. When X : Y, we may call X a *non-species* of Y, and Y a *non-genus* of X. When X . Y we may call X an *exclusive* or *excludent* of Y, or else a *non-participant;* and also Y of X. When XY, we may say that each is *participant*, or *non-exclusive*, of the other. When x . y, which means that X and Y together fill up, or more than fill up, the universe, we may say that they are *complemental* names. When x y, which only means that X and Y do not between them contain the universe, we may call them *non-complemental*. We have then

Inconvertibles.		Name of X with respect to Y.
A_1	X)Y	species, or subaffirmative.
O_1	X:Y	non-species, or particular subnegative.
A'	Y)X	genus, or superaffirmative.
O'	Y:X	non-genus, or particular supernegative.

Convertibles.		Name of X and Y with respect to each other.
E_1	X.Y	Exclusives, or non-participants, or subnegatives.
I_1	XY	Non-exclusives, or participants, or particular subaffirmatives.
E'	x.y	Complements, or supernegatives.
I'	xy	Non-complements, or particular superaffirmatives.

The following exercises in these terms, really contain the description of all the syllogisms in the next chapter.

Inclusion in the species is inclusion in the genus; and inclusion of the genus is inclusion of its parts (species or not).

Exclusion from the genus is exclusion from the species; and exclusion of the genus is exclusion of its parts (species or not).

Inclusion or exclusion of the species is part inclusion or exclusion of the genus.

When the species is complemental, so is the genus: and when the genus is not complemental, neither is the species.

Exclusion from one complement is inclusion in the other.

Complements of the same are participants.

[76] Two species of one genus, are not complements; neither are two exclusions from the same.

The complement of a genus is a non-species; and the complement is a non-species of the non-complement.

CHAPTER V.

ON THE SYLLOGISM.

A SYLLOGISM is the inference of the relation between two names from the relation of each of those names to a third. Three names therefore are involved, the two which appear in the conclusion, and the third or *middle term*, with which the names, or terms, of the conclusion are severally compared. The statements expressing the relations of the two *concluding* terms to the *middle* term, are the two *premises.* In this chapter, no ratio of quantities is considered except the definite *all* and the indefinite *some.*

A syllogism may be either *simple* or *complex.* A syllogism is *simple* when in it two simple propositions produce the affirmation or denial of a third: or the affirmation of a third, we may say, since every denial of one simple proposition is the affirmation of another. A *complex* syllogism is one in which two complex propositions produce the affirmation or denial of a third complex proposition.

It might be supposed that we ought to begin with the simple syllogism, and from thence proceed to the complex. On this point I have some remarks to offer, in justification of following precisely the reverse plan.

Hitherto the complex syllogism has never made its appearance in a work on logic, except in one particular case, in which it is allowed to be treated as a simple syllogism, though most obviously it is not so. I allude to the common *à fortiori* argument, as in "A is greater than B, B is greater than C, therefore A is greater than C." There is no middle term here: the predicate of the first proposition is "a thing greater than B," the subject of the second proposition is "B."

Admitting fully that the quality of the premises,—that which [77] entitles the conclusion to be made, as it is said, *à fortiori*—marks this argument out as, if anything, stronger, clearer, and (could such a thing be) truer, than a simple syllogism; yet it is plain that the very additional circumstance on which this additional clearness depends, takes the argument out of a syllogism, as defined by all writers. By beginning with the complex syllogism, and thence descending to the simple one, it will be seen that we begin with cases which present this *à fortiori* and clearer character. I think I shall shew that the complex syllogism is easier than the simple one.

Next, the syllogism hitherto considered has never involved any contrary terms; the consequence of which has been that various legitimate modes of inference have been neglected. Moreover, several of the usual syllogisms are more strong than need be in the premises, in order to produce the conclusion. Thus Y)X and Y)Z being admitted as premises, the necessary conclusion is XZ. But if Y)X be weakened into YX, the same conclusion follows. If we call a syllogism *fundamental*, when neither of its premises are stronger than is necessary to produce the conclusion, it is obvious that every fundamental syllogism which has a particular premise, gives at least as strong a conclusion when that particular is strengthened into a universal. But, except when strengthening the premise also enables us to strengthen the conclusion, in which case we have a new and different syllogism, it seems hardly systematic to mix with fundamental arguments syllogisms which have quality or quantity more than is necessary for the conclusion.

The use of the complex syllogism will, as we shall see, give an independent and systematic derivation to these strengthened syllogisms, as well as to the rest.

Let X and Z be the terms of the conclusion; and let Y be the middle term. Let the premise in which X and Y are compared come first of the two. Let the order of reference in each case be that of the alphabet

XY YZ XZ

So that by stating what X is with respect to Y, and what Y is with respect to Z, our syllogism involves the statement of what X therefore must be, or therefore cannot be, with respect to Z. We can, in every case, express the result in simple words. Thus, [78] one of our syllogisms being what I shall represent by $D_{,}D_{,}D_{,}$ is as follows. If X be a subidentical of Y, and Y a subidentical of Z, then X is a subidentical of Z. But all this merely amounts to the following "A subidentical of a subidentical is a subidentical."

We have then to examine every way in which $D_{,}$ or D' or $C_{,}$ or C' can be combined with $D_{,}$ or D' or $C_{,}$ or C', giving sixteen cases in all, and all conclusive in one way or the other. Instead of taking an accidental order, and afterwards classifying the results, it will be better to predict the order which will give classification. That order will be to take 1. a D followed by another of the same preposition 2. a C followed by another of different preposition 3. a D followed by another of a different preposition. 4. a C followed by another of a like preposition. This arrangement gives us

1.	$D_{,}D_{,}$	$D'D'$	$D_{,}C_{,}$	$D'C'$	3.	$D_{,}D'$	$D'D_{,}$	$D_{,}C'$	$D'C_{,}$
2.	$C_{,}D'$	$C'D_{,}$	$C_{,}C'$	$C'C_{,}$	4.	$C_{,}D_{,}$	$C'D'$	$C_{,}C_{,}$	$C'C'$.

Each of these cases will be examined by a method similar to that proposed in page 61. But a clear perception of the meaning of the words will at once dictate the sixteen results, which are as follows, preceded by the mode in which the syllogisms are to be expressed.

$D_{,}D_{,}D_{,}$	Subidentical of subidentical is subidentical.
$D'D'D'$	Superidentical of superidentical is superidentical.
$D_{,}C_{,}C_{,}$	Subidentical of subcontrary is subcontrary.
$D'C'C'$	Superidentical of supercontrary is supercontrary.

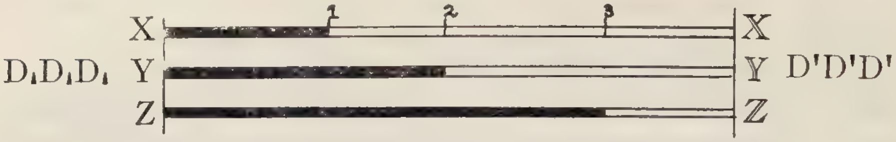
$C_{,}D'C_{,}$	Subcontrary of superidentical is subcontrary.
$C'D_{,}C'$	Supercontrary of subidentical is supercontrary.
$C_{,}C'D_{,}$	Subcontrary of supercontrary is subidentical.
$C'C_{,}D'$	Supercontrary of subcontrary is superidentical.
$D_{,}D'{:}C'$	Subidentical of superidentical is *not* supercontrary.
$D'D_{,}{:}C_{,}$	Superidentical of subidentical is not subcontrary.
$D_{,}C'{:}D'$	Subidentical of supercontrary is not superidentical.
$D'C_{,}{:}D_{,}$	Superidentical of subcontrary is not subidentical.
$C_{,}D_{,}{:}D'$	Subcontrary of subidentical is *not* superidentical.
$C'D'{:}D_{,}$	Supercontrary of superidentical is not subidentical.
$C_{,}C_{,}{:}C'$	Subcontrary of subcontrary is not supercontrary.
$C'C'{:}C_{,}$	Supercontrary of supercontrary is not subcontrary.

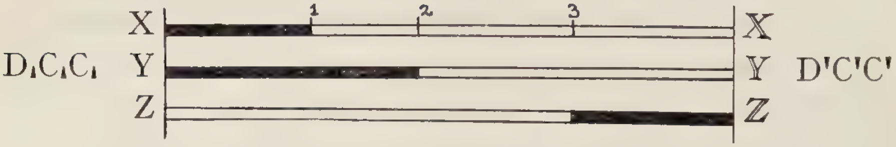
[79] In the denials, the extreme limit is included: in the affirmations it is not. Thus "not superidentical" and "not subidentical" both include "not identical;" and the same of contraries. In the affirmations, extreme limitation of *one* premise does not alter the conclusion: but that of *both* reduces the conclusion to its extreme limit. Thus

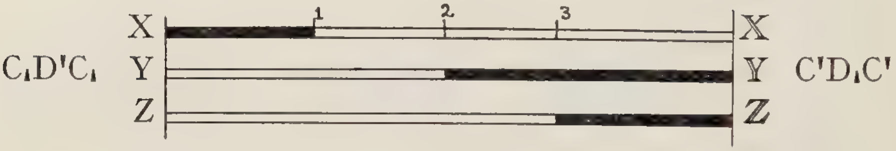
Subcontrary of identical is subcontrary.
Contrary of superidentical is subcontrary.
Contrary of identical is contrary.

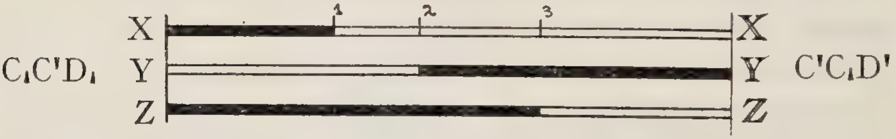
and so on. The rules of this species of syllogism are as follows. *For affirmatory conclusions*;—(1.) Like names in the premises give D in the conclusion, and unlike names C. (2.) D in the first premise requires premises of the same preposition; C in the first premise, of different prepositions. (3.) The preposition of the conclusion agrees with that of the first premise. *For negatory conclusions*, the preceding rules are reversed. These rules will do for the present, but they afterwards merge in others.

The sixteen forms of complex conclusion above given are of the clearness of axioms, as soon as the terms are distinctly apprehended. The following diagrams will assist, and should be used until the propositions suggest their own meaning. Though there be four, yet these four are really but one, as will be shown.









[80] In each diagram are three lines, partly thick and partly open: these are meant to be laid over one another, but are kept separate for distinctness. A point on the first line signifies a X or a x; and one on the second or third, a Y or a y, and a Z or a z. The universe of the propositions is supposed to be the whole breadth. Points which come under one another are supposed to represent the same object of thought, variously named. Thus in the first diagram, when the *thick* lines contain the points named X, Y, and Z, it is shown that we mean to say there are objects to which all the three names apply: for there are points under one another in the thick part of all the three lines.

When we read by the letters on the left, the thick lines are meant to represent the parts in which the Xs, Ys, and Zs must be placed: and when by those on the right, the open lines. Accordingly, looking at the third diagram, and at the left, we see $C_{,}D'C_{,}$: while in the diagram, it is clear that X is a subcontrary of Y, or that X . Y and x y; and that Y is a superidentical of Z, or that Z) Y and Y : Z.

And the conclusion is equally manifest, namely, that X is a subcontrary of Z. But, looking at the left, and seeing C'D͵C', we take the open parts to represent the spaces in which Xs, Ys, and Zs are found, and the thick parts for those in which xs, ys, and zs are found. Here then we see that X is a supercontrary of Y, that Y is a subidentical of Z, and that, *consequently*, X is a supercontrary of Z.

Some attempts at laying down the premises so as to evade the conclusions, will be instructive to any one who does not immediately see the latter. And formal demonstration is always practicable. Thus if X be a subcontrary of Y, that is, if X and Y do not fill the universe, and have nothing in common; and if Y be a superidentical of Z, or entirely contain Z, without being filled by it: then it is clear that X must be more a subcontrary of Z than of Y, by all the instances which there are of a Y not being a Z. The diagram, however, is so much clearer than this sort of demonstration, that the reader, until he has great command of the language, may as well look to the former to see that he is right in the latter.

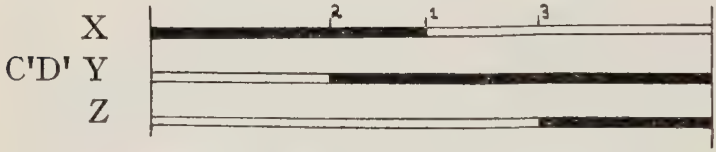
It may be convenient, as a matter of language, to speak of a name as a kind of collective whole, consisting of instances. And thus we may talk of one name being entirely in another, or partly in and partly out &c., as in fact we have already done.

[81] All the complex syllogisms which conclude by affirmation are obviously of the *à fortiori* character: I should rather say, those of the first three diagrams properly and obviously, those of the fourth by an easy extension of language. The marks 1 2 3 in the middle of the diagrams show how this is. In the first, on the left, X is more of a subidentical of Z than it is of Y: the instances in which its *sub*-identity appears consist of all those which prove the subidentity of X to Y, *together with* all those which prove the subidentity of Y to Z. In the third, read from the right, X is more supercontrary to Z than it is to Y, by all the instances which show the subidentity of Y to Z. In

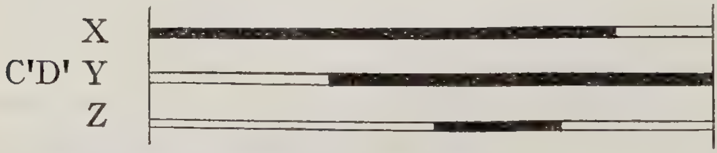
the fourth diagram (from the left) we cannot say that X is more subidentical of Z than of something else, simply because there is no previous subidentity among the relations. But still the distinguishing characteristic of the conclusion takes its quantity from the addition of those of both the premises.

If either of the premises be brought to the limit which separates it from the relation of an opposite preposition; that is, if C' or $C_{,}$ be changed into C, or else D' or $D_{,}$ into D: the nature of the conclusion is not altered, except by the loss of the *à fortiori* character. One of the quantities which have hitherto contributed to the quantity of the conclusion, now disappears. Thus $C_{,}$ D gives $C_{,}$ as well as $C_{,}$ D'; and C D' gives $C_{,}$ as well as $C_{,}$ D'; $C_{,}$ C gives $D_{,}$ as well as $C_{,}$ C'.

Let one of the premises pass over the limit, and take the opposite preposition. Choose $C_{,}$ D', which gives $C_{,}$, and continues to give it, though weakened, when the first $C_{,}$ becomes C. Then let $C_{,}$ become C': so that our premises are C' D'. The diagram is then as follows



The *quantity* of the conclusion now depends upon the *difference* between the number of instances in (12) and (23) and its *quality* upon whether (12) has fewer instances than (23), or the same number, or more. As I have drawn it, $C_{,}$ is the conclusion, still: strengthen the first premise still more, and the conclusion **[82]** will pass through C into C' or else into P, and in the second case may pass into D', as in the following diagram



Nothing is impossible except $D_{,}$ or D. Hence C' D' enables us only to deny $D_{,}$ and its limit D. Treat the other cases in the same manner, and, remembering that denial is to include denial up to the limit (while affirmation only affirms to any thing short of the limit) we have

$D_{\prime}\,D^{\prime}$ denies $C^{\prime}$	$D^{\prime}\,D_{\prime}$ denies $C_{\prime}$
$D_{\prime}\,C^{\prime}$ . . $D^{\prime}$	$D^{\prime}\,C_{\prime}$ . . $D_{\prime}$
$C_{\prime}\,D_{\prime}$ . . $D^{\prime}$	$C^{\prime}\,D^{\prime}$ . . $D_{\prime}$
$C_{\prime}\,C_{\prime}$ . . $C^{\prime}$	$C^{\prime}\,C^{\prime}$ . . $C_{\prime}$

The rules given above in page 79 may be collected from the instances.

As long as we keep contraries out of view, the ultimate element of inference is of a twofold character. It is either "X and Z are both Y; therefore X is Z" or else "X is Y and Z is not Y; therefore X is not Z": X, Y, Z, being single instances of three names; and Y the same instance in both premises. But the use of contraries enables us to give an affirmative form to the latter case. It is "X is Y, and not-Z is Y"; therefore "X is not-Z."

Connected with this change of expression is the following theorem: that all the eight affirmatory complex syllogisms are reducible to any one among them: and the same of the negatory ones. The reader may trace this theorem to the order of the figures 1, 2, 3, being the same in all the four diagrams. Taking $D_{\prime}D_{\prime}D_{\prime}$ as the most simple and natural form, and looking at the diagram of $C_{\prime}D^{\prime}C_{\prime}$, we see the last as $D_{\prime}D_{\prime}D_{\prime}$ in "X is subidentical of y; y is subidentical of z; therefore X is subidentical of z." If we write the terms of the syllogism after its descriptive letters, as in $D_{\prime}D_{\prime}D_{\prime}$ (XYZ) we have the following results;—

$D_{\prime}D_{\prime}D_{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(XYZ)$	$D^{\prime}D^{\prime}D^{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(xyz)$
$D_{\prime}C_{\prime}C_{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(XYz)$	$D^{\prime}C^{\prime}C^{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(xyZ)$
$C_{\prime}D^{\prime}C_{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(Xyz)$	$C^{\prime}D_{\prime}C^{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(xYZ)$
$C_{\prime}C^{\prime}D_{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(XyZ)$	$C^{\prime}C_{\prime}D^{\prime}\,(XYZ)=D_{\prime}D_{\prime}D_{\prime}\,(xYz)$

[83] Thinking of the first description only as to relations, and of the second only as to terms, we see the following rules of connexion. In the first and second premises and

terms, there are X and Y in the terms, or their contraries, according as there are subaccents or superaccents in the relations. But in the conclusion, the term is Z for $D_{\prime}$ and $C_{\prime}$, z for D' and $C_{\prime}$. And we may thus reduce any syllogism involving any one of the eight varieties of relation combined with any one of the varieties of terms, either to $D_{\prime}D_{\prime}D_{\prime}$ or to XYZ. Thus $C_{\prime}D'C_{\prime}$ (XyZ) is $D_{\prime}D_{\prime}D_{\prime}$ (XYz), or $D_{\prime}C_{\prime}C_{\prime}$ (XYZ). Not to load the subject with demonstration of forms, I will give at once the general rules by which changes of accent and letter are governed: remarking that they apply throughout the whole of my system.

The varieties in question are eight:

XYZ, xyz; xYZ, Xyz; XyZ, xYz; XYz, xyZ.

in which (thinking of XYZ) all are kept; or all changed; or one only kept; or one only changed. Learn to connect each letter with the propositions in which it occurs; marking the propositions, premises and conclusion, as 1, 2, 3. Connect X with 1,3; Y with 1,2; Z with 2,3. Keeping all, or changing all, makes no alteration of letters: keeping only one, or changing only one, alters the letters in the premises in which that one occurs. Thus, be the accents what they may, if in DDD we change only the first letter into its contrary, the syllogism becomes CDC; and the same if we keep only the first letter unchanged.

As to accents, remember that change of Z produces no effect: look then only at X and Y. When either letter is changed into its contrary, change the accents belonging to the premises in which that letter comes first; 13 for X, 2 for Y, 123 for XY. For example, what is $C_{\prime}C'D_{\prime}$ (Xyz). Here, as to letters, X alone (1,3) is unchanged: then CCD becomes DCC. As to accents, Y is changed, which comes first only in 2: change C' into $C_{\prime}$. Hence $C_{\prime}C'D_{\prime}$ (Xyz) $=$ $D_{\prime}C_{\prime}C_{\prime}$ (XYZ). Here we have passed from a syllogism in Xyz to the corresponding equivalent in XYZ: the rules equally hold for the inverse process, and for all combinations of letters. For the change of XYZ into Xyz, and that of Xyz into XYZ, have only one description: the first only left unchanged. Now suppose it required to know [84] what syllogism in xYz answers to $D_\prime C_\prime C_\prime(Xyz)$. The key words are, *the third only unchanged*. Alter then DCC into DDD by the first rule, and change all the accents. Thus $D_\prime C_\prime C_\prime\ (Xyz) = D'D'D'\ (xYz)$. The independent rules are that change of subject only, changes *both* letter and accent; predicate only, letter; subject and predicate, accent. Thus to find what $D'C'C'(xYz)$ is, expressed in XYz, the changes are, in the three premises S, neither, S, and $D'C'C'(xYz) = C_\prime C'D_\prime(XYz)$. The following table may be verified for exercise: it shows the effect of all changes except that of the middle term.

XYZ	xYZ	XYz	xYz
$D_\prime D_\prime D_\prime$	$C'D_\prime C'$	$D_\prime C_\prime C_\prime$	$C'C_\prime D'$
$C'D_\prime C'$	$D_\prime D_\prime D_\prime$	$C'C_\prime D'$	$D_\prime C_\prime C_\prime$
$D_\prime C_\prime C_\prime$	$C'C_\prime D'$	$D_\prime D_\prime D_\prime$	$C'D_\prime C'$
$C'C_\prime D'$	$D_\prime C_\prime C_\prime$	$C'D_\prime C'$	$D_\prime D_\prime D_\prime$

Similarly, $D'D'D'$ would have $C_\prime D'C_\prime$ $D'C'C'$ &c. When the middle term only is changed, the table may stand thus;—

XYZ	$D_\prime D_\prime D_\prime$	$C'D_\prime C'$	$D_\prime C_\prime C_\prime$	$C'C_\prime D'$
XyZ	$C_\prime C'D_\prime$	$D'C'C'$	$C_\prime D'C_\prime$	$D'D'D'$

It will of course have been observed that the eight syllogisms go in pairs, each one of a pair differing from the other in accentuation, and nothing else. When we take sets of four, the ones put together should be those in which the first premise, or the second, or the conclusion (whichever we take for a standard) has $D_\prime$ and C', or else has D' and $C_\prime$.

The same rules of transformation apply to negatory complex syllogisms; thus $D'D_\prime{:}C_\prime(XYZ)$ is $C'D'{:}D_\prime(Xyz)$. In fact these rules do not depend upon the character of the inference, nor even upon its validity, but merely on the effects produced in the single propositions by changes of term. Thus the statement $D'D_\prime C_\prime\ (XYZ)$, an invalid inference, is the same statement (equally invalid of course) as is expressed in $D_{\prime}C'D'(xyZ)$.

An examination of the complex particular relation $P = I_{\prime} + I' + O_{\prime} + O'$, whether by the diagram or by unassisted thought, will show that when this relation exists between X and Y, it also exists between x and Y, X and y, x and y. Hence PC, CP, PD, DP, [85] give P. Moreover, two complex particulars give no possibility of any conclusion, all being equally possible. Thus PP may give $C_{\prime}$ or C or C' or $D_{\prime}$ or D or D'.

Now combine one of the others, as $D_{\prime}$, with P: examine $PD_{\prime}$ and $D_{\prime}P$. It will be found that the complex particular of a subidentical may be either complex particular, subidentical, or supercontrary; or that $PD_{\prime}$ may be either P, $D_{\prime}$ or C'. Examine all the cases, and the rules will be found in

$$\begin{array}{ll} (D_{\prime}C_{\prime})P & P(D_{\prime}C') \\ (D'C')P & P(C_{\prime}D') \end{array}$$

thus interpreted. Either premise from between the parentheses, with P, in order as written, may have either, and must have one, of the three for its conclusion. That $D_{\prime}P$ must give either $D_{\prime}$ $C_{\prime}$ or P, and so must $C_{\prime}P$: but $PC_{\prime}$ must have either P, $C_{\prime}$, or D'.

Before proceeding to the simple syllogism, as I have called it, I will state that I much doubt the propriety of the terms *simple* and *complex*. Undoubtedly the phrases are historically just, for each of the syllogisms which I propose to call *complex* is, as we shall see, necessarily composed of three of those which are always called *simple*. But in another point of view, the phraseology ought to be reversed; the simple syllogism is the affirmation of the existence of one out of several of the complex ones. Thus XY) + Y)Z =X)Z, or $A_{\prime}A_{\prime}A_{\prime}$, is really ($D_{\prime}$ or D, not known which) ($D_{\prime}$ or D, not known which) ($D_{\prime}$ or D, not known which) and asserts that there is either $D_{\prime}D_{\prime}D_{\prime}$ or $D_{\prime}DD_{\prime}$ or $DD_{\prime}D_{\prime}$ or DDD.

But it will be said, surely the complex proposition requires the *conjunctive* existence of two simple ones: $D_1 = A_1 + O'$; and is therefore *compound* at least. I answer that, on the other hand, the simple proposition requires the *disjunctiv* existence of two complex ones: as $A_1 = D_1$ or D. Which is most simple, *both*, or *one or the other?* to me, I think, the first. Certainly the syllogism $D_1D_1D_1$ is one which I more readily apprehend than $A_1A_1A_1$. Indeed, to most minds, the latter is the former, if they are left to themselves: and the cases D_1DD_1, &c. are only admitted when produced and insisted on.

But further, is the simple proposition properly called *simple?* Is there in it but one assertion to deny or admit? Is but one [86] question answered? When I affirm "Every X is Y," I affirm 1. Comparison of X and Y. 2. Coincidences. 3. The greatest possible amount of them. 4. That *every* X has been used in obtaining them. In "Some Xs are Ys" the first two of the preceding are employed. In "No X is Y," we have, 1. Comparison of Xs and Ys. 2. Exclusions. 3. The greatest amount. 4. The comparison of *every* X with every Y. And "Some Xs are not Ys" omits the third, and substitutes Xs for every X in the fourth.

Now the subidentical, for instance, only contains, besides what is in the subaffirmative, the notion that there are more Ys than Xs in existence. The subcontrary consists, over and above what is in the subnegative, in that Xs and Ys are not every thing that the proposition might have applied to: and so on. On these considerations, I think it may be allowed to treat the words simple and complex as only of historical reference, and to consider the first as disjunctively connected with the second, the second as conjunctively connected with the first, in the manner above noted. I think I shall make it clear enough, that the passage from the conjunctions to the disjunctions is better suited to a demonstrative system than the converse. If the plan which I propose should gain any reception, I should imagine that *disjunctive* and *conjunctive* would be the names given to the classes which I have called simple and complex: the conjunctive composed of several of the disjunctive, the disjunctive consisting of one or the other out of several of the conjunctive.

When a proposition R, is the necessary consequence of two others, P and Q, it necessarily follows that the denial of R, must be the denial of one at least of P and Q. For every proposition admits but of affirmation or denial : and he who affirms *both* P and Q must *affirm* R. If then P be affirmed and R denied, the denial of Q must follow : if Q be affirmed and R denied, the denial of P must follow.

A *simple syllogism* is one, the two premises and conclusion of which are to be found among the simple propositions $A_{\prime}$, $E_{\prime}$, $I_{\prime}$, $O_{\prime}$, A', E', I', O'. Thus we have $A_{\prime}E_{\prime}E_{\prime}$ or $X)Y+Y.Z=X.Z$, as an instance. The order of reference is always XY, YZ, XZ.

The following theorems will be necessary ;—1. *A particular premise cannot be followed by a universal conclusion.*

[87] If possible, let $A_{\prime}I_{\prime}$ for example, have a universal conclusion. Take the complex premises $D_{\prime}P$ or $(A_{\prime}+O')\,(I_{\prime}+I'+O_{\prime}+O')$. All that can be inferred is that one of *three* conclusions (page 85) is valid, and neither D nor C : either $D_{\prime}$ or P or C'. But if a universal be true, one of *two* conclusions must be valid (page 69) and one of them D or C. If then $A_{\prime}$ and $I_{\prime}$ alone yielded a universal conclusion, quite as much must $D_{\prime}P$: or a form which is indifferent to three conclusions, and not having D nor C, is necessarily productive of one of two conclusions, one of which is D or C. This contradiction cannot exist : or $A_{\prime}I_{\prime}$ cannot yield a universal conclusion.

2. *From two particular premises no conclusion can follow.* If possible, let $I_{\prime}I_{\prime}$ yield a conclusion ; which by the last theorem, must be only particular. Now PP or $(I_{\prime} + I' + O_{\prime} + O')\,(I_{\prime} + I' + O_{\prime} + O')$ is indifferent to all complex conclusions : quite as much is $I_{\prime}I_{\prime}$. But if these premises yield a particular conclusion, two complex conclusions are denied (page 69). This contradiction cannot exist : or particular premises can yield no conclusion.

Let a simple syllogism with premises and conclusion all universal, be called *universal:* and with either premise (and therefore the conclusion) particular, be called *particular.* Then every universal syllogism has two particular syllogisms deducible from it. Thus if $A_{,}E_{,}E_{,}$ be valid, then $A_{,}$ joined with the denial of $E_{,}$ gives the denial of $E_{,}$: or $A_{,}I_{,}I_{,}$ *seems to be valid.* But the alteration of the places of the propositions requires us to say that it is $A'I_{,}I_{,}$ which is valid: and this point requires close attention.

Take $A_{,}E_{,}E_{,}$ or $X)Y + Y.Z = X.Z$. Then $X)Y$ with the denial of $X.Z$ (or XZ) gives the denial of $Y.Z$ (or YZ); and we have

$$X)Y + XZ = YZ.$$

This is valid, if the first be (as it is) valid: but its symbol is not $A_{,}I_{,}I_{,}$. For the middle term is, in our notation, made *middle* in the order of reference, which is therefore YX, XZ, YZ: and the syllogism is $A'I_{,}I_{,}$. Similarly we have

$$XZ + Y.Z = X : Y$$

produced by coupling the denial of $X.Z$ with $Y.Z$. But this is $I_{,}E_{,}O_{,}$: for the order of reference is now XZ, ZY, XY, and [88] $E_{,}$ is not changed by change of order. The rule is as follows. When the denials of the conclusion and of a premise are made to take the places of that premise and the conclusion, the order of reference remains undisturbed as to the transposed terms, and is changed as to the standing term. This last must therefore have the preposition of the inconvertible proposition changed; but not that of the convertible proposition.

Thus $E'A_{,}E'$, if valid, gives $E'I'O_{,}$ and $I'A'I'$. Again, in a similar way it may be shown that from each particular syllogism follows a universal: thus $I_{,}E'O'$, if valid, shows that denial of O', and E', give denial of $I_{,}$ or $A'E'E_{,}$. In this case neither is valid. And $E'I'O_{,}$, besides $E'A_{,}E'$, also gives $A_{,}I'I'$.

Such classification of these *opponent forms* as is useful, will presently be given.

Since there are eight forms of assertion, with reference to each of the orders XY YZ, it follows that there are sixty-four combinations of a pair of premises each. But of these the only ones which have a chance of yielding a conclusion are, 1. sixteen with premises both universal ; 2. thirty-two with one universal and one particular. If, for a moment, *U* stand for universal and *P* for particular, the form of a syllogism is either *UUU*, *PUP*, *UPP*, or *UUP*. Of these, the first, second, and third are so related that each form has its opponents in the set[14] : but the fourth has its own form in each of its opponents.

Now examine one of the complex affirmative syllogisms, say $D_\prime D_\prime D_\prime$, by the diagram in page 79. The premises are $A_\prime + O'$ and $A_\prime + O'$, giving the four combinations $A_\prime A_\prime$, $A_\prime O'$, $O'A_\prime$, and $O'O'$. The conclusion is $A_\prime + O'$: but it is not merely twofold, but threefold : for the *à fortiori* character explained in page 81, shows that O' is obtainable on two different grounds, and is the sum, as it were, of two different and necessary parts of the conclusion. That every X is Z, follows from X)Y and Y)Z, or we have the syllogism

$$A_\prime A_\prime A_\prime \quad X)Y + Y)Z = X)Z$$

But as far as the Zs which are below (12) are concerned, it follows that they are not Xs because they are the Ys which are not Xs : or we have

$$O'A_\prime O' \quad Y : X + Y)Z = Z : X$$

[89] and as to the Zs below (23) they are not Xs because they are not Ys, among which are all the Xs. Accordingly we have

$$A_\prime O'O' \quad X)Y + Z:Y = Z:X$$

or $D_\prime D_\prime D_\prime$ requires the coexistence of $A_\prime A_\prime A_\prime$, $O'A_\prime O'$, $A_\prime O'O'$. Apply this reasoning to the contraries x, y, z, or else examine D'D'D' in the same way, and we find that D'D'D' requires the coexistence of A'A'A', $O_\prime A'O_\prime$, $A'O_\prime O_\prime$.

By applying the preceding results to x, Y, Z, &c. as in page 82, or, as is better at first, by examining all the cases of the diagram in page 79, we get the following table of derivations from the eight affirmatory complex syllogisms. The first column shews the terms which must be used, to deduce all from $D_{\prime}D_{\prime}D_{\prime}$

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[14] [See Table of Contents, additions and corrections to Chapter V.—Ed.]

XYZ	$D_{\prime}D_{\prime}D_{\prime}$ ..	$A_{\prime}A_{\prime}A_{\prime}$	X)Y + Y)Z = X)Z	
		$O'A_{\prime}O'$	Y:X + Y)Z = Z:X	(12)
		$A_{\prime}O'O'$	X)Y + Z:Y = Z:X	(23)
x y z	$D'D'D'$ ..	$A'A'A'$	Y)X + Z)Y = Z)X	
		$O_{\prime}A'O_{\prime}$	X:Y + Z)Y = X:Z	(12)
		$A'O_{\prime}O_{\prime}$	Y)X + Y:Z = X:Z	(23)
xYZ	$C'D_{\prime}C'$ ..	$E_{\prime}A_{\prime}E'$	x. y + Y)Z = x . z	
		$I_{\prime}A_{\prime}I_{\prime}$	XY + Y)Z = XZ	(12)
		$E'O'I_{\prime}$	x . y + Z:Y = ZX	(23)
X y z	$C_{\prime}D'C_{\prime}$ ..	$E_{\prime}A'E_{\prime}$	X.Y + Z)Y = X.Z	
		$I'A'I'$	xy + Z)Y = xz	(12)
		$E_{\prime}O_{\prime}I'$	X.Y + Y:Z = xz	(23)
X Y z	$D_{\prime}C_{\prime}C_{\prime}$ ..	$A_{\prime}E_{\prime}E_{\prime}$	X)Y = XZ	
		$O'E_{\prime}I'$	Y:X + Y.Z = xz	(12)
		$A_{\prime}I'I'$	X)Y + yx = xz	(23)
x y Z	$D'C'C'$ ..	$A'E'E'$	Y)Z + y.z = x.z	
		$O_{\prime}E'I_{\prime}$	X:Y + y.z = XZ	(12)
		$A'I_{\prime}I_{\prime}$	Y)X + YZ = XZ	(23)
x Y z	$C'C_{\prime}D'$ ..	$E'E_{\prime}A'$	x . y + Y.Z = Z)X	
		$I_{\prime}E_{\prime}O_{\prime}$	XY + Y.Z = X:Z	(12)
		$E'I'O_{\prime}$	x . y + yz = X:Z	(23)
X y Z	$C_{\prime}C'D_{\prime}$ ..	$E_{\prime}E'A_{\prime}$	X.Y + y.z = X)Z	
		$I'E'O'$	xy + y.z = Z:X	(12)
		$E_{\prime}I_{\prime}O'$	X.Y + YZ = Z:X	(23)

Before forming any rule, or making any remark, I proceed to **[90]** collect the results of the remaining cases. And first, let a premise be brought to its limit, D or C: say that $D_{\prime}D_{\prime}D_{\prime}$ becomes $DD_{\prime}D_{\prime}$. In the diagram it immediately appears that one of the particular conclusions is lost; not contradicted, but nullified: for (12) disappears, because X and Y are identical names. That is, $A_{\prime}A_{\prime}A_{\prime}$ remains, and $A_\prime O'O'$: but the conclusion of $O'A_\prime O'$ is nullified. But this very circumstance creates, not a new conclusion, for it is only a part of one already existing, but a new form of deduction. The premises are now $A_\prime + A'$ and $A_\prime + O'$, and the conclusion is $A_\prime + O'$. The syllogisms $A_\prime A_\prime A_\prime$ and $A_\prime O'O'$ are as before, and for the same reasons: but there is now the combination $A'A_\prime$ among the premises, which produces the conclusion $I_\prime$, and we have

$$A'A_\prime I_\prime \quad Y)X+Y)Z=XZ$$

This syllogism, though new as far as $D_\prime D_\prime D_\prime$ is concerned, is only a strengthened form of $I_\prime A_\prime I_\prime$, a concomitant of $E'A_\prime E'$. For (page 65) $I_\prime$ is true whenever A' is true, so that $A'A_\prime$ includes $I_\prime A_\prime$ and its necessary consequence $I_\prime$. But if $I_\prime$ had been strengthened into $A_\prime$ instead of A', we should have had $A_\prime A_\prime I_\prime$ which though perfectly valid, yet admits of a stronger conclusion, as seen in $A_\prime A_\prime A_\prime$.

Of the two modes of strengthening a particular proposition (as $I_\prime$ into $A_\prime$ or A') there is one which strengthens the quantity of the first form of the proposition, and another that of the second. Thus XY or $I_\prime$ becomes X)Y or $A_\prime$ when the first form, and Y)X or A', when the second form, if strengthened. Similarly $O_\prime$ or X:Y becomes X.Y or $E_\prime$, and y.x or E', according as the form strengthened is X:Y or y:x. The preposition remains the same, or changes, according as the first or second form is strengthened. If the *first* form of the *second* premise[15] of a syllogism, or the *second* form of the *first* premise, be strengthened, no strength is added to the conclusion. Thus, as far as the syllogisms in this chapter are concerned, $I_\prime A_\prime$ gives as much as $A'A_\prime$, and $E_\prime O_\prime$ as $E_\prime E_\prime$. But if the first form of the first premise, or the second form of the second, be strengthened, the conclusion has its first form strengthened.

A very simple and obvious theorem contains all these results. The concluding terms are, in our order of reference, the first **[91]** term of the first premise and the second term of the second. The conclusion is never strengthened by augmenting the quantity of the middle term, nor *only* weakened (it may be altogether destroyed) by weakening the middle term. A wider field of comparison does not by itself give more comparisons: nor can more comparisons arise except by augmenting the number of things compared in that field. Since the conclusion can obviously speak of no more than was in the premises, no term of that conclusion can be augmented in quantity, until the same thing has taken place in its premise. But no strengthening of a proposition strengthens both terms: consequently, to make such a thing effective, it must be the concluding, and not the middle, term which is strengthened.

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[15] [For De Morgan's reasons for adopting this unusual spelling see "Table of Contents."—Ed.]

The following table is only worth inserting as a collection of exercises. The fourth column shows the eight strengthened particular syllogisms, as I will call them, having universal premises but only a particular conclusion, not stronger than might have been inferred from the particular syllogism itself.

Alteration of	into	removes	and substitutes	strengthened from	occurring in
D,D,D,	DD,D,	O'A,O'	A'A,I,	I,A,I,	C'D,C'
D'D'D'	D'DD'	A'O,O,		A'I,I,	D'C'C'
D,D,D,	D,DD,	A,O'O'	A,A'I'	I'A'I'	C,D'C,
D'D'D'	DD'D'	O,A'O,		A,I'I'	D,C,C,
D,C,C,	DC,C,	O'E,I'	A'E,O,	I,E,O,	C'C,D'
D'C'C'	D'CC'	A'I,I,		A'O,O,	D'D'D'
D,C,C,	D,CC,	A,I'I'	A,E'O'	I'E'O'	C,C'D,
D'C'C'	DC'C'	O,E'I,		A,O'O'	D,D,D,
C,D'C,	CD'C,	I'A'I'	E'A'O,	O,A'O,	D'D'D'
C'D,C'	C'DC'	E'O'I,		E'I'O,	C'C,D'
C,D'C,	C,DC,	E,O,I'	E,A,O'	O'A,O'	D,D,D,
C'D,C'	CD,C'	I,A,I,		E,I,O'	C,C'D,
C,C'D,	CC'D,	I'E'O'	E'E'I,	O,E'I,	D'C'C'
C'C,D'	C'CD'	E'I'O,		E'O'I,	C'D,C'
C,C'D,	C,CD,	E,I,O'	E,E,I'	O'E,I'	D,C,C,
C'C,D'	CC,D'	I,E,O,		E,O,I'	C,D'C,

I will now examine the negatory complex syllogisms, premising however that we cannot get any new conclusions from them. [92] For we have now got all the sixteen cases in which both premises are universal: and we know that there can be no syllogism with a particular premise, except it have one of those with universal premises for its opponents.

Take $D_{\prime}D':C'$ or $A_{\prime}+O'$ and $A'+O_{\prime}$ together deny $E'+I_{\prime}$, that is, deny the coexistence of E' and $I_{\prime}$, that is, deny either E' or $I_{\prime}$, that is, assert either I' or $E_{\prime}$. This syllogism then may be written thus,

$$(A_{\prime}+O')\ (A'+O_{\prime})\ (\text{either } E_{\prime} \text{ or } I')$$

Now the fact is that this disjunction is superfluous; it is I' which is always asserted, and $E_{\prime}$ is never a necessary consequence of $D_{\prime}D'$. For $A_{\prime}A'$ gives I' as already shown, and $A_{\prime}O_{\prime}$ and $O'A'$ are inconclusive (and $O'O_{\prime}$ of course). And the rationale of the inference is as follows: since X is a subidentical of Y, and Y a superidentical of Z, it follows that Y is superidentical both of X and Z; consequently, Y not filling the universe (our supposition throughout) it follows that there are things which are neither Xs nor Zs, namely, all which are not Ys. Again, in $C_{\prime}C_{\prime}:C'$, which the same reasoning shows to be only $C_{\prime}C_{\prime}I'$, none either of X or of Z is in Y, therefore every instance in Y is both x and z. And thus it will appear that in every negatory complex conclusion *the whole middle term*, or the *whole of its contrary*, makes the subject matter of the strengthened particular syllogism which is all that can be collected.

Our conclusion is that no negatory complex syllogism is of any more logical effect than the strengthened particular derived from it. Thus we may say that, so far as the extent and character of the inference is concerned, the former is the latter.

I will now pass to the general rules of the complete system of syllogisms;—

The reader must take pains to remember two rules of formation, perfect contraries of each other, for the dependence of the *accents* (or *prepositions*) on the *sign* (affirmative or negative character) of the first premise. I express them in the briefest way possible.

Direct Rule. *Affirmation* (in the first premise) makes the second *premise* agree with both the other propositions, or *isolates* nothing: negation makes the second premise differ from both the **[93]** others, or isolates the second premise. *Inverse rule.* Affirmation *isolates the first premise,* makes the first premise differ from both the others in preposition: negation *isolates the conclusion,* makes the conclusion differ from both the others. These rules might be expressed so as to make their contrariety more complete. Thus in the $\begin{matrix}\text{direct}\\ \text{inverse}\end{matrix}$ rule, affirmative commencement shows $\begin{matrix}\text{like}\\ \text{unlike}\end{matrix}$ prepositions in the two premises, and the conclusion $\begin{matrix}\text{agreeing}\\ \text{differing}\end{matrix}$ $\begin{matrix}\text{with}\\ \text{from}\end{matrix}$ the first premise in preposition: but negative commencement shows $\begin{matrix}\text{unlike}\\ \text{like}\end{matrix}$ prepositions in the two premises, and the conclusion $\begin{matrix}\text{agreeing with}\\ \text{differing from}\end{matrix}$ the first premise in preposition.

The subjects of the following rules are,

1. The eight affirmatory complex syllogisms.
2. The eight universal simple syllogisms.
3. The eight strengthened particular simple syllogisms.
4. The sixteen particular simple syllogisms.

Omit the negatory complex syllogisms, as fully contained in the third of this enumeration, and the complex syllogisms which contain the unaccented D or C, as carrying a momentary accent for the rule, to be expunged when the formation is completed. Consider $D_{\prime}$, D, D', $A_{\prime}$, A', $I_{\prime}$, I', as of the affirmative signs, and $C_{\prime}$, C, C', $E_{\prime}$, E', $O_{\prime}$, O', as negative.

Rule 1. In the complex syllogism all parts are complex;

in the universal simple syllogism all parts are universal; in the strengthened particular only the conclusion is particular; in the particular only a premise is universal.

Rule 2. Premises of like sign have an affirmative conclusion; of unlike sign, a negative.

Rule 3. The complex, the universal, the particulars which begin with a particular, follow the direct rule; the strengthened particulars, and the particulars which begin with a universal (all that commence with a universal, and conclude with a particular) follow the inverse rule. [Or thus; all which begin and end alike, follow the direct rule; all which begin and end differently, the inverse.]

The complex syllogisms and universals are easily remembered **[94]** by rule: the particulars almost as easily. The following sub-rules may be noted, as far as these last are concerned.

Sub-rule 1. *First and second premises.* A and O in the first premise demand unlike prepositions in the two premises: E and I demand like prepositions. Thus $A_{\prime}O_{\prime}$ must be inconclusive: $A_{\prime}O^{\prime}$ must be conclusive. But $E_{\prime}O_{\prime}$ must be conclusive: and $E_{\prime}O^{\prime}$ must be inconclusive.

Sub-rule 2. *First premise and conclusion.* A universal in the first premise demands an unlike preposition in the conclusion: a particular first premise, a like preposition in the conclusion.

Sub-rule 3. *Second premise and conclusion.* Every second premise demands its own preposition in a conclusion of like sign: and the other preposition in a conclusion of unlike sign.

As far as the four species are concerned, every syllogism formed according to the three rules is valid; and every one not so formed is invalid. The following remarks are partly recapitulatory, partly new.

Remark 1. Every complex syllogism gives one universal syllogism * and two particular ones, its concomitants: and the concomitants are formed by changing one of the premises of the universal and the conclusion, into their particular concomitant propositions (page 63).

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* *Syllogism*, not preceded by *complex*, means simple syllogism.

Remark 2. Every syllogism has its *contranominal,* which asserts of the contraries in the same manner as the first does of the direct terms: and contranominals have all their accents different, as in $O'A_\iota O'$ and $O_\iota A'O_\iota$ (page 62).

Remark 3. Every syllogism has *two* opponents, made by interchanging the contradictories of one premise and of the conclusion, and altering the accent of the remaining premise, if inconvertible (A or O) (page 88).

Remark 4. Every complex syllogism has two such opponents formed in the same way, the Ds being the inconvertibles, the Cs the convertibles. Thus (:) meaning *denial of,* the opponents of $C_\iota D'C_\iota$ are $C_\iota{:}C_\iota{:}D'$ and $:C_\iota D_\iota C_\iota$. The first of these is

$$(E_\iota + I')\quad (I_\iota\ \textit{or}\ E')\quad (O'\ \text{or}\ A_\iota)$$

containing the valid syllogisms $E_\iota E'A_\iota$, $E_\iota I_\iota O'$, $I'E'O'$; being **[95]** $E_\iota E'A_\iota$ and its concomitants. And $:C_\iota D_\iota{:}C_\iota$ gives $E'A_\iota E'$ (the contranominal of $E_\iota A'E_\iota$) and its concomitants. And the same of the rest.

Remark 5. Each universal syllogism has two weakened forms, made by weakening one premise and the conclusion. When the *first* premise is weakened, it is without change of preposition: but when the second, with change. Thus the weakened forms of $E_\iota A'E_\iota$ are $O_\iota A'O_\iota$ and $E_\iota I_\iota O'$.

Remark 6. Each particular syllogism has two strengthened forms, one of which is a universal, the other only a strengthened particular. Thus the strengthened forms of $O_\iota A'O_\iota$ are $E_\iota A'E_\iota$ and $E'A'O_\iota$.

Remark 7. In every syllogism except the strengthened particular, the middle term is universal in one premise, and particular in the other: and its contrary is therefore the same. But in the strengthened particular, the middle term is universal in both premises, or particular in both. This affords a complete criterion of syllogism, as will be noticed hereafter: in fact, the completeness of this system crowds us with relations, from many of which general rules might be deduced, though they need only appear here by casual remark.

In $O'A_{\prime}O'$, $A'O_{\prime}O_{\prime}$, $I_{\prime}A_{\prime}I_{\prime}$, $E_{\prime}O_{\prime}I'$, $O'E_{\prime}I'$, $A'I_{\prime}I_{\prime}$, $I_{\prime}E_{\prime}O_{\prime}$, $E_{\prime}I_{\prime}O'$, the middle term enters universally in the universal, and particularly in the particular. In all the others it enters particularly in the universal, and universally in the particular. In the first set, the convertible premises are all *subs*, the inconvertibles are *subs* in the second premise, and *supers* in the first. In the second set, these rules are inverted.

Remark 8. Of the twelve possible pairs of premises AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, OA, OE, which *can* give a conclusion, each one *will*, in two ways, which two ways are inverted in their accents. Thus EO appears in $E'O'I_{\prime}$ and $E_{\prime}O_{\prime}I'$. The two premise-letters and one accent dictate all the rest : thus $I'A$ can belong to nothing but $I'A'I'$. When the system is well learnt, it will be found unnecessary to write more than $I'A$, for the symbol of $I'A'I'$. I now speak only of fundamental syllogisms : the strengthened syllogism $A_{\prime}A'I'$ might be signified by $A'A'$.

Remark 9. The syllogisms of the three first classes are all really **[96]** specimens of one, those of the fourth of two, among them, with the eight variations XYZ, xYZ, XYz, xYz, XyZ, xyZ, Xyz, xyz. The rules for conducting these changes are

Change of subject is change of both accent and letter.
Change of predicate is change of letter.
Change of both is change of accent.

thus to pass from $E'E_{\prime}A'$ to $A_{\prime}E_{\prime}E_{\prime}$ we note in XY change of subject, in YZ change of neither, in XZ change of subject : therefore xYZ is the set of terms into which XYZ must be changed : and the $E'E_{\prime}A'$ syllogism of either set is the $A_{\prime}E_{\prime}E_{\prime}$ syllogism of the other.

The 24 syllogisms, which are 24 with reference to the order XY, YZ, XZ, are only 12 if the order ZY, YX, ZX, be allowed. Thus $A_{\prime}I'I'$ of the first is the $I'A'I'$ of the second. These syllogisms are essentially the same in the mode of inference they afford. To change a syllogism into another of the same mode of inference, invert the premises and change the preposition of all the inconvertibles. Thus $A^{\prime}O_{\prime}O_{\prime}$ and $O^{\prime}A_{\prime}O^{\prime}$ are of the same inference. The pairs which in this point of view are identical are

$A_{\prime}A_{\prime}A_{\prime}=A^{\prime}A^{\prime}A^{\prime}$	$E^{\prime}A_{\prime}E^{\prime}=A^{\prime}E^{\prime}E^{\prime}$	$E_{\prime}A^{\prime}E_{\prime}=A_{\prime}E_{\prime}E_{\prime}$	$E^{\prime}E_{\prime}A^{\prime}=E_{\prime}E^{\prime}A_{\prime}$
$O^{\prime}A_{\prime}O^{\prime}=A^{\prime}O_{\prime}O_{\prime}$	$I_{\prime}A_{\prime}I_{\prime}=A^{\prime}I_{\prime}I_{\prime}$	$I^{\prime}A^{\prime}I^{\prime}=A_{\prime}I^{\prime}I^{\prime}$	$I_{\prime}E_{\prime}O_{\prime}=E_{\prime}I_{\prime}O^{\prime}$
$A_{\prime}O^{\prime}O^{\prime}=O_{\prime}A^{\prime}O_{\prime}$	$E^{\prime}O^{\prime}I_{\prime}=O_{\prime}E^{\prime}I_{\prime}$	$E_{\prime}O_{\prime}I^{\prime}=O^{\prime}E_{\prime}I^{\prime}$	$E^{\prime}I^{\prime}O_{\prime}=I^{\prime}E^{\prime}O^{\prime}$

[16] The ninth remark admits of considerable extension. The "*some*" of a logical proposition may have a much more definite character in some cases than in others. It may be a selected, or at least a distinguishable *some*, which wants nothing but a nominal distinction to make the particular proposition easily and usefully universal. Whether it can be done more or less easily, and more or less usefully, is no question of formal logic. If it be supposed done, the particular is converted into a universal. In "some Xs are Ys," if we make a name for every X which is Y, say M, we have then "Every M is Y." This proposition may be purely identical, or it may not. If we call every X which is Y by the name M merely because it is Y, then our universal is only "Every *X which is Y* is Y." But if the name M be conferred from any other circumstance, which distinguishes the Xs that are Ys from other Xs, then the change from the particular to the universal by **[97]** means of the new restriction imposed by the new name, is the expression of new knowledge.

The quantities in the conclusion are of two kinds. There are those which are brought in with the terms, and which continue in the conclusion such as they were introduced in the premises: and there are those which depend on the union of the premises, and which are what they are only in virtue of the joint existence of the premises. For example, in $I_1A_1I_1$ we have "some Xs are Ys, but every Y is Z, therefore some Xs are Zs": if we ask, what Xs are Zs, the answer is, those which are Ys, and no others, so far as this conclusion affirms. But when we look at $O'A_1O'$ or "some Ys are not Xs, and every Y is Z; therefore some Zs are not Xs:" and if we then ask *what* Zs are not Xs; the answer is, that this quantity does not enter with Z, but depends upon the other premise, namely, upon the number of Ys which are not Xs. In a particular syllogism, let us call the quantity of the subject in the conclusion *intrinsic* or *extrinsic* according as it is that of the premise which introduces that subject, or of the other premise. Examination will show that in every particular syllogism which concludes in I_1 or I', in which both terms are particular, the quantities of the terms are, of the one intrinsic, of the other extrinsic: but that where the conclusion is in O_1 or O', either the quantity of the subject is intrinsic and that of the contrary of the predicate extrinsic, or *vice versâ.*

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[16] [Here should be inserted the additional sentence given in the Table of Contents. "The inverted forms of the strengthened syllogisms are omitted: of these, four are their own inversions, namely $A_{\prime}A^{\prime}I^{\prime}$, $A^{\prime}AI_{\prime}$, $E^{\prime}E^{\prime}I_{\prime}$, and $E_{\prime}E_{\prime}I^{\prime}$: of the remainder, $A_{\prime}E^{\prime}O^{\prime}$ and $E^{\prime}A^{\prime}O_{\prime}$ are inversions; and also $A^{\prime}E_{\prime}O_{\prime}$ and $E_{\prime}A_{\prime}O^{\prime}$."—Ed.]

When the quantity of a particular term in the conclusion is intrinsic, the invention of a name will convert the syllogism into a universal. Thus $I_1A_1A_1$ or $XY+Y)Z=XZ$, if M be taken to represent all those Xs which are Ys, and nothing else, becomes $M)Y+Y)Z=M)Y$, of the form $A_1A_1A_1$. Again, $O'A_1O'$ or $Y{:}X+Y)Z=Z{:}X$, thrown into the form $x{:}y+z)y=x{:}z$, becomes $m.y+z)y=m.z$, of the form $E_1A_1E_1$, when the xs which are ys are distinguished from the rest of the universe by the name m. There is nothing either illegitimate or uncommon in distinguishing by a peculiar name *certain some* (or even *uncertain some*, if *certainly always the same some*) of another name. Again, since we know that every universal syllogism is reducible to the form $A_1A_1A_1$ by use of contraries, we have now reason to know that there is no fundamental inference, of the kind treated in this chapter, which is any other than that in $A_1A_1A_1$, or, the *contained* [98] *of the contained is contained.* And there is no better exercise than learning to read off each of the syllogisms, universal and particular, into this one form, by perception, and without use of rules. Take as an instance X:Y+y.z=XZ: what is the container, what is the contained, and what is the middle container of one and contained of the other. It is a parcel of Xs which are contained in y, all y in Z, and therefore *that parcel* of Xs in Z.

This general principle suggests a notation for all the complex, universal, and fundamental particular, syllogisms. If we abbreviate X)Y+Y)Z=X)Z into XYZ), and if we denote by XYZ, without), that it is only a parcel of Xs (all or some, defined or undefined, but always the same), we have the following,

For $A_{\prime}A_{\prime}A_{\prime}$ read XYZ) or zyx)	For $A'A'A'$ read xyz) or ZYX)
— $O'A_{\prime}O'$ — xYZ	— $O_{\prime}A'O_{\prime}$ — Xyz
— $A_{\prime}O'O'$ — Zyx	— $A'O_{\prime}O_{\prime}$ — zYX
For $E'A_{\prime}E'$ read xYZ) or zyX)	For $E_{\prime}A'E_{\prime}$ read Xyz) or ZYx)
— $I_{\prime}A_{\prime}I_{\prime}$ — XYZ	— $I'A'I'$ — xyz
— $E'O'I_{\prime}$ — ZyX	— $E_{\prime}O_{\prime}I'$ — zYx
For $A_{\prime}E_{\prime}E_{\prime}$ read XYz) or Zyx)	For $A'E'E'$ read xyZ) or zYX)
— $O'E_{\prime}I'$ — xYz	— $O_{\prime}E'I_{\prime}$ — XyZ
— $A_{\prime}I'T'$ — zyx	— $A'I_{\prime}I_{\prime}$ — ZYX
For $E'E_{\prime}A'$ read xYz) or ZyX)	For $E_{\prime}E'A_{\prime}$ read XyZ) or zYx)
— $I_{\prime}E_{\prime}O_{\prime}$ — XYz	— $I'E'O'$ — xyZ
— $E'I'O_{\prime}$ — zyX	— $E_{\prime}I_{\prime}O'$ — ZYx

Here, using P,Q,R, as general terms, PQR) denotes that all Ps are Qs, and all Qs are Rs, whence all Ps are Rs: while PQR only denotes that there is a parcel of Ps among the Qs, and all Qs are among the Rs, whence that parcel of Ps is among the Rs.

The rules for the connection of these systems are not complicated, considering the extent of the cases they are to include. Let the letters A,E, &c. be called *proponents;* X,Y,Z, *nominals:* and by the *order* of the nominals we always mean that X is first, &c. both in XYZ, and ZYX.

The nominals being *direct* (X,Y,Z) and *contrary* (x,y,z), remember that, *first*,

[99]

$$\text{An affirmative}\left\{\begin{array}{l}\textit{first}\\ \textit{second}\\ \textit{third}\end{array}\right.\text{proponent denotes that the}\left\{\begin{array}{l}\textit{first}\text{ and second}\\ \textit{second}\text{ and third}\\ \textit{third}\text{ and first}\end{array}\right.$$

nominals agree (are both direct or both contrary).

$$\text{A negative}\left\{\begin{array}{l}\textit{first}\\ \textit{second}\\ \textit{third}\end{array}\right.\text{proponent denotes that the}\left\{\begin{array}{l}\textit{first}\text{ and second}\\ \textit{second}\text{ and third}\\ \textit{third}\text{ and first}\end{array}\right.$$

nominals differ (are one direct, one contrary).

Thus EIO must give Xyz or xYZ or zyX or ZYx
IEO must give XYz or xyZ or zYX or Zyx

Secondly, whether the middle term be Y or y depend only on the accent of the middle proponent: a *sub*-accent gives Y, a *super*-accent gives y. In the universal syllogism however, either gives either.

Thirdly, the XYZ syllogisms are the particulars which begin with a particular: and the ZYX syllogisms are the particulars which begin with a universal.

For example, required $O_{\prime}E^{\prime}I_{\prime}$. Seeing the particular $O_{\prime}$, at the beginning, take the order XYZ, seeing the superaccent in $E^{\prime}$ make it XyZ. Seeing the negative $O_{\prime}$, let the existing disagreement of the first and second nominals continue: and the same of the second and third from the negative $E_{\prime}$. Consequently XyZ is the syllogism expressed in nominals. Or the rationale of the inference in $O_{\prime}E^{\prime}I_{\prime}$ is that a parcel of Xs are among the Zs because among the ys which are all among the Zs.

Again, required the nominal mode of expressing $E^{\prime}I^{\prime}O_{\prime}$. Seeing the universal E at the beginning, write down ZYX: for the superaccent in $I^{\prime}$, write down ZyX; for the negative in $E^{\prime}$, continue yX; for the affirmative in $I^{\prime}$, write zy; hence zyX is the nominal form of $E^{\prime}I^{\prime}O_{\prime}$.

Required the proponent mode of expressing xYz. Here xY, Yz, show us that the premises are negative forms, and the direction of the order x, Y, z, that the first premise is particular. Then OE are the premises, and I the conclusion. And Y tells us that the middle proponent has a subaccent. Whence OE‚I is, so far as it goes, the proponent expression. And, by the laws of form, the other accents must be as in O'E‚I', since the syllogism follows the direct rule (page 93).

[100] Required the proponent mode of expressing XYz. Here we note in succession—universal commencement—first premise negative—second, affirmative—middle accent sub. This gives EI‚O of the inverse rule, or E‚I‚O'.

Required the proponent notation for the universal xYZ) or zyX). We see at once EA‚E, or E'A‚E'.

The concomitants of a universal are found by changing the first nominal into the contrary, in each of the forms, and throwing away the sign of universality [ )]. Thus the concomitants of XyZ) or zYx) are xyZ and ZYx.

The weakened forms of a universal are found by merely throwing away the symbol of universality [ )] from the two forms of the universal. Thus the weakened forms of XYZ) which is also zyx) are XYZ and zyx.

But we have not yet reached the climax of symbolic simplicity in the mere representation of syllogisms. An algebraist would say that the structure of the inference, as now considered, does not depend upon the names; but only upon their reference to the names in the fundamental form XYZ). He would therefore propose a simple symbol to represent *letting alone*, and another to represent *changing into the contrary*. These, with a sign of complete universality, and another of inversion of order, are all that he would find necessary. Let 0 and 1 signify letting alone and changing into the contrary: let the terminal parenthesis denote complete universality, as before, and let inversion of order be denoted by a negative sign prefixed. Thus XYZ or I‚A‚I‚, would be denoted by 000; Zyx or A‚O'O' by −011; A‚E‚E‚ or XYz) by 001) or its equivalent −011). Thus −011 tells us that some of the Zs are ys, all the ys are xs, whence some of the Zs are xs. To write its proponent form, observe that — instructs us to write a universal first; 11 to make it affirmative; 1 in the middle to superaccent the middle proposition; 01 to make the second premise negative. We have then $A_{\prime}O'O'$ or $X)Y+Z:Y=Z:X$ which is $Zy+y)x=Zx$, as asserted.

All that relates to universals in the preceding, applies to the complex syllogisms. Let a couple of parentheses imply a complex syllogism: thus $D_{\prime}D_{\prime}D_{\prime}$ may be (XYZ) or (000). Then in (010) or (XyZ,) we are to see that X is a subidentical of y, and y of Z, whence X is the same of Z. But Xy and yZ warn us to write **[101]** *contraries* for the first and second premises and y to superaccent the middle letter: whence $C_{\prime}C'D_{\prime}$ is the syllogism expressed by the names XYZ. The equivalent forms—(101) and (zYx) express it by saying that z is a subidentical of Y and Y of x, whence z is a subidentical of x.

I now look at the strengthened particular syllogisms. All inference which is fundamental, that is, which will come from nothing weaker than the premises given, has been reduced to the one easy case of "the contained of the contained is contained." The strengthened particular, the type of which is $A'A_{\prime}I_{\prime}$, obeying the inverse rule of formation, and written at more length in $Y)X+Y)Z=XZ$, may be stated thus "all names are common as to what they contain in common." If we denote this strengthened syllogism by XYZ|, a symbol intended to imply something between XYZ and XYZ) in the amounts of quantity introduced, we shall find that the eight strengthened syllogisms must be represented by

$A'A_{\prime}I_{\prime}$ = XYZ|
$A'E_{\prime}O_{\prime}$ = XYz|
$E'A'O_{\prime}$ = Xyz|
$E'E'I_{\prime}$ = XyZ|
$A_{\prime}A'I'$ = xyz|
$A_{\prime}E'O'$ = xyZ|
$E_{\prime}A_{\prime}O'$ = xYZ|
$E_{\prime}E_{\prime}I'$ = xYz|

The rules of connexion are precisely those for the particular syllogisms: and inversion is absolutely ineffective. Thus $XYZ^{|}=ZYX_{|}$.[17]

A few words will serve to dispose of the *mixed complex syllogisms* in which a complex premise is combined with a simple one, universal or particular. First, when a complex and a universal are premised, and signs and accents are as in the *direct rule* (page 92), the conclusion is as it would be if the A were heightened into D, or E into C. Thus $E_{\prime}D'$ gives $C_{\prime}$, the same as $C_{\prime}D'$. For $E_{\prime}$ is C or $C_{\prime}$, and both CD' and $C_{\prime}D'$ give $C_{\prime}$, but with different quantities. But if the premises be constructed on the inverse rule, there is no more inference than can be obtained when the complex premise is lowered into a universal: or we have only a strengthened particular. Thus in $D_{\prime}E'$ or $(A_{\prime}+O')E'$, $A_{\prime}E'$ gives the strengthened particular $A_{\prime}E'O'$, and $O'E'$ is inconclusive. And when the complex premise is combined with a particular, we have only what would follow if the complex premise were lowered into a universal. Thus $D_{\prime}I'$, or **[102]** $(A_{\prime}+O')I'$ can only give $A_{\prime}I'I'$; and $D'I'$ or $(A'+O_{\prime})I'$ gives no conclusion, for $A'I'$ is inconclusive.

The classification of opponent forms may be thus treated. We know that opponent forms of AEE, for instance, be it $A_{\prime}E_{\prime}E_{\prime}$ or $A'E'E'$, must be IEO and AII. Now whether $A_{\prime}E_{\prime}E_{\prime}$ shall have $I_{\prime}E_{\prime}O_{\prime}$ or $E_{\prime}I_{\prime}O'$, whether $A'I_{\prime}I_{\prime}$ or $I_{\prime}A_{\prime}I_{\prime}$, depends upon the introduction of a new and arbitrary notion of the order to be adopted. Our first syllogism being described by XY, YZ, XZ, the opponent which ends in the contradiction of the first premise is in XZ, YZ, XY; which, keeping Z middle, is either to be described with reference to XZ, ZY, XY, or to YZ, ZX, YX. Now in adopting the first of these three orders, there is nothing which compels us therefore to prefer the second to the third, or *vice versâ*.

The effect of the change of order which consists in the interchange of Z and X is as follows. The premises change places; A and O with altered accents, altered also in the conclusion, E and I with unaltered accents. Thus $A_\prime I^\prime I^\prime$ becomes $I^\prime A^\prime I^\prime$; $E^\prime O^\prime I_\prime$ becomes $O_\prime E^\prime I_\prime$. Accordingly, it is matter of new arrangement whether for instance, $I_\prime E_\prime O_\prime$ or $E_\prime I_\prime O^\prime$ shall be called the opponent of $A_\prime E_\prime E_\prime$; and I prefer to give the name to both. The consequence is, the following distribution of opponents;—

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[17] [The following instruction is given in the Table of Contents. "Read the symbols of the strengthened syllogisms so as to begin from the middle term in both premises: thus, $Xyz^{|}$ is $y)X + y)z = Xz$."—Ed.]

AA	AO	OA	AE	EA	AI	IA	EI	IE.
EE	EO	OE						

The three sets represent letters combined in representation of premises: the first two containing six syllogisms each, the third twelve. The third must be divided into two sets of six each, in one of which the subaccents are in greater number, in the other the superaccents. There are then four sets in all. Pick any two out of a set, which only differ in change of order: these two have the same opponent forms, namely, the other four of the set. For instance, $A^\prime I_\prime I_\prime$ and $I_\prime A_\prime I_\prime$, in which subaccents predominate. Take AE, EA, EI, IE, and complete syllogisms in such manner as to make subaccents predominate: giving $A_\prime E_\prime E_\prime$, $E_\prime A^\prime E_\prime$, $E_\prime I_\prime O^\prime$, $I_\prime E_\prime O_\prime$. The last four are the opponents of the first two.

In the set of strengthened particulars the opponent forms will be found to be universals weakened in the conclusion without [103] being weakened in the premises. Thus $A_\prime A^\prime I^\prime$ has $A^\prime E^\prime O^\prime$ for one of its opponents: but $A^\prime E^\prime$ may produce the universal conclusion $E^\prime$ as well as its weaker form $O^\prime$.

Some readers, particularly those who have a tincture of algebra, are more helped by symbolic notation than by language: with others it is the converse. To suit the latter, observe that the language of page 78 may easily be adapted to simple syllogisms. Thus $A_\prime$ being subaffirmation, $I_\prime$ may be some subaffirmation, $O^\prime$ may be some supernegation; and so on. Thus instead of $E^\prime I^\prime O_\prime$ we may say that "supernegation of some superaffirmation gives some subnegation." Practice in this language would make the phrase suggest something more than the notation it is derived from. The phrase refers to Z: there is a term partially superaffirmed of Z, namely Y; and a complete subnegative of Y, namely X. The partial subaffirmation declares some things neither Y nor Z; the complete supernegation declares that whatever is not Y is X. Consequently there are some Xs which are not Zs: or X is a partial subnegative of Z. This subject will be resumed.

In what precedes are two views of the deduction of all the varieties of syllogism. The first, taking the complex syllogism as the source, connects the strengthened syllogisms and the particular ones with the universals, and thus in fact reduces every thing to the constituents of $D_{\prime}D_{\prime}D_{\prime}$ or $DD_{\prime}D_{\prime}$. The second proceeds from $A_{\prime}A_{\prime}A_{\prime}$, $A'A_{\prime}I_{\prime}$, $A_{\prime}I'I'$, and $I_{\prime}A_{\prime}I_{\prime}$ and forms the classes of universal, strengthened, and particular, syllogisms by substituting contraries in every way in which it can be done. These two systems have close connexion, but not so close as might perhaps be thought: for $I_{\prime}A_{\prime}I_{\prime}$ is not one of those which are connected with $A_{\prime}A_{\prime}A_{\prime}$ in the formation of a complex syllogism.

The two new views which I now proceed to give are also closely connected, and different from the former ones, in which we held it equally admissible to refer one of the concluding terms to the middle, as in X)Y, or the middle to one of the concluding terms, as in Y)X. But now I ask whether it be not possible so [104] to construct the system, that we may first lay down the middle term and its contrary, as constituting the universe of the syllogism, and then complete the premises and their conclusion, by properly laying down the concluding terms in their places. We may succeed, if, in the first instance, we consider none but convertible propositions. And this we can do; for universal exclusion and particular inclusion comprehend all assertion. Thus universal inclusion is only universal exclusion from the contrary, and particular exclusion is only particular inclusion in the contrary.

Setting out then with the middle term and its contrary, and restricting ourselves to E and I, let E signify (universal) exclusion from the middle term, and e from its contrary; let I signify (particular) inclusion in the middle term, and i in its contrary. Choosing a pair of concluding terms, we reject II, Ii, and ii on grounds already demonstrated, and very easily seen in this view, and proceed to consider Ee, EE and ee, EI and ei, Ei and eI.

Ee. From this a universal conclusion must follow. If one term be completely excluded from the middle and the other from its contrary, the terms are completely excluded each from the other. The fundamental forms are,

$$E_{\prime}A^{\prime}E_{\prime}, \quad X.Y+Z.y=X.Z\,; \quad A_{\prime}E_{\prime}E_{\prime}, \quad X.y+Z.Y=X.Z$$

and by use of XZ, Xz, xZ, xz, we thus bring out the eight universal syllogisms.

EE and ee. From these a particular inclusion must follow. Exclusion of both terms from a third, gives partial inclusion of their contraries in each other: for all that third term belongs to the contraries of the other two. The fundamental forms are,

$$E_{\prime}E_{\prime}I^{\prime}, \quad X.Y+Z.Y=xz\,; \quad A_{\prime}A^{\prime}I^{\prime}, \quad X.y+Z.y=xz$$

from which, as before, the eight strengthened syllogisms are deduced.

EI and ei. From these a particular inclusion must follow. The exclusion of one term from a third, and the inclusion of part of a second term in that third, tell us that part of the particularized term is in the contrary of the universalized term. The fundamental forms are,

[105] $$E_{\prime}I_{\prime}O^{\prime}, \quad X.Y+ZY=Zx\,; \quad A_{\prime}O^{\prime}O^{\prime}, \quad X.y+Zy=Zx$$
$$I_{\prime}E_{\prime}O_{\prime}, \quad XY+Z.Y=Xz\,; \quad O_{\prime}A^{\prime}O_{\prime}, \quad Xy+Z.y=Xz$$

from which the sixteen particular syllogisms are deduced.

Ei and eI. From these no conclusion can be drawn. All that is signified is that one concluding term is wholly excluded from a third, and the second partially excluded (or included in the contrary).

It thus appears that a syllogism with one particular premise is valid when the premises reduced to convertible forms, show the middle term in both or the contrary of it in both; otherwise, invalid. Also, that the conclusion in its convertible form, takes directly from the particular premise and contrariwise from the universal.

It also appears that a syllogism with both premises universal is always valid; with a universal conclusion when the premises (made convertible) show one the middle term and the other its contrary; with a particular conclusion when both show the middle term or both its contrary. And the convertible form of the conclusion takes directly from both in the first case, and contrariwise from both in the second.

The other view which I here propose is really a different mode of looking at that just given. By the time we have made every name carry its contrary, as a matter of course, we become prepared to take the following view of the nature of a proposition. A name by itself is a sound or a symbol: its relation to things (be they objects or ideas) is twofold. There may be *in rerum natura* that to which the name applies, or there may not. I do not here speak of how many things there may be to which a name applies: it is not essential to know whether they be more or fewer, either absolutely or relatively. The introduction of *contraries* may be made the expulsion of *quantity*. With reference to application, then, let a name be called *possible* or *impossible* according as the thing to which it applies can be found or not.

A name may be compounded of others; the compound name being that of everything to which all the components apply. Thus *wild animal* is the name of all things to which both the names *wild* and *animal* apply. To call this compound name [106] impossible is to say that there is not such a thing as a wild animal: to call it possible is to say that there is such a thing.

X and Y being two names, the compound name may be represented by XY when possible, and by XY) when impossible. This does not alter the meaning of our symbol XY, as hitherto used: as yet it has been "there are Xs which are Ys" and now it is "XY, the name of that which is both X and Y, is the name of some thing or things;" and these two are the same in meaning, so far as their use in inference is concerned. Nor need XY), as just defined, be treated as a departure from, otherwise than as an extension of, the use of X)Y. In X)Y, we assert that X is something, namely Y: in X) we assert that X is *nothing whatever*. The proper notation, however, for indicating that the name X has no application, is X)u, u being the contrary of U, which last includes everything in the universe spoken of; so that u may denote nonexistence.

The proposition "Every X is Y" asserts that Xy is the name of nothing, or X)Y=Xy). Similarly "No X is Y" asserts that XY is the name of nothing, or X.Y=XY). But "Some Xs are Ys" and "Some Xs are not Ys" merely assert the possibility of the names XY and Xy.

A syllogism, then, is the assertion that from the possibility or impossibility of the names produced by compounding X or x, Z or z, each with Y or y, may be inferred the possibility or impossibility of a name compounded of X or x with Z or z. The rules of the last system are now so easily changed into the language of the present one, that it is hardly worth while to state more than one for example. Thus, if X compounded with Y, and Z compounded with y, both give impossible names, then X compounded with Z gives an impossible name. This is XY)+Zy)=ZX) or X.Y+Z.y=Z.X, or $E_{\prime}A^{\prime}E_{\prime}$.

The view here taken of compound names will be extended in the next chapter.

[107]

CHAPTER VI.

ON THE SYLLOGISM.

WHEN the premises of a syllogism are true, the conclusion is also true, and when the conclusion is false, one or both of the premises are false. There are two kinds of modifications which it may be useful to consider: those which concern the entrance of the proposition into the argument; and those which affect the connexion of the subject and predicate.

As to the proposition itself, it may be true or false absolutely, or it may have any degree of truth, credibility, or probability. This relation will be hereafter considered; and, according to the principles of Chapter IX. so far as the proposition is probable it is credible, and so far as it is credible, it is true. But as to other modes of looking at the syllogism, are we entitled to say that every thing which can be announced as to the premises may be announced in the same sense as to the conclusion? The answer is, that we cannot make such announcement absolutely; but of the premises *as derived from that conclusion* we can make it. In what manner soever two premises are applicable, their conclusion as from those premises is also applicable: because the conclusion is in the premises. For instance, in the syllogism "all men are trees, all trees are rational, therefore all men are rational," the premises are absurd and false, and the conclusion taken independently is rational and true: but that conclusion, as from those premises, is as absurd as the premises themselves. Again, in "all pirates are convicted, all convicts are punished, therefore all pirates are punished," the premises are *desirable*, and so is the conclusion with those premises. But the conclusion is not desirable in itself: as that pirates should be punished with or without trial. Neither may we say "X ought to be Y and Y ought to be Z, therefore X ought to be Z" except in this manner, that we affirm X ought to be Z in a particular way. We may not even say that when "X ought to be Y, and Y *is* Z" it follows that "X ought to be Z," for it may be that Y ought not to be Z. Thus a royalist, in 1655, would say that the hundred excluded [108] members of Cromwell's parliament ought to be allowed to take their seats, and also that all who took any seats in that parliament were rebels; but he would not infer that the hundred members ought to be rebels. There is nothing which, being the property of the premises, is necessarily the independent property of the conclusion, except *absolute truth.* It should be noted that in common language and writing, the usual meaning of conclusions is that they are stated as of their premises and to stand or fall with them, even as to truth. Though a conclusion may be true when its premises are false, the proponent does not mean, for the most part, to claim more than his premises will give, nor that any thing should stand longer than the premises stand.

Next, we are not to argue from what we may say of a proposition to what we may say of the instances it contains, except as to what concerns the truth of those instances, or else to what concerns the instances as parts of a whole. If I say "Every X is Y" I assert, no doubt, of each X independently of the rest: that is, the truth of "Every X is Y" involves the truth of "this X is Y." But if, to take something else, I maintain "Every X is Y" to be a desirable rule, I do not therefore assert "this X is Y" to be a desirable case, except upon an implied necessity that there should be a rule. And if I say that "every X is Y" is unintelligible, I do not say that "this X is Y" is unintelligible; and so on. Thus, where there must be a rule, as in law, "every man's house is his castle" is desirable, because there is but one alternative "no man's house, &c." But the proposition, by itself, may not be desirable as to the instance of a generally reputed thief or receiver.

There is one case, however, in which a term cannot be applied to the general proposition, unless it can be applied in a higher degree to the instances. The proposition "Every X is Y" cannot be announced as of any degree of probability, unless each instance has a much higher degree of probability. If μ, ν, ϱ, &c. be the probabilities of the several instances, supposed independent, that of the proposition (Chapter IX.) is $\mu\nu\varrho$ . . . which product must be less than that of any one of the fractions of which it is formed.

I now come to the consideration of circumstances which modify the internal structure of the premises themselves. And first of conditions.

[109] A *conditional* proposition is only a grammatical variation of the ordinary one; as in "If it be X, then it is Y." The common form of this, "Every X is Y," is called *categorical*, or *predicative*. Of the two forms, categorical and conditional, either may always be reduced to the other; as follows,

"Every X is Y" or "If X, then it is Y"
"No X is Y" or "If X, then it is not Y"

The particular propositions might be given conditionally in various ways, but the transformation is not so common. Thus "some Xs are Ys" might be "if X, then it may be Y" or "if X, then Y must not therefore be denied of it," &c.

Of the two common subject-matters of names, ideas and propositions, it is most common to apply the categorical form to the first, and the conditional form to the second: in truth we might call the conditional form a grammatical convenience for the expression of dependence of propositions on one another, and of names which require complicated forms of expression. Thus in pages 2 and 3, the conditional forms, containing *if*, are more simple than the corresponding categorical forms.

A condition may be either *necessary*, or *sufficient*, or both.

A necessary condition is that without which the thing cannot be ; a sufficient condition is one with which the thing must be. In pages 73, 74, I have sufficiently pointed out the completeness of the connexion between the conditional and the categorical forms. In any one case the sufficient must contain all that is necessary, and may contain more.

After what is said in page 23, it is not necessary to dwell on the reduction of a conditional * syllogism to a categorical one. The premises contain the conclusion : whatever gives us the premises, gives us the conclusion. But I think that the reduction of conditional to categorical forms, though just, and, for inference, complete, is not the representation of the whole of what passes in our minds.

As an example of what I mean, look forward to the numerical system of Chapter VIII. Precedent to all propositions, **[110]** there are the numerical conditions which prescribe the limits of the universe under consideration. Say there are 250 instances in that universe : this is the first condition. Of these 100 are Xs and 200 are Ys ; giving a second and third condition. If we take a proposition, as 20XY, and ask whether it be spurious or not, we have reference to the three conditions understood. But this is not necessary : for it would be possible categorically to express these conditions by " 20Xs out of 100 in a universe of 250 instances containing 200 Ys are to be found among those 200 Ys." It is of course the rule of brevity not to drag about these conditions with every proposition which is employed, but rather to state them once for all. There is however something more. The conditions are a restriction upon the arguments intended to be introduced, and a restriction throughout. The attachment of them to each individual proposition does not express this : if they be seen in twenty consecutive propositions, there is no more than a presumption that they are to be seen in the twenty-first. It is better that the limits allowed should be marked out by one boundary than that the several arguments should each have a description of the boundary to itself.

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* Wallis, as far as I know, was the first who asserted that all syllogisms are, or can be made, categorical. He did this in the second thesis attached to his logic, headed *Syllogismi Hypothetici, aliique Compositi, referendi sunt omnes ad Aristotelicos Categoricorum Modos.*

Just as a universe of names is defined by specifying one or more names to constitute collectively the *summum genus*, or *universe*, so one of propositions may be defined by stating propositions which are to be true, or which are not to be contradicted, as the case may be. These propositions may be conditions preceding all, or some only, of the premises which are used in argument ; or some may precede some, and others others. In analysing arguments, it would be found that many propositions which enter as premises, enter each with a condition understood, and well understood, to be granted. Whatever the conditions may be, so long as the consequent propositions act logically together to produce the final result, then that same result depends at last only on the conditions, and must be affirmed when the conditions, and their connexion with their consequents, are affirmed. But then it must be understood that the result also stands upon the conditions, and may fall with them. Let us now examine the common syllogism, and see whether there be any preceding conditions, on which the result depends.

On looking into any writer on logic, we shall see that *existence* [111] is claimed for the significations of all the names. Never, in the statement of a proposition, do we find any room left for the alternative, *suppose there should be no such things*. Existence as objects, or existence as ideas, is tacitly claimed for the terms of every syllogism. The existence of an idea we must grant whenever it is distinctly apprehended, and (therefore) not self-contradictory : we cannot for instance admit the notion of a lamp which is both metal and not metal ; but, as an idea, we are at liberty to figure to ourselves such a lamp as that with which Aladdin made his fortune. An attempt at a self-contradicting idea is no idea ; we have not that apprehension of it in which an idea consists : but in no other way can we say that the attempt to produce an idea fails. It may then be more convenient here to dwell on *objective* definition of terms, as more easily conceived with relation to existence and non-existence. Accordingly, let us take the propositions X)Y and X.Y, of the character of which the particulars must partake, as to the point before us. By the meaning of y, in relation to Y, it follows that every thing is either Y or y: if we say that Y does not exist, then every thing is y. If then X exist, and Y do not, the proposition X)Y, or X.y is false, and X)y, or X.Y is true. If neither X nor Y exist, I will not so far imitate some of the questions of the schools as to attempt to settle what nonexisting things agree or disagree. If Y exist, but not X, then y)x is certainly true, but not thence X)Y, for when x is, as here, the whole universe, the proof of y)x=X)Y fails to present intelligible ideas, that is, fails to be a proof. But Y)x or Y.X is true.

If all my readers were mathematicians, I might pursue these extreme cases, as having interest on account of their analogy with the extreme cases which the entrance of zero and of infinite magnitude oblige him to consider. But as those who are not mathematicians would not be interested in the analogy, and those who are can pursue the subject for themselves, I will go on to say that the preceding order is not the natural one. We cannot, to useful purpose, laying down the truth of the proposition, *first*, then proceed to enquire how the non-existence of one or both terms affects the proposition. The existence of the terms must be first settled, and then the truth or falsehood of the proposition. The affirmative proposition requires the existence of both terms: **[112]** the negative proposition, of one; being necessarily true if the other term do not exist, and depending upon the matter, as usual, if it do exist.

Let us make the existence of the terms to be preceding conditions of the propositions. The syllogism $A_1A_1A_1$ is then as follows,

If X and Y both exist,   Every X is Y
If Z also exist   Every Y is Z
Therefore If X, Y, Z all exist   Every X is Z.

As to the concluding terms, X and Z, they remain, as it were, to tell their own story. Whatever conditions accompany their introduction unto[18] the premises, these same conditions may be conceived to accompany them in the conclusion. But the middle term disappears: and, not showing itself in the conclusion, the conditions which accompany it must be expressly preserved. The conclusion then is "every X is Z, if Y exist" which may be thrown into the form of a dilemma, "Either every X is Z, or Y does not exist."

But taking X and Z to exist, let us consider the following syllogism, *as it appears to be,*

Every X is (Y, if Y exist)
Every (Y, if Y exist) is Z
Therefore Every X is Z.

If this be not a valid syllogism, what *expressed* law of the ordinary treatises does it break? The middle term, a curious one, is strictly middle: but there is no rule for excluding middle terms of a certain degree of singularity. That it does break, and very obviously, an implied rule, I grant. And as to this work, the rule laid down in Chapter III. is broken in its second condition (page 50). The two uses of the word *is* do not amount to one such use as is made in the conclusion. That X is (conditionally) Y which is (on the same condition) Z, gives that X is (on the same condition) Z. Accordingly, the absolute conclusion is only true upon such conditions as give the middle term absolute existence.

But it must be particularly noted that it is enough if this ex-[113]istence be given to the middle term by the fulfilment of the conditions which precede the entrance of one of the concluding terms. The condition of the act of inference is, that the comparison must be really made, if the terms to be compared with the middle term really exist, or, which is the same, if the conditions under which they are to enter be satisfied. The other terms being ready, there must

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[18] [Presumably a misprint for "into."—Ed.]

then be a real middle term : and there will be, if the mere entrance of one of the concluding terms be proof of the existence of a middle term ; while, if the other terms cannot be brought in, from nonexistence, there is no occasion to inquire about a middle term, for it is otherwise known that the comparison cannot be completed. I will take two concrete instances, in the first of which one of the concluding terms, if existing, is held to furnish a middle term as real as itself, and in the second of which no such supposition occurs. Of course I have nothing here to do with the truth of the premises.

Philip Francis, (if the author of Junius), was an accuser whose silence was simultaneous with a government appointment : an accuser &c. reflects disgrace upon the government (if they knew that their nominee *was* the accuser) : therefore Francis (if &c.) reflects disgrace upon government (if &c.).

Homer (if there were such a person) was a perfect poet (if ever there were one) : a perfect poet (if &c.) is faultless in morals : therefore Homer (if &c.) was faultless in morals.

The first inference is good, even though we grant that our only possible mode of knowing of the existence of an accuser &c. is by establishing that Francis was Junius : it is even good against one who should assert that the accuser &c. is a contradiction in terms in every actual and imaginable case except that of Junius.

In the second case, we put it that the man Homer (if he ever existed ; some critics having contended for the contrary) was a perfect poet, if ever there were one. There may never have been one ; and then Homer (existent or nonexistent) was not a perfect poet. There is no condition here, which being fulfilled, is held to amount to an assertion that the middle term must have existed : but the condition of the existence of the middle term is independent. Accordingly, the second inference is not good : it should be Homer (if &c.) was a perfect poet, if ever there were one : that is, or else there never was a perfect poet.

[114] These points refer to the matter of a syllogism, and not to the form; or rather, perhaps, hold a kind of intermediate relation.

There is another process which is often necessary, in the formation of the premises of a syllogism, involving a transformation which is neither done by syllogism, nor immediately reducible to it. It is the substitution, in a compound phrase, of the name of the genus for that of the species, when the use of the name is particular. For example, " man is animal, therefore the head of a man is the head of an animal " is inference, but not syllogism. And it is not mere substitution of identity, as would be " the head of a man is the head of a *rational animal* " but a substitution of a larger term in a particular sense.

Perhaps some readers may think they can reduce the above to a syllogism. If *man* and *head* were connected in a manner which could be made subject and predicate, something of the sort might be done, but in appearance only. For example, " Every man is an animal, therefore he who kills a man kills an animal." It may be said that this is equivalent to a statement that in " Every man is an animal; some one kills a man; therefore some one kills an animal," the first premise, and the second premise *conditionally*, involve the conclusion as *conditionally*. This I admit: but the last is not a syllogism: and involves the very difficulty in question. " Every man *is* an animal; some one *is* the killer of a man ": here is no middle term. To bring the first premise into " Every killer of a man is the killer of an animal " is just the thing wanted. By the principles of chapter III, undoubtedly the copula *is* might in certain inferences be combined with the copula *kills*, or with any verb. But so simple a case as the preceding is not the whole difficulty. If any one should think he can syllogize as to the instances I have yet given, let him try the following. " Certain *men*, upon the report of certain other *men* to a third set of *men*, put a fourth set of *men* at variance with a fifth set of *men*." Now every man is an animal:

and therefore "Certain *animals*, upon the report of certain other *animals*, &c." Let the first description be turned into the second, by any number of syllogisms, and by help of "Every man is an animal."

The truth is, that in the formation of premises, as well as in their use, there is a postulate which is constantly applied, and therefore of course constantly demanded. And it should be demanded openly. It contains the *dictum de omni et nullo* (see the next chap-[115]ter), and it is as follows. For every term used universally *less* may be substituted, and for every term used particularly, *more*. The species may take the place of the genus, when all the genus is spoken of: the genus may take the place of the species when some of the species is mentioned, or the genus, used particularly, may take the place of the species used universally. Not only in syllogisms, but in all the ramifications of the description of a complex term. Thus for "men who are not Europeans" may be substituted "animals who are not English." If this postulate be applied to the unstrengthened forms of the Aristotelian Syllogism, (page 17) it will be seen that all which contain A are immediate applications of it, and all the others easily derived.

I now pass to the consideration of the invention of names, and of the distinctions which are made to exist for the want of it.

Any one may invent a name, that is, may choose a sound or symbol which is to apply to any class of ideas or of objects. The class should, no doubt, be well defined: but small caution is here necessary, for invented words are generally much more definite than those which have undergone public usage. They come from the coiner's hand as sharp at the edge as a new halfpenny: and in process of time we look in vain for any edge at all. The right of invention being unlimited, and the actual stock having been got together without any uniform rule of formation, *there can be no reason why we should admit any distinction which can be abrogated by the invention of a name*, so far as inference is concerned. I do not dispute that the modes of supplying the want of names may be of importance in many points of view: what I deny is, that they create any peculiar modes of inference.

The invention of names must either be by actually pointing out objects named, or by description in terms of other names. With the former mode of invention, as "let this, that, &c. (showing them) be called X" we can have nothing to do. As to the latter, we may make a symbolic description of the process by joining together the names to be used, with a symbol indicative of the mode of using them, in extension of the system in page 106. Thus, P, Q, R, being certain names, if we wish to give a name to everything which is all three, we may join them thus, PQR: if we wish to give a name to every thing which is either of the three (one or more of them) we may write P,Q,R: if we want to signify any thing that is either both P and Q, or R, we have **[116]** PQ,R. The contrary of PQR is p,q,r; that of P,Q,R is pqr; that of PQ,R is (p,q)r: in contraries, conjunction and disjunction change places. This notation would enable us to express any complication of the preceding conditions: thus, to name that which is one and one only of the three, we have Pqr, Qrp, Rpq; for that which is two and two only, PQr, QRp, RPq. Thus, XY includes the instances common to X and Y; but X,Y includes all X and all Y: accordingly X,Y is a wider term than XY, except when X and Y are identical. As in page 106, XY, the term, supposed to exist, is XY, the proposition of chapter IV; if we wish to distinguish, we may make X-Y the term, and XY the proposition, the hyphen having its common grammatical use. Thus, X-Y P-Q tells us the same as XYP-Q, both meaning, *for inference*, no more than that there exist objects or ideas to which the four names are applicable. But the first tells it thus, some XYs are PQs; and the second thus, some things are Xs, Ys, and PQs.

With respect to this and other cases of notation, repulsive as they may appear, the reader who refuses them is in one of two circumstances. Either he wants to give his assent or dissent to what is said of the form by means of the matter, which is easing the difficulty by avoiding it, and stepping out of logic: or else he desires to have it in a shape in which he may get that most futile of all acquisitions, called a *general idea*,* which is truly, to use the contrary adjective term as colloquially, *nothing particular*, a whole without parts.

If the difficulty of abstract assertion be to be got over, the easiest way is by first conquering that of abstract expression, to the extent of becoming able to make a little use of it.

Suppose we ask for the alternative of the following supposition, "Both X, and either P, or Q and one of the two R or S." This is no impossible complication: for instance, "He was rich, and if not absolutely mad, was weakness itself subjected either to bad advice or to most unfavourable circumstances." The representation of the complex term is $X\{P, Q(R,S)\}$; of the contrary, **[117]** $x, p(q,rs)$ or x,pq,prs. If not the above, he was either not rich, or both not mad and not very weak, or neither mad nor badly advised, nor unfavourably circumstanced.

When a name thus formed, whether conjunctively or disjunctively, enters a simple inference, it gives rise to what have been called the *copulative* syllogism, the *disjunctive* syllogism, and the *dilemma*. The two last are not well distinguished by their definitions as given: the disjunctive syllogism seems to be that in which *names* are considered disjunctively, the dilemma that in which *propositions* are so used. But a proposition entering as part of a proposition, enters merely as a name, the predicates being usually only *true* or *false*, or some equivalent terms. A proposition may only enter for its matter, or it may enter in such a way that its truth is the matter: in this last case it is only as a name that it is the subject of inference. Thus, " It is true that he was fired at " is " the assertion (that he was fired at) is a true assertion." I believe the best way would be to apply the term *disjunctive* argument so as to include the dilemma, marking by the latter word (as a term rather of rhetoric than of logic) every argument in which the disjunctive proposition is meant to be a difficulty for the opponent on every case, or *horn*, of it.

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* "Je vous avoue, dit ...., que j'ai cru en deviner quelque chose, et que je n'ai pas entendu le reste. L'abbé de .... à ce discours, fit réflexion que c'était ainsi que lui-même avait toujours lu, et que la plupart des hommes ne lisaient guere autrement."

Whatever has right to the name P, and also to the name Q, has right to the compound name PQ. This is an absolute identity, for by the name PQ we signify nothing but what has right to both names. According $X)P+X)Q=X)PQ$ is not a syllogism, nor even an inference, but only the assertion of our right to use at our pleasure either one of two ways of saying the same thing instead of the other. But can we not effect the reduction syllogistically ? Let Y be identical with PQ ; we have then PQ)Y and Y)PQ, and also Y)P and Y)Q. Add to these X)P and X)Q, and we have all the propositions asserted. But we cannot deduce from them alone X)Y, the result wanted, by any syllogistic combination of the six. Nor must it be thought surprising that we cannot, by a train of argument, arrive at demonstration of it being allowable to give to anything which has right to two names, a third name invented expressly to signify that which has such right. We might as well attempt to syllogize into the result, that a person who sells the meat he has killed is a butcher.

[118] I lay stress upon this, to an extent which may for a moment appear like diligently grinding nothing in a mill which might be better employed, for two reasons. First, the young mathematician is very apt to try, in algebra, to make one principle deduce another by mere force of symbols : and the above attempt may show him what he is liable to. Secondly, I am inclined to suppose that the distinction drawn between the classes of syllogisms to which I presently come, and the ordinary categorical ones, is due to what must be described in my language as a want of perception of the absolute, *less than inferential* (so to speak) identity of X)P+X)Q and X)PQ. But all other propositions of the kind, however simple, may be made deductions. For instance, "if X be both P and Q, and if P be R, and Q be S, then X is both Q and S" is thus deduced: X)P+P)R =X)R, and X)Q+Q)S=X)S, and X)R+X)S is X)RS. Even P)R+Q)R=P,Q)R is deducible; being P)R+Q)R =r)p+r)q=r)pq=P,Q)R. Thus it is seen that, as soon as the *conjunctive* postulate is laid down, the identity of the corresponding disjunctive postulate with it may be shown. Next, if X must be either P or Q, or X)P,Q, and if P be always R, and Q be always S, then X)R,S may be *deduced* from the preceding.

First, that X)P and Y)Q give XY)PQ can be deduced; evident as it may be, it is a succession of applications. XY)X+X)P gives XY)P, and XY)Y+Y)Q gives XY)Q, and XY)P+XY)Q is XY)PQ by the postulate. Next, X)P,Q is pq)x, and P)R is r)p, and Q)S is s)q, whence, as just proved rs)pq. Now, rs)pq+pq)x=rs)x, which is X)R,S. It will be a good exercise for the reader to translate this proof into ordinary language.

I may now proceed to extend this idea and notation relative to propositions of complex terms. The complexity consists in the terms being conjunctively or disjunctively formed from other terms, as in PQ, that to which both the names P and Q belong conjunctively; and as in P,Q that to which one (or both) of the names P and Q belong disjunctively. The contrary of PQ is p,q; that of P,Q is pq. *Not both* is either not one or not the other, or not either. *Not either P nor Q* (which we might denote by :P,Q or .P,Q) is logically "*not* P *and not* Q" or pq: and this is then the contrary of P,Q.

[119] The disjunctive name is of two very different characters, according as it appears in the universal or particular form: so very different that it has really different names in the two cases, *copulative* and *disjunctive*. This distinction I here throw away: opposing *disjunctive*, (having one or more of the names) to *conjunctive*, (having all the names). The disjunctive particle *or* has the same meaning with the distributive copulative *and*, when used in a universal. Thus, "Every thing which is P or Q is R or S" means "Every P *and* every Q is R *or* S." But PQ is always "both P and Q in one." Accordingly

Conjunctive	PQR uses *and* collectively.
Disjunctive	P,Q,R in a universal uses *and* distributively, P,Q,R in a particular uses *or* disjunctively, in the common sense of that word.

"Either P or Q is true," is an ambiguous phrase, which is P,Q)T or T)P,Q according to the context.

The manner in which the component of a name enters, whether conjunctively or disjunctively, is to pass as it were for a part of the quality of the name itself. Thus the contrary of P (conjunctive, as indicated by the absence of the comma) is ,p (disjunctive, as indicated by the comma). To test this assertion about the mode of making contraries, let us ask what is that of "one only of the two P or Q?" We know it of course to be "both or neither." The name proposed is Pq, Qp and its contrary is (p,Q)(q,P), that is, one of the two p,Q, *and* one of the two q,P. It is then either pq, pP, qQ, or PQ: the second and third cannot exist, therefore it is pq, PQ, as already seen. I need hardly have remarked that (P,Q)(R,S) is PR, PS, QR, QS.

Observe that though X)PQ gives X)P, and that XPQ gives XP, we may not say that XY)P gives X)P, nor that X)P,Q gives X)P. But any disjunctive element may be rejected from a universal term, and any conjunctive element from a particular one. Thus P)QR gives P)Q and P,Q)R gives P)R. Also P.Q,R gives P.Q and PQ:R,S gives P:R. All these rules are really one, namely that PQ is of the same extent at least as PQR. This will appear from our rules of transposition presently given.

[120] Let change from one member of the proposition to the other be called *transposition*. I proceed to inquire how many transpositions the various forms will bear, and what they are. It will however be necessary to complete our forms by the recognition, as a proposition, of the simple assertion of existence or non-existence. By XU we mean that there are in the universe things to which the name X applies, and we speak only of such things under the name. Accordingly X)U and XU do not differ in meaning. By u, the contrary of U, we can only denote non-existence; thus X.U or X)u throws the name X out of consideration. Thus Y)X=U)X,y; Y.X=YX)u, &c. To signify, for instance, that X and Y are complements (contraries or subcontraries, page 75) we have U)X,Y, which our rules will transpose into xy)u, or x.y.

Having to consider subject and predicate, conjunctive and disjunctive, affirmative and negative, universal and particular, we must think of sixteen different forms. Thus the four forms of the universal affirmative are

XY)PQ; X,Y)PQ; XY)P,Q; X,Y)P,Q

It will be best here to neglect the contranominal converses of A and O equally with the simple converses of E and I: thus XY)PQ may be read as identical with p,q)x,y. There is also one obvious transposition which we must not merely neglect but throw out; since it does not give a result identical with its predecessor. I mean the transposition of M)PQ into MP)Q: the second follows from the first but not the first from the second. Also the corresponding change of M.P,Q into Mp.Q, for the same reason.

This being premised, the following are the rules;—

Direct transposition is the change from one member to the other without alteration of name or junction: *contrary*, with alteration of both.

The convertibles (E,I) allow direct transposition of conjunctive elements either way, from subject to predicate, or from predicate to subject: and these are the only direct transpositions. Thus X.YZ=XY.Z, and X-YZ=XY-Z.

The inconvertibles (A,O) allow contrary transposition of conjunctive elements from subject to predicate, and of disjunctive **[121]** elements from predicate to subject: best remembered by allowing SP to stand for *conjunctive* and PS for *disjunctive*. And these are the only contrary transpositions. Thus XY)M=X)M,y and M)X,Y=My)X.

An element that can be rejected cannot be transposed, and *vice versâ*. Thus X,Y)M gives X)M, and Y cannot be transposed.

The following table exhibits the varieties of the forms A and E, equivalents being written under one another, and conversions, contranominal or simple, opposite.

XY)P,Q	pq)x,y	XY.PQ	PQ.XY
Xp)Q,y	Yq)P,x	XP.QY	QY.XP
Xq)P,y	Yp)Q,x	XQ.PY	PY.XQ
X)P,Q,y	pqY)x	X.PQY	PQY.X
Y)P,Q,x	pqX)y	Y.PQX	PQX.Y
p)Q,x,y	XYq)P	P.QXY	QXY.P
q)P,x,y	XYp)Q	Q.PXY	PXY.Q
XYpq)u	U)P,Q,x,y	XYPQ.U	U.XYPQ
XY)PQ	p,q)x,y	XY.P,Q	P,Q.XY
X)PQ,y	[p,q]Y)x	X.[P,Q]Y	[P,Q]Y.X[19]
Y)PQ,x	[p,q]X)y	Y.[P,Q]X	[P,Q]X.Y
XY[p,q])u	U)[x,y],PQ	XY[P,Q].U	U.XY[P,Q]
X,Y)P,Q	pq)xy	X,Y.PQ	PQ.X,Y
[X,Y]p)Q	q)xy,P	[X,Y]P.Q	Q.[X,Y]P
[X,Y]q)P	p)xy,Q	[X,Y]Q.P	P.[X,Y]Q
[X,Y]pq)u	U)xy,P,Q	X,Y]PQ.U	U.[X,Y]PQ
X,Y)PQ	p,q)xy	X,Y.P,Q	P,Q.X,Y
[X,Y][p,q])u	U)xy,PQ[20]	[X,Y][P,Q].U	U.[X,Y][P,Q]

If for) we write (:) in the left hand divisions, and erase the (.) and use the hyphens of page 116,[21] on the right, we have the transpositions of O and I. And if we write p and q for P and Q on the left, and change the form X)Y into X.y, we thereby change the forms of A into those of E. If more than two elements were used, the transpositions would now be perfectly easy.

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[19] [Edition 1 by an oversight has [P, Q]YX.—Ed.]

[20] [Edition 1 by a misprint has [x,y]. See Table of Contents.—Ed.]

[21] [The first edition has 115 by an oversight.—Ed.]

It appears that there are no less than sixteen A forms into [122] which XY)P,Q may be varied: the reason is that both subject and predicate are transposibly constructed. But XY)PQ shows only a transposible subject; X,Y)P,Q only a transposible predicate: and these have only four forms each. Lastly, X,Y)PQ, having neither transposible, has only two forms. By transposibly constructed, I mean capable of having the elements separated by transposition. The whole term is always transposible: that is, the complete subject, or the complete predicate, may be looked on as conjunctive or disjunctive, at pleasure. Thus in X)Y, if we consider this as XU)Y,u, we may make this yU)x,u or y)x. So that the ordinary contranominal conversion may be considered as a case of the more general rule. Just as, in arithmetic, a number, 5, may be made to obey the laws of *a*+*b* as 0+5, or of *ab* as 1×5.

Syllogisms of complex terms might be widely varied, even if we chose to consider only each first case of the preceding table as fundamental. Thus

XY)P,Q+VW)P,Q=(x,y)-(v,w)   A,A'I'

would give sixty-four varieties of premises. I now proceed to show that the ordinary disjunctive and dilemmatic forms are really common syllogisms with complex terms, reducible to ordinary syllogisms by invention of names.

Example 1. Every S is either P, Q, R; no P is S; no Q is S; therefore every S is R. Let S represent "the true proposition" (singular), and let P, Q, R be names of propositions, and this then represents a very common form, which would be expressed thus "either A is B, or C is D, or E is F; but A is not B, C is not D; therefore E is F." I say that, where the necessary names exist, the final step of this could not be distinguished from a common syllogism; which accordingly it becomes by invention of names.

We have S)P,Q,R, whence Spq)R. But S.P and S.Q or S)p and S)q give S)pq, with which S)S combined gives S)pqS. And S)pqS+pqS)R=S)R. Let M be the name of what is S and not P and not Q, and the thing required is done. Here then is a syllogism of the ordinary kind, to one premise of which we are led by a use of the *conjunctive postulate* (page 116): the necessity for which is the distinction between the class we are considering and others. It happens here that two of the terms of our final syllogism are [123] identical: for Spq is of no greater extent than S. But the use made of S)S is perfectly legitimate.

Example 2. "If A be B, E is F; and if C be D, E is F; but either A is B or C is D; therefore E is F." This can be reduced to

$$P)R+Q)R+S)P,Q=S)R$$

which is immediately made a common syllogism by changing P)R+Q)R into P,Q)R.

Example 3. "From P follows Q; and from R follows S; but Q and S cannot both be true; therefore P and R cannot both be true." This may be reduced to

$$\begin{aligned} &P)Q+R)S+T.QS=T.PR \\ \text{or }\ &PR)QS \quad +T.QS=T.PR \end{aligned}$$

Example 4. "Every X is either P, Q, or R; but every P is M, every Q is M, every R is M; therefore every X is M." This is a common form of the dilemma; it is obviously reducible to P,Q,R)M+X)P,Q,R=X)M.

Example 5. "Every X is either P or Q, and every Q is X." This is wholly inconclusive, and leads to an identical result, as follows; X)P,Q gives Xp)Q, which with Q)X gives Xp)X, a necessary proposition.

Example 6. If we throw X)R into the form X)R,R, we have Xr)R, or "Every X which *is not* R *is* R," a contradiction in terms. But it evidently implies that there can be no Xs which are not Rs; and thus also we return to

X)R. Take " every X is either P, Q, or R ; every P is M ; every Q is M ; and every M is R." Here X)P,Q,R=Xr)P,Q, which with P,Q)M gives Xr)M, which with M)R gives Xr)R or X)R.

Example 7. " Every X is either P or Q, and only one." This gives two propositions, X)P,Q+X.PQ. Now X)XP,XQ is identical with X)P,Q, and this may be looked on as an extreme case of

$$X)P,Q+X)Y=X)PY,QY$$

but X.PQ gives XP)q and XQ)p, from which we can obtain

$$\begin{aligned} &X)XP,XQ+XP)q+XQ)p=X)p,q \\ \text{Hence } &X)P,Q+X)p,q=X)[P,Q,][p,q.] \\ &\qquad\qquad =X)Pp,Pq,Qp,Qq=X)Pq,Qp \end{aligned}$$

since Pp and Qq are subject to X.Pp and X.Qq. All this being **[124]** worked out in syllogistic detail, shows us that the transition from " Every X is P or Q, and no X is both " to " Every X is either P and not Q, or Q and not P " is capable of being made syllogistically. The student of logic may thus acquire the idea, which so soon becomes familiar to the student of mathematics, of perfectly self-evident propositions which are deducible from one another, as distinguished from those which are not.

Example 8. " Every X is one only of the two, P or Q ; every Y is both P and Q, except when P is M, and then it is neither ; therefore no X is Y." Here is a case in which it is the fact of the exception and not its nature which determines the inference : M may be anything. This ought to appear in our reduction : and it does appear in this way. From X)P,Q it is obvious that X)P,Q,R,S, and syllogistically demonstrable from X)P,Q, and Xrs)X. Now in the second premise we have

$$\begin{aligned} &Y)PQm,pqM, \text{ or } [p,q,M][P,Q,m])y \\ \text{or } &\quad pQ,Pq,PM,QM,pm,qm)y \end{aligned}$$

from which, by rejection, follows pQ,Pq)y. And the first premise is X)Pq,Qp. Whence X)y or X.Y.

It is not necessary to multiply examples: I will conclude this part of the subject by pointing out that the ordinary propositions X)Y, &c. are, with reference to their instances, disjunctively composed: the difference between the universal and particular lying in the latter being indefinite in the number of its instances. Thus, if there be three Xs and four Ys, the four propositions are, applying the name to each instance, as seen written at length in

X,X,X)Y,Y,Y,Y; X,X,X.Y,Y,Y,Y; (X,X,X)(Y,Y,Y,Y);
and (X,X,X):Y,Y,Y,Y.

The proposition in page 25, is a case of the preceding method. I leave the reader to show it, and also that the hypothesis is slightly overstated.

I now come to the *sorites*, the *heap* or chain of syllogisms, in which the conclusion of the first is a premise of the second, and so on. Take a set of terms, P, Q, R, S, &c. and let the order of reference be PQ, QR, RS, &c. Then $A_1A_1A_1A_1$ &c. is a sorites, and the only one usually considered thus,

$$P)Q+Q)R+R)S+S)T=P)T$$

[125] The first two links give P)R, which with the third gives P)S, which with the fourth gives P)T. Thus we have *links*, *intermediate conclusions*, and a *final conclusion*.

A great number of different sorites may be formed, under the following conditions,

The first particular proposition which occurs, be it link or conclusion, prevents any future link from being particular: for all the conclusions thence become particular.

Examine the cases of syllogism which proceed by the first rule of accentuation (page 92), that is, which have beginning and ending both universal, or both particular: these only can occur in a sorites, except at the end, or in the place where a particular proposition first enters. It will be found that the conclusion, when the argument goes on, must come after something connected with that which comes after it by the first rule of accentuation: except at the place where a particular conclusion comes in for the first time. For instance, $E_\prime E'$ gives $A_\prime$, which, still keeping conclusions universal, must be followed by $A_\prime$ or $E_\prime$, which follow E' by the first rule. Again, take $O_\prime E'$, which gives $I_\prime$; this must be followed either by $A_\prime$ or $E_\prime$, which follow E' by the same rule: and so on. Accordingly,

Any chain of universals, in which affirmation is followed by a like preposition, and negation by a different one, as $A_\prime A_\prime E_\prime A'$ $E' A_\prime E_\prime E'$, &c. may be part of the chain of a sorites. And the chain must be either of this kind wholly, or once only broken in one of two ways: either by the direct entrance of a particular proposition, or by a breach of the rule. In a chain of this kind, unbroken, the conclusions are affirmative or negative, according as an even or odd number of negatives goes to the formation of them. All the conclusions have the same accent as the first link.

Let a particular premise be introduced, as in $A_\prime E_\prime E' I'$ &c. The accent of the particular introduced must be the same as or contrary to that of the first link, according as the preceding number of negatives is odd or even. For the accent of the first link remains as long as the conclusion is universal, and a syllogism with the second premise particular follows the second rule. Thus, inserting the intermediate conclusions, the above is $A_\prime E_\prime (E_\prime) E' (A_\prime) I' (I')$. And after (I') must come A' or E', so that the first rule still continues. But the accent of the conclusions changes.

[126] Now let the rule of accentuation be broken. The *accent* of the conclusion still requires the first rule to be resumed. Thus, $E_\prime E'$ (rule unbroken) gives $A_\prime$, and $E_\prime E_\prime$ (rule broken) gives I', and $A_\prime$ requires $A_\prime$ or $E_\prime$ to follow E', while I' requires A' or E' to follow $E_\prime$. This one breach of rule only changes the conclusion from universal to particular. The accent of the conclusion changes as before.

The links of a sorites, then, are either a chain of universals following the first rule of accentuation, or such a chain with *one* breach of the rule, or such a chain with one particular inserted, of the same or contrary accent to the first link, according as the preceding negatives are odd or even, and made the commencement of the resumption of the rule (if broken). In all the cases the conclusion is affirmative or negative according as the preceding negatives are even or odd in number: the unbroken chain has a universal conclusion with the accent of the first link, and the broken one a particular with the contrary accent.

$A'E'E_{\prime}A'E'$	$E_{\prime}A'A_{\prime}E_{\prime}E'A_{\prime}$	$A_{\prime}E_{\prime}A'O_{\prime}A'E'$
$E'A'A'E'$	$E_{\prime}O'I'O'O'$	$E_{\prime}E_{\prime}I'I'O'$

Here are examples of the three kinds. The chain is in the first row, the intermediate and final conclusions in the second. Thus the second example presents the syllogisms $E_{\prime}A'E_{\prime}$, $E_{\prime}A_{\prime}O'$, $O'E_{\prime}I'$, $I'E'O'$, $O'A_{\prime}O'$; and at length is

P.Q+R)Q+R)S+S.T+t.u+U(V=V:P

The sorites usually considered are only $A_{\prime}A_{\prime}A_{\prime}$. . . . and $A'A'A'$. . . . . To these might be added without abandoning the Aristotelian syllogism, such as $A_{\prime}E_{\prime}A'A'A'$. . . ., $A_{\prime}E_{\prime}A'A_{\prime}A_{\prime}$. . . . . But it would not be very easy to follow the chain in thought without introducing the intermediate conclusions, and thus destroying the specific character of the process.

And just as the ordinary universal syllogism can be reduced to $A_{\prime}A_{\prime}A_{\prime}$, so the universal sorites can always be reduced to a chain of $A_{\prime}$. Thus $A'E'E_{\prime}A'E'$ or

Q)P+q.r+R.S+T)S+t.u =p.u
is u)T+T)S+S)r+r)Q+Q)P =u)P

[127]

CHAPTER VII.

ON THE ARISTOTELIAN SYLLOGISM.

FROM the time of Aristotle until now, the formal inference has been a matter of study. In the writings of the great philosopher, and in a somewhat scattered manner, are found the materials out of which was constructed the system of syllogism now and always prevalent: and two distinct principles of exclusion appear to be acted on. Perhaps it would be more correct to say that the followers collected two distinct principles of exclusion from the writings of the master, by help of the assumption that everything not used by the teacher was forbidden to the learner. I cannot find that Aristotle either limits his reader in this manner, or that he anywhere implies that he has exhausted all possible modes of syllogizing. But whether these exclusions are to be attributed to the followers alone, or whether those who have more knowledge of his writings than myself can fix them upon the leader, this much is certain, that they were adopted, and have in all time dictated the limits of the syllogism. Of all men, Aristotle is the one of whom his followers have worshipped his defects as well as his excellencies: which is what he himself never did to any man living or dead; indeed, he has been accused of the contrary fault.

The first of these exclusions is connected with the celebrated *dictum de omni et nullo*, namely, that what is distributively affirmed or denied of all, is distributively affirmed or denied of every some which that all contains. It is there said that in every syllogism the middle term must be universal in one of the premises, in order that we may be sure that the affirmation or denial in the other premise may be made of some or all of the things about which affirmation or denial has been made in the first. This law, as we shall see, is only a particular case of the truth: it is enough that the two premises together affirm or deny of more than all the instances of the middle term. If there be a hundred boxes, into which a hundred *and one* articles of two different kinds are **[128]** to be put, not more than one of each kind into any one box, some one box, if not more, will have two articles, one of each kind, put into it. The common doctrine has it, that an article of one particular kind must be put into every box, and then some one or more of another kind into one or more of the boxes, before it may be affirmed that one or more of different kinds are found together. This exclusion is a simple mistake, the mere substitution of the assertion that none but a certain law of inference *can* exist, for the determination that no other *shall* exist. Any one is at liberty to limit the inferences he will use, in any manner he pleases: but he may err if he declare his own arbitrary boundary to be a natural limit imposed by the laws of thought.

The other exclusion may involve, on the same terms, an error of the same kind; or may equally be the expression of arbitrary will: but there is what is more reasonably matter of opinion about it. Aristotle will have no contrary terms: not-man, he says, is not the name of anything. He afterwards calls it an indefinite or *aorist* name, because, as he asserts, it is both the name of existing and non-existing things. If he had here made the distinction between ideal and objective, he would have seen that *man* and *not-man* equally belong to both (objectively) existing and non-existing things: *man*, for example, belongs as a name to Achilles and the seven champions of Christendom, whether they ever existed in objective reality or not: and *not-man* belongs, in either case, to their horses. I think, however, that the exclusion was probably dictated by the want of a definite notion of the extent of the field of argument, which I have called the *universe* of the propositions. Adopt such a definite notion, and, as sufficiently shown, there is no more reason to attach the mere idea of negation to the contrary, than to the direct term.

The exclusion of contraries throws out the propositions E' and I', or x.y and xy, which cannot be expressed without either contraries, as in x.y=x)Y=y)X, and xy=x:Y=y:X, or reference to things not named by X and Y, as in " Every *thing* is either X or Y " and " Some things are neither Xs nor Ys," the most natural readings of " No not-Xs are not-Ys," and " Some not-Xs are not-Ys." There remain then six modes of connexion of X and Y, namely X)Y and Y)X, X:Y and Y:X, and XY(=[129]YX) and X.Y(=Y.X). These six are made eight; for in the common system, XY and YX are considered as distinct in form, and also X.Y and Y.X. But these eight are only treated as four: for reference to order is not made in the simple proposition. Thus X)Y and Y)X are both denoted by A, XY and YX by I, X.Y and Y.X by E, and X:Y and Y:X by O. But the standard of order which is neglected as to the proposition by itself, is adopted in the syllogism in the following manner.

The *predicate* of the conclusion is called the *major* term, and the subject of the conclusion the *minor* term. This language is fashioned upon the idea of an affirmative proposition, in which major and minor have reference to *magnitude*. In " every X is Z," Z is a name which entirely contains X and is therefore *at least as great* as X, *greater than or equal to* X. Here is, before it was introduced into mathematics, the idea now so familiar to the mathematician, of allowing his language to include the extreme limit of its meaning. When the same terms are applied to negative propositions, the notion of magnitudinal inclusion is lost; and major and minor, being still retained, must be presumed to refer to real or supposed importance. The premises are called major and minor, according as they contain the major or minor term of the conclusion: and the major premise is always written first. Accordingly, Z and X being the major and minor terms, there are four possible arrangements, which are called the four figures. Aristotle gives three, and tradition has it that Galen supplied the fourth in number and order.

$$1.\ \begin{array}{c}\mathrm{YZ}\\ \mathrm{XY}\\ \hline \mathrm{XZ}\end{array}\qquad 2.\ \begin{array}{c}\mathrm{ZY}\\ \mathrm{XY}\\ \hline \mathrm{XZ}\end{array}\qquad 3.\ \begin{array}{c}\mathrm{YZ}\\ \mathrm{YX}\\ \hline \mathrm{XZ}\end{array}\qquad 4.\ \begin{array}{c}\mathrm{ZY}\\ \mathrm{YX}\\ \hline \mathrm{XZ}\end{array}$$

To me, the most simple arrangement is that which takes up what was left off with, as in the fourth figure: and "X is in Y, Y is in Z, therefore X is in Z" is more natural than "Y is in Z, X is in Y, therefore X is in Z."

It is now plain, that whenever one only of the three propositions is convertible, there are two distinct ways in which the syllogism may be written: when two only, four: and when all three (if there were such a thing), eight.

[130] The system rejects all conclusions which may be made stronger: thus when X.Z follows, it does not allow X:Z to make a distinct form. But when X)Z is the conclusion, it does not reject ZX, for, not considering ZX as identical with XZ, it does not consider X)Z as a strengthened form of ZX. But it does not reject syllogisms in which as strong a conclusion can be deduced from a weaker premise: accordingly, we must search for Aristotelian forms among the strengthened syllogisms of chapter V, as well as among the fundamental ones. Now, taking all the forms which show neither E' or I', let us write down the symbols of them, and the number of cases we may expect from each. Moreover, since transformation of order makes no difference here, I put the syllogisms together as in page 96, into twelve pairs.

Fundamental $A_\prime A_\prime A_\prime$, $A'A'A'$, 1; $O'A_\prime O'$, $A'O_\prime O_\prime$, 1; $A_\prime O'O'$, $O_\prime A'O_\prime$, 1; $E'A_\prime E'$, $A'E'E'$, rejected; $I_\prime A_\prime I_\prime$, $A'I_\prime I_\prime$, 4; $E'O'I_\prime$, $O_\prime E'I_\prime$, rejected; $E_\prime A'E_\prime$, $A_\prime E_\prime E_\prime$, 4; $I'A'I'$, $A_\prime I'I'$, rejected; $E_\prime O_\prime I'$, $O'E_\prime I'$, rejected; $E'E_\prime A'$, $E_\prime E'A_\prime$, rejected; $I_\prime E_\prime O_\prime$, $E_\prime I_\prime O'$, 4; $E'I'O_\prime$, $I'E'O'$, rejected.

Weakened $A_\prime A_\prime I_\prime$, 1.

Strengthened $A'A_\prime I_\prime$, 1; $A_\prime A'I'$, rejected; $A'E_\prime O_\prime$,

$E_,A_,O'$, 2; $A_,E'O'$, $E'A'O_,$, rejected; $E'E'I_,$, rejected; $E_,E_,I'$, rejected.

There are then fifteen fundamental, one weakened, and three strengthened, forms of syllogism in the received system. I now put them down, with their derivations, forms of expression in full, ordinary symbols, figures into which they fall, and the magic words by which they have been denoted for many centuries, words which I take to be more full of meaning than any that ever were made.

Fundamental.

$A_,A_,A_,$	$A'A'A'$	Y)Z+X)Y=X)Z	AAA	I	*Barbara*
$O'A_,O'$	$A'O_,O_,$	Y:Z+Y)X=X:Z	OAO	III	*Bokardo*
$A_,O'O'$	$O_,A'O_,$	Z)Y+X:Y=X:Z	AOO	II	*Baroko*
$I_,A_,I_,$	$A'I_,I_,$	Y)Z+XY =XZ	AII	I	*Darii*
—	—	Y)Z+YX =XZ	AII	III	*Datisi*
—	—	ZY +Y)X=XZ	IAI	IV	*Dimaris*
—	—	YZ +Y)X=XZ	IAI	III	*Disamis*

[131] *Fundamental.*

$E_,A'E_,$	$A_,E_,E_,$	Y.Z+X)Y=X.Z	EAE	I	*Celarent*
—	—	Z.Y+X)Y=X.Z	EAE	II	*Cesare*
—	—	Z)Y+Y.X=X.Z	AEE	IV	*Camenes*
—	—	Z)Y+X.Y=X.Z	AEE	II	*Camestres*
$E_,I_,O'$	$I_,E_,O_,$	Y.Z+XY =X:Z	EIO	I	*Ferio*
—	—	Z.Y+XY =X:Z	EIO	II	*Festino*
—	—	Y.Z+YX =X:Z	EIO	III	*Ferison*
—	—	Z.Y+YX =X:Z	EIO	IV	*Fresison*

Weakened.

$A_,A_,I_,$	$A'A'I_,$	Z)Y+Y)X=XZ	AAI	IV	*Bramantip*

Strengthened.

$A'A_,I_,$	$A'A_,I_,$[22]	Y)Z+Y)X=XZ	AAI	III	*Darapti*
$A'E_,O_,$	$E_,A_,O'$	Y.Z+Y)X=X:Z	EAO	III	*Felapton*
—	—	Z.Y+Y)X=X:Z	EAO	IV	*Fesapo*

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[22] [So in Edition 1, but apparently by an oversight for $A_,A'I_,$—ED.]

The words which represent the different *moods* (as they are called) are usually collected under their figures in the following lines.

Barbara, Celarent, Darii, Ferioque prioris.
Cesare, Camestres, Festino, Baroko, secundæ.
Tertia Darapti, Disamis, Datisi, Felapton,
Bokardo, Ferison habet. Quarta insuper addit [23]
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

The vowels of the different words give the symbol of the syllogism; thus A,A,A, are seen in *Barbara*. The consonants in the first figure have no special meaning: but in the other figures every consonant except T and N (which are only euphonic) has its meaning as follows;—every mood of every figure can (with two exceptions) in one way or another, be reduced to a mood of the first figure: and the letters show the way of doing it. The initial tells to which mood the reduction brings us: thus Cesare is reduced to Celarent, and also Camestres; Festino is reduced to Ferio, and so on. The two exceptions are denoted by the letter K (as in Baroko and Bokardo); we shall presently notice them further. And S means that the preceding premise is to be simply converted. P, that what was called conversion *per acci-*[132]*dens* is to be made, ZX for X)Z, or X)Z for ZX: accordingly, P only occurs in the weakened or strengthened syllogisms. M means that the premises are to be transposed. Thus the meaning of the word *Disamis* is nothing less than what follows. "There is a syllogism in which the middle term is the subject of both premises, and when reduced to the first figure it becomes *Darii*: the major premise, which must be converted in reduction, is a particular affirmative: the minor premise, which must become the major one in reduction, is a universal affirmative: and the conclusion, which must be converted in reduction, is a particular affirmative." Thus,

YZ + Y)X = XZ *Disamis*
becomes Y)X + ZY = ZX *Darii*

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[23] [As has been already observed, the names of the moods Baroco and Bocardo are more correctly spelt with a c.—Ed.]

The moods *Baroko* and *Bokardo* do not admit of reduction to the first figure, by any fair use of the phrase: but the logicians[24] were determined they should do so, and they accordingly hit upon the following plan, which they called reduction *per impossibile.* AOO and OAO being the opponent forms (pages 88, and 102) of AAA, the two moods in question were connected with *Barbara* (whence their letter B) by showing that the latter would make the denial of their conclusions force one premise to contradict the other. Thus, *Baroko,* or if Z)Y and X:Y then X:Z was proved in the first figure as follows. If under these premises, X:Z be not true, then X)Z is true; but Z)Y is true: and Z)Y+X)Z, by *Barbara,* gives X)Y. But X:Y: therefore, if *Baroko* be not a legitimate form, X)Y and X:Y are both true at once, which is absurd. Had contraries been used, Z)Y+X:Y=X:Z would have been thrown into the first figure as y)z+Xy=Xz, *Darii,* or y.Z+Xy=X:Z, *Ferio.* And Y:Z+Y)X=X:Z, *Bokardo,* is seen reduced to the first figure in Y)X+zY=zX, *Darii.*

Aristotle did not use the fourth figure, considering it, as is said, to be only an inversion of the first. The introduction of it among the figures is attributed to Galen, and it does not often appear in ordinary works of logic before the beginning of the last century. If the order of the premises be inverted, so as to make the first figure appear, the major and minor terms will appear wrongly placed in the conclusion. The words used for these [133] indirect moods of the first figure were usually the fifth and following ones in

Barbara, Celarent, Darii, Ferio, Baralip-*ton*
Celantes, Dabitis, Fapesmo, Frisesom-*orum*

the final syllables in Italics being only euphonic (Frisesmo-orum would have been more correct). Some used the words *Faresmo* and *Firesmo.*

In calling the moods of the fourth figure by the name of indirect moods of the first figure, notice was taken of the circumstance that a transposition of the premises would give the arrangement of the first figure, in every thing but the proper arrangement of major and minor terms, which is inverted. A little consideration will show the reader that the earlier Aristotelians were wiser than the later ones in this matter. Consider the fourth and first figures as coincident, and the arbitrary notion of arrangement by major and minor vanishes. It was not till this mere matter of discipline was made an article of faith that the fourth figure had any ground of secession from the first.

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[24] [The reduction of Baroco and Bocardo *per impossibile* is given by Aristotle, *Analytica Priora,* Bk. I. c. 5 (Baroço), 6 (Bocardo).—Ed.]

It might seem as if the union of the first and fourth figures would demand that of the second and third: the first pair containing all the moods in which the middle term occupies different places in the two premises, the second pair those in which it has the same place in both. If this were done, each of the two main subdivisions must be itself subdivided into two. And this would perhaps have been the more skilful mode of division.

The distinction of figures has been condemned by many, and particularly by Kant. Whether attacked or defended, it is essential that the true grounds of the side taken should be more explicitly stated than is often done. The root of the distinction of figure is undoubtedly the distinction between the two forms XY and YX, X.Y and Y.X. It would be equally absurd, either to deny the identity of XY and YX, considered as material of inference, or to deny their difference in many other points of view. In this work I am concerned only with what can be inferred, and to what extent of quantity, and accordingly the distinction is to me immaterial. But if I had not merely to study the way of using premises, but also that of arriving at them, it might very well happen that the aspects under which the same [134] inference is seen in different figures would give it very different shades of character. A simple instance will show that though the comparison, and its extent, are all that can be attended to in forming the conclusion, these points of meaning are not the only ones. A person who wished to contest the old use of the word *green*, as applied to unripe fruit, would say that "some green fruits are ripe," if he wanted specially to show the misapplication of the word. But if he rather wanted to show the badness of the method of denying ripeness, he would say "some ripe fruits are green." The propositions are endless in which, X and Y being the terms, it is at one time X which is brought to Y for comparison, and at another Y to X. The subject of a proposition is always the object of examination; whether the form be X)Y, X.Y, XY, or X:Y, we examine and report upon the Xs.

If we arrange the four figures separately, we shall better see their several peculiarities.

First Figure.

Barbara	Y)Z+X)Y=X)Z	*Celarent*	Y.Z+X)Y=X.Z
Darii	Y)Z+XY =XZ	*Ferio*	Y.Z+XY =X:Z

What is here declared, is in every case the *dictum de omni et nullo* in its simplest form, in a manner which justifies the preference given to this figure. The middle term being completely contained in, or completely excluded from, the major term; such inclusion or exclusion then follows of all such part of the minor term as is declared in the second premise to be in the middle term. The inference then is in this sentence "What is true of the whole middle term, is true of its part." And it is obvious that in this figure the major premise must be universal, the minor premise affirmative. The four forms are all found among the conclusions. I think that the inversion of the premises which the system of chapter V. employs will be found to give the forms which are most easily translated into language independent of the middle term. The sentence "All (or some) of the Xs are what must be Zs, therefore all (or some) of the Xs are Zs" includes *Barbara* and *Darii*: and "All (or some) of the Xs are what cannot be Zs, and therefore cannot be Zs," contains *Celarent* and *Ferio*.

[135] *Second Figure.*

Cesare	Z.Y+X)Y=X.Z	*Camestres*	Z)Y+X.Y=X.Z
Festino	Z.Y+XY =X:Z	*Baroko*	Z)Y+X:Y=X:Z

In this figure (in which only negatives can be proved) the appearance of the dictum is not so direct. The terms of the conclusion are both objects of examination, and one is wholly included, and the whole or part of the other excluded (Cesare, Camestres, and Baroko) or one is wholly excluded, and the whole or part of the other included (Cesare, Camestres, and Festino). Or rather, to justify the distinction, we should say that the whole of the major term is $\begin{matrix}\text{included}\\ \text{excluded}\end{matrix}$ and the whole of the minor $\begin{matrix}\text{excluded}\\ \text{included}\end{matrix}$ which gives $\begin{matrix}\text{Camestres}\\ \text{Cesare}\end{matrix}$ in which the whole of the minor is therefore excluded from the major; or else the whole of the major is $\begin{matrix}\text{included}\\ \text{excluded}\end{matrix}$ and part of the minor $\begin{matrix}\text{excluded}\\ \text{included}\end{matrix}$ which gives $\begin{matrix}\text{Baroko}\\ \text{Festino}\end{matrix}$ in which that part of the minor is excluded from the major. And it is evident enough why the premises must be of different signs.

In the first figure, though all the forms be essentially one, (page 98), the reduction of either to the form *Barbara* requires either the explicit use of contraries, or invention of a name subidentical to X. Accordingly, no mood of that figure is reducible to any other by the usually admitted reductions. But this cannot be said of any of the other figures. In the one before us, Cesare and Camestres are identical, even without changing the figure. That which is *Cesare* when X is major and Z minor, is *Camestres* when X is minor and Z major. In the first figure, the same attempt on *Celarent* or *Darii*, removes them into another figure.

Third Figure.

Darapti	Y)Z+Y)X=XZ	*Felapton*	Y.Z+Y)X=X:Z
Disamis	YZ +Y)X=XZ	*Bokardo*	Y:Z+Y)X=X:Z
Datisi	Y)Z+YX =XZ	*Ferison*	Y.Z+YX =X:Z

The first and second figures contain a pair of universals each, [136] with one particular derived from each, by a legitimate weakening of one premise and the conclusion at the same time: but in no instance is the quantity of the middle term weakened. And all the syllogisms in these two figures are fundamental (page 77). In the case now before us, both the leading syllogisms are not fundamental, but strengthened, and capable of being weakened in two different ways. The middle term is here examined in both premises: if it be wholly included in, or excluded from, one of the concluding terms, and wholly or partly included in, or excluded from, the other (but not so that there shall be exclusion from both) we have it that the whole or part mentioned in one case is included in, or excluded from, that which the whole is included in, or excluded from, in the other. There can be none but particular conclusions.

Fourth Figure.

Bramantip	Z)Y+Y)X=XZ	*Camenes*	Z)Y+Y.X=X.Z
Dimaris	ZY +Y)X=XZ	. . . .	

Fesapo Z.Y+Y)X=X:Z
Fresison Z.Y+YX =X:Z

We have now one universal syllogism in a form which does not admit of being weakened *in this figure*, and two strengthened syllogisms, each of which has one weakened form, one of them, *Bramantip*, admitting a stronger conclusion in another figure. Every conclusion except A appears. The mode of inference of the three first syllogisms has been described in the other figures. In Fesapo and Fresison, the perfect exclusion of the major term from the middle, accompanied by the total or partial inclusion of the middle in the minor, secures the exclusion from the major, of as much of the minor as it has in common with the middle.

I shall now proceed to the rules usually given, and to some remarks on the degree in which they apply to the more general system in chapter V. Aldrich gives them as follows—

Distribuas medium: nec quartus terminus adsit:
Utraque nec præmissa negans, nec particularis:
Sectetur partem conclusio deteriorem;
Et non distribuat, nisi cum præmissa, negetve.[25]

[137] These rules, I need hardly say, are perfectly correct when the contraries of the terms are excluded, and also all notion of quantity except all, or the indefinite some. Taking them in the natural order, which verification has a little disturbed, we have;—

1. There are to be but three terms, of which it is understood two only appear in the conclusion, the excluded or middle term appearing in both of the premises. This is true in my system, when by terms are understood also contraries of terms. I should suppose that there can be no objection to the admission of contraries, unless there be one to the conception of a contrary. Any one may, with Aristotle, object to the word not-man, as not the name of anything: on the grounds which immediately induced him to call it an aorist, or indefinite, name. But it can hardly be affirmed that any one admitting not-man as a name, should thereupon refuse to recognise the identity of "horse *is not* man," with "horse *is* not-man."

2. The middle term is to be *distributed* in one or the other of the premises. By distributed is here meant *universally spoken of*. I do not use this term in the present work, because I do not see why, in any deducible meaning of the word *distributed*, it can be applied to universal as distinguished from particular. In using a name, it seems to me that we always distribute: that is, scatter as it were, the general name over the instances to which it is to apply. When I say some horses are animals, I distribute certain horses among the animals; and when all, all. Leaving the word, the principle is one which clearly must be true whenever we are restricted in quantity to all or some

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[25] [For an earlier form of these verses, due to Petrus Hispanus, afterwards Pope John XXI., see Mansel's note on the passage in his edition of Aldrich's *Artis Logicae Rudimenta*, p. 62.—Ed.]

(indefinite), and when contraries are not admitted. In the former case we have, in one form or another, to make $m+n$ greater than η (chapter VIII.) when we cannot know what relation either m or n has to η, unless one of them, or both, be equal to η. We have no alternative then, but to require that m or n shall be η. The cases in which there is apparently no dependence on η will be discussed in the next chapter.

But when contraries are introduced, this rule is not universally true. The exception is seen in

$$A_{\prime}A'I' \text{ or } X)Y+Z)Y=xz.$$

If all the Xs be Ys, and also all the Zs, it follows that there are things which are neither Xs nor Zs, namely, all which are not [138] Ys. It is here, as elsewhere, implied that the middle term is not the universe of the proposition.

When we come, then, to use contraries, the simple rule of the middle term is no longer universally true. What other rule are we to put in its place? We know, of course, that every syllogism can be reduced to an Aristotelian syllogism, and even to one or other of two among them, $A_{\prime}A_{\prime}A_{\prime}$ or $I_{\prime}A_{\prime}I_{\prime}$, or to the first of these, if we be at liberty to use invention of names (page 97). Again, each term, or its contrary, is mentioned universally in every proposition: so that there is certainly one way in which every pair of premises may be made to exhibit a middle term universally used in one of them. The rule to be substituted for the *distribuas medium* is, that all pairs of universals are conclusive, but a universal and a particular require that the middle term should also be a universal and a particular, that is, universal in one and particular in the other. Thus, in $X)Y+Z)Y$, as it stands, the middle is particular in both; transpose into $y)x+y)z$ and the middle is now universal in both, by which we see the Aristotelian conclusion. Again, in $X)Y+ZY$, which is of the same kind, the transposition gives $y)x+Z{:}y$, which is faulty, because, though there be a particular premise, there is not anywhere a particular middle term. The cases in which the middle is of the same name in both places (universal in four, particular in four), are the strengthened syllogisms only. There is nothing to be surprised at in its thus appearing that the particularity of the middle term is just as much a test of a good syllogism as its universality: of every name and its contrary, one enters universally, and one particularly, in every proposition which contains it; and the system in chapter V. is as much concerned with contrary as with direct terms. It is thence visible beforehand, to the mathematician at least, that any test must be defective, unless universal and particular enter into it in the same manner.

The above contains a complete canon of validity, as soon as the law of the three terms is understood, which is only a law of definition. We may state it as follows: Two premises conclude—when both are universal, always; when one only is universal, so often as it happens that the middle term (be it Y or y) is once only universal; when neither is universal, never. By this rule alone the thirty-two conclusive cases can be distinguished from the thirty-two inconclusive ones.

[139] 3. When both premises are negative, there is no Aristotelian syllogism. In the system completed by contraries, there are eight such syllogisms, as many in fact, as there are with premises both affirmative. But a pair of negative premises never conclude with both terms of the premises, but with the contrary of one or both: and this must be substituted, as a rule of conclusion, for the one just named.

4. Both premises must not be particular. This rule, which relates wholly to quantity, must be preserved in every system which admits the definite ratio, except that of one to one, or *all* (pages 56, 57). I cannot learn that any writer on logic ever propounded even the very simple case of "Most Ys are Xs, most Ys are Zs, therefore some Xs are Zs," as a legitimate inference. And this, though it is certain

that the quantitative prefix *most* (*plurimi*) has before now excited discussion as to whether it belonged to a universal or a particular.

5. By *sectetur partem conclusio deteriorem* it is understood that the negative is called weaker or lower (*deterior*) than the affirmative, and the particular than the universal ; and that the conclusion is to be as weak as negative, or as particular, if there be a premise which is negative or particular. This rule must be preserved, when contraries are introduced, so far as relates to particulars. But so far as negatives are concerned, the rule must be that *one* negative premise gives a *negative* conclusion, and *two* an affirmative one.

7. The last line, *et non distribuat, nisi cum præmissa, negetve,* spoils the symmetry to procure a verse. The conclusion is not to be negative without a negative premise : that is, affirmative premises give an affirmative conclusion. Also, no term is to be distributively, (i.e. universally) taken in the conclusion, unless it were so taken in its premise. A breach of this rule would be equivalent to drawing a conclusion about what was not (or about more than was) introduced into the premises.

When contraries are introduced, the distinction between positive and negative is made to appear, what it really is, one of language, or rather one of choice of names. But the distinction of form is not abolished, but is exactly what it was before. We cannot lay down any rules for the formation of the conclusion unless, in our eight standard forms, we preserve the mode of **[140]** writing which belongs to the fundamental derivation of the forms (page 61). Thus, the order being XY, A' is x)y and not Y)X, and O' is x:y and not Y:X. This method of writing being restored, when necessary, in pages 89 and 91, it follows immediately that the rule of accentuation in the notation gives the rule by which we determine whether the conclusion takes the terms from the premises, or prefers contraries. According as the preposition of the conclusion agrees with or differs from that of a premise, so does the conclusion take a term from that premise, or its contrary. Thus, $\mathrm{A}_\prime\mathrm{A}_\prime\mathrm{A}_\prime$ takes both terms from the premises, but $\mathrm{A}_\prime\mathrm{A}'\mathrm{I}'$ takes a contrary from the first premise only. This last we see if we write the syllogism as X)Y+y)z=xz. Accordingly, we have—

Syllogisms taking both concluding terms direct from the premises. Universals which begin with A; particulars which begin with I: eight in number; being all which isolate no accent.

Taking the first term only from the premise. Universals beginning with E; particulars beginning with O: eight in number; being all which isolate the middle accent.

Taking the second term only from the premise. Strengthened forms and particulars which begin with A: eight in number, being all which isolate the first accent.

Taking neither term from the premises. Strengthened forms and particulars which begin with E: eight in number, being all which isolate the third accent.

This is a new mode of stating the law of accentuation (pages 92–3) which I have preferred to place here, for fear of overloading chapter V. with rules. I have not stated one half of those which suggested themselves. This multiplicity of relations is a presumption of the completeness of the system.

In the Aristotelian system, there is multiplication of the same modes of inference, under the varieties of figure. In that which I propose, there is a reduplication of most of the essential cases; for whatever case is found, the same is also found with X and Z interchanged, and also the order of the premises. Again, whatever case is found, it is found contranominally; or with all the accents (or prepositions) altered. There are other ways (and many of them) in which the system is only in one half a duplicate of what it is in the other. If all these modes of dividing the system into
[141] two correlative parts divided it into the same two parts, there can be no question that one alone of those parts should have been presented as the object of consideration. But

this does not happen in any instance: so that it is impossible to dispense with the whole of the thirty-two cases. The Aristotelian cases do not form or include any half whatever of this system.

CHAPTER VIII.

ON THE NUMERICALLY DEFINITE SYLLOGISM.

In the last chapter I considered no other quantity in names except all and some: the latter meaning "one or more, it may be all." To this extent of quantity we are limited in most kinds of reasoning, by want of knowledge of the definite extent of our propositions: and the few phrases (page 58), as "most," "a good many," &c. by which we endeavour to establish differences of extent in ordinary conversation, have been hitherto held inadmissible into logic. In this science it seems to have been always intended that the bases on which its forms are constructed shall be nothing but the supposition of the most imperfect and inaccurate knowledge. Though in geometry we are permitted to assume as the object of reasoning the ideal straight line, the "length without breadth" of Euclid, which has no objective prototype, and though we see the advantage of reasoning upon ideas, and allowing the essential inaccuracies of material application to produce no effect except in material application,—yet in the consideration of the pure forms of thought, the learner has always been denied the advantage of studying the more perfect system of which his inferences are the imperfect imitation.

The ordinary universal propositions are of a certain approach to definite character, both of them with respect to their subjects, and the negative one with respect to its predicate also. In X)Y for example, what is known is as much known of any one X as of any other. Perfect definiteness would consist in a more exact degree of description, and would require a higher degree of knowledge. But in this chapter I speak only of *numerical* definite-[142]ness, of the supposition that we know *how many* things we are talking about. We may be well content to examine what we should do if we were a step or two higher in the scale of creation, if by so doing we can manage to add something to our methods of inference in the highest to which we have as yet attained.

A numerically definite proposition is of this kind. Suppose the whole number of Xs and Ys to be known: say there are 100 Xs and 200 Ys in existence. Then an affirmative proposition of the sort in question is seen in "45 Xs (or more *) are each of them one of 70 Ys": and a negative proposition in "45 Xs (or more) are no one of them to be found among 70 Ys."

But it must be particularly noticed that in speaking of a number of Xs, as 45 Xs, I do not mean *certain* 45 Xs which can be distinguished from all the rest, so that of any X it is possible to be known whether it belong to the 45 of the proposition, or to the remaining 55. This degree of definiteness is one step higher than that which I here propose to consider, and which is described by "there are 45 Xs which are contained among 70 Ys, it not being known which Xs are the 45 Xs, nor which Ys are the 70 Ys:" or else by "there are 45 Xs which are not any of them identical with any one of 70 Ys, the precise Xs and Ys in question being unknown."

It cannot of course be disputed that if any thing should necessarily follow from *any* 45 Xs being found among *any* 70 Ys, it will not the less follow from our knowing which are the Xs and which are the Ys. But this last supposition only brings us to really universal propositions. If, there being 100 Xs, 45 of them can be specifically separated from the rest, so as to be known, the process of separation is equivalent to putting them [143] under a separate name subidentical to X, and the rest, which are equally distinguishable, under another name, also subidentical to X, and contrary of the first name, when the universe is X. Whether the name be long or short, does not matter, nor whether it carry the separating distinction in its etymology or not. To separate in any way instance from instance by language, is to name.

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* These words (or more) show that the word *definite* has reference only to the *lower* boundary. Of course nothing can be shown in right of "45 or more, *perhaps*" except what is true in right of the 45. It is desirable that as the premises, so should be the conclusion, of a syllogism: this would not be the case if we used premises definite both ways. For example, there being 100 Ys in existence, it will presently appear that "*Exactly* 55 Ys are Xs and *exactly* 60 Ys are Zs," though it enable us to say that "15 Xs are Zs" does not allow us to say "Exactly 15 Xs are Zs," but only "15 Xs or (more) are Zs."

If then 45 definite Xs were known to be contained among 70 definite Ys, and if these Xs were each named M, and those Ys each N, and if the rest of the Xs and Ys were named P and Q, we should have the following propositions,

M)X, P)X, N)Y, Q)Y, M)N, M.P, N.Q,[26]

and all inferences. Moreover, in each case, we should have the total number of instances which are contained under each name; the numbers carrying with them evidence that every X is either M or P, and every Y either N or Q. Substitute M.N for M)N and we have the corresponding negative proposition.

But if 45 unseparated and inseparable Xs be supposed known each to be among 70 similarly situated Ys, there is no immediate method of making any other proposition out of the terms X and Y except its converse, that 45 of these 70 Ys are 45 Xs, and (if the whole number of Ys be known, say 200) that there are 45 Xs which are not any one among 200—70, or 130 Ys. This is then a simple proposition, which becomes of a highly complex character, when the Xs and Ys named in it are taken as definitely separable from the rest. I shall call it the *simple numerical* proposition.

The distinction may be easily illustrated by example. "All the planets but one" is a particular proposition; it is "some planets:" there is no one planet of right included in it. But " all the planets except Neptune " is a universal proposition: " a-planet-not-Neptune " is a name of Mercury, of Venus, &c.; and of every planet it can be stated whether it be in the name or not. That which is true inferentially of " all the planets but one " left particular, is true of " all the planets but Neptune: " but that which is true of the latter is not necessarily true of the former.

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[26] [" Supply the propositions X)M,P and Y)N,Q as deducible from the number of instances in the several names." De Morgan, "Table of Contents."—ED.]

[144] Taking X, Y, Z as the terms of the syllogism, ξ the number of Xs in existence, η the number of Ys, and ζ the number of Zs, and v the number of instances in the universe, there are of course sixteen possible cases of knowledge, more or less, of these primary quantities, from all unknown to all known. Of these sixteen cases, it will be requisite to consider two only. First, when the extent η of the middle term is known, and all the rest unknown; secondly, when all are known. The *algebraical* formulæ of the latter case will enable us to point out how the supposition of any less degree of knowledge would affect our power of inference.

I propose the following notation. Let mXY denote either of the equivalent propositions, that m Xs are to be found among the Ys, or that m Ys are to be found among the Xs. Let mX:nY denote either of the equivalent propositions, that there are m Xs which are not any one among n Ys, or n Ys which are not any one among m Xs.

The symbol 10X is the algebraical symbol for ten equal Xs added together, X being a magnitude: it is then a *collective* symbol. In this work, X being a name, it implies every one out of ten instances of that name, *distributively*, but not *collectively*. This distinction is very material, not only in this chapter, but throughout every part of logic. " Every X is Y " is distributively true, when, by " Every X " we mean each one X: so that the proposition is " The first X is Y, *and* the second X is Y, *and* the third X is Y, &c." In this case the subject is X, and the word *every* belongs to the quantity of the proposition. But " every X is Y " is *collectively* true, when we do not mean that any one X is a Y, nor that any number of Xs are Ys, but that all the Xs make a Y. In this case the proposition is *singular:* there is but one instance of the subject mentioned, that subject being, not X, but the collection "all the Xs." Thus "the ten men are members of a committee" is distributive: "the ten men are a committee" is collective.

If, in such a proposition as 10XY, we were to suppose the 10 Xs specifically separated from the rest, being certain assignable ten individuals from among all the Xs, then 10X *becomes a name* for each of the ten, as much as X, and may be considered as a universal term. And now 10XY and {10X})Y mean the same things.

[145] Let η be known, and η only of the four, v, ξ, η, ε. The only collections of premises which it is necessary to consider are

$$mXY + nYZ$$
$$mXY + nZ{:}sY$$
$$mX{:}rY + nZ{:}sY$$

Without some knowledge of the number of ys, of which by supposition we have none, it would be useless to attempt to draw an inference from a pair in which Y and y enter together, partially quantified, as in mXY + nZ:ry. And nZy merely amounts to nZ:ηY.

The above three are all we need consider: and even of these the third is incapable of inference, since both premises are negative, and moreover, not reducible to a positive form by use of contraries, the only way in which negative premises really acquire a conclusion in chapter V.

Let us first consider the premises mXY + nYZ. They tell us that among the η Ys we find m Xs and n Zs: accordingly, neither m nor n exceeds η. If m and n together fall short of η, nothing can be inferred: Y is extensive enough (that is, there are instances enough of Y) to hold the m Xs and the n Zs without any coincidence of an X with a Z. As to other Xs or Zs, we do not know whether they exist; or, if they exist, we do not know that any one of them is a

Y. But if m and n together exceed η, it is impossible that m Xs and n Zs can find place among η Ys, except by putting either two Xs or two Zs, or an X and a Z, with *one* of the Ys. Now as by the nature of the suppositions, there cannot be two Xs, nor two Zs, to one Y, we must have the inference 1XZ as often as there are units in the excess of $m+n$ over η. That is,

$$mXY+nYZ=(m+n-\eta)\ XZ$$

Next, let us take $mXY+nZ{:}sY$. There may be two inferences, perfectly distinct from each other, the connexion of which can only be explained in the more general system to which we shall presently come. First, let m and s together exceed η. Then $m+s-\eta$ of the Ys have the common property of being Xs, and of being clear of the n Zs. Accordingly, we have

$$mXY+nZ{:}sY=(m+s-\eta)X{:}nZ$$

[146] Next, let $n+s$ be greater than η. Take the s Ys among which no one of the n Zs is found. Because $n+s$ is greater than η, n is greater than $\eta-s$, the number of Ys left. Accordingly, $n-(\eta-s)$ of the n Zs cannot be *any* Ys, and therefore cannot be any of the m Xs which are Ys. Hence we have

$$mXY+nZ{:}sY=mX{:}(n+s-\eta)Z$$

In the appendix to this chapter (at the end of the work) will be seen the manner in which all the Aristotelian syllogisms can be brought under the first case, and the first* inference of the second case. No Aristotelian syllogism can be deduced from the second inference except when $s=\eta$, in which case it agrees with the first. For, when s is not η, we must, to make such a syllogism, have $m=\eta$, and then, to make $nZ{:}sY$ Aristotelian, s not being η, we must have all the Zs in n, or $n=\zeta$. We thus get $Y)X+sY{:}Z$, the premises of *Bokardo*. But the conclusion is $\eta X{:}(\zeta+s-\eta)Z$, that of *Bokardo* being $sX{:}Z$. And this will be found to be the only Aristotelian syllogism which has this second and numerically quantified inference, depending upon the number of Zs exceeding the number of Ys unnamed in the particular premise.

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* I was not in possession of the second inference till I had written what is in page 157.

I now proceed to suppose that all the quantities are taken into account. Some preliminary considerations will be useful, as follows.

Let two propositions be called identical, when, either of them being true, the other must be true also: so that nothing can be inferred from the one, which does not equally follow from the other. Such propositions are X.Y and Y.X, such are X)Y and y)x, and so on. Again, two propositions may be identical relatively to a third: thus, P being true, Q and R may either follow from the other; accordingly, as long as it is understood that P is true, Q and R may, relatively to that supposition, be treated as identical.

The word *identical*, as applied to *propositions*, is here made to mean more than usual, but not with more license than when the word is applied to *names*. Thus, *man* and *rational animal* are [147] not identical names, *quà* names, for they neither spell nor sound alike: the identity understood is that of meaning; where one applies, there shall the other apply also. Similarly, as to propositions (of which subject, predicate, and copula are the material parts, just as spelling and sound are those of names), identity does not consist in sameness of parts, nor in reducibility to sameness, but in simultaneous truth or falsehood, so that what either is, be it true or false, the other is also, in every case. Thus two propositions, one of which signifies that an end has been gained, and the other that the sole and sufficient means of gaining it have been used, are identical.

All the theory of *names*, their *application* or *non-application*, may be applied to *propositions*, their *truth* or *falsehood*. To say that a proposition is true in a certain case, is to say that a certain name applies to a certain case: to say that it is false, is to say that a certain name does not apply, but that its *contrary* does. That contrary is what logicians usually call *contradictory*: and the *name* is not simply *true* or *false*, but the adjective attached to the proposition. The conditions under which we are to speak limit us to a number of cases which constitute what we may now call, not the *universe* of the *names in the propositions*, but the universe of the *truth or falsehood* of the propositions. Thus we shall suppose ourselves now to be speaking, not of all instances to which the *name* U applies, but of all in which the *proposition* U is *true*, or in which the name "true U" applies. A case in which a proposition P is true may be marked P, one in which it is false, p. We may now apply the names subidentical, &c. and the symbols, together with all the syllogisms, complex and simple; but on each a remark may be necessary.

Subidentical, identical, and *superidentical.* If P be a proposition subidentical of Q, that is, if every case in which P is true be one in which Q is true, but so that Q is sometimes true when P is not, the proposition Q is usually mentioned as *essential* to P, and as a *necessary consequence* of it. Whenever P is true, Q is true; Q necessarily follows from P; if Q be false, P cannot be true; Q is essential to P; are all mere synonymes. Accordingly "*necessary consequent*" and "*superidentical or identical*" are synonymous terms: that is (page 68), *necessary consequent* and *superaffirmative.* Identity of course consists in *each* proposition **[148]** being true when the other is true. I think that, according to general notions, it would be held more just to say that a proposition *contains* its necessary consequence than that it *is contained*: but a moment's consideration will show that the latter analogy is at least as sound. If the second be true whenever the first is true, it may be true in other cases also: so that we only say the second contains the first, and it may be more.

Subcontrary, contrary, and *supercontrary.* It is usual to call "No X is Y" and "Every X is Y" by the name of contraries, and to say that "contraries may be both false, but cannot be both true." This is a technical use of the word: in common language we should say that either a proposition or its contrary must be true; "have you any thing to say to the *contrary*" generally means what a logician would express by putting the word *contradictory* in the place of *contrary*. I am compelled to use the words contrary and contradictory as synonymous: at which compulsion I am well pleased, never having seen any good reason why, in the science which considers the relations of *dicta*, the *contraria* should be any thing but the *contra dicta*. The proper word for contrary, commonly used to express the relation of X)Y and X.Y, is *subcontrary*. Here are two propositions P and Q which cannot both be true, but may both be false: here is a pair which can never be asserted of the same instance, and of which, in many instances, neither can apply. In the same manner, the propositions XY and X:Y, usually called *subcontrary* (for no reason that I can find except that they are written *under* the so called *contraries* in a scheme or diagram very common in books of logic) should be called *supercontrary*: they are never both false, and may be both true. This is a complete inversion of the usual prepositions: an inversion which seems to me imperatively required, if only my use of *sub* and *super* in Chapter IV. be allowed.

In applying these names to propositions, it must be remembered that we make the same sort of ascent which we make in passing from specific to universal arithmetic, in using a symbol to stand for any number at pleasure. For instance;—Perhaps it may be thought that XY and X:Y may sometimes be only contraries, and not supercontraries, because there may be names which make one only true and not both. But this is not correct: **[149]** for we are considering the proposition itself *as an instance among propositions*, not the proposition as subdivisible into instances, in which name is compared with name. In speaking of propositions, it is change from use of one name to use of another, or from use of one number to use of another, which is change of instance: not change from one instance of name to another. And just as in a universe of names, every name introduced is supposed to belong, or not to belong, to every instance in that universe: so in a universe of propositions, I suppose every proposition, or its contrary, to apply (whether it be or be not known which applies) in every instance. We have never considered such a thing as the universe U, in which there are cases in which neither X nor x applies: we suppose there is always a power of declaring that the name X must either belong or not belong to each instance. In like manner, all the propositions in each universe now considered, are supposed to be connected with all the names in question: so that X, Y, being two of them in their order of reference, $A_{\prime}$ or $O_{\prime}$ is true in each case, and A' or O', $E_{\prime}$ or $I_{\prime}$, and E' or I'. We might, if we pleased, enter upon a wider system. For though we cannot imagine of any object of thought, but that it is either X or not X, be X what *name* it may, yet we can imagine of *propositions* that they may be wholly inapplicable, as being neither true nor false. The first assertion is all the more true, that it could hardly be exemplified without exciting laughter: as I should do if I reminded the reader that a book is either a cornfield or not a cornfield. We have never considered names under more predicaments than two; never, for instance, as if we were to suppose three names X_1, X_2, X_3, of which everything must be one or the other, and nothing can be more than one. But we should be led to extend our system if we considered propositions under three points of view, as true, false, or inapplicable. We may confine ourselves to single alternatives either by introducing not-true (including both false and inapplicable) as the recognized contrary of true: or else by confining our results to universes in which there is always applicability, so that true or false holds in every case. The latter hypothesis will best suit my present purpose.

This digression is somewhat out of place here, but I have preferred to retain the matter of it until I had occasion to use it. **[150]** I now proceed to assert that the simple numerical proposition has no occasion for a numerically definite predicate. Let us consider first an affirmative proposition, say " Of 10 Xs, each is to be found among some 15 Ys." Of course it is supposed there are 15 or more Ys in existence. With this let us compare " 10 Xs are to be found among the Ys." These two propositions are identical: if 10 Xs be among 15 Ys, there are 10 Xs among the Ys: and if 10 Xs be among the Ys they are certainly 10 Ys; put on 5 more Ys at pleasure, and they can be said to be among 15 Ys in just as many ways as we can choose 5 more Ys to make up the 15. Note, that if the 10 Xs were among certain specified 15 Ys, then, though the first proposition would give the second, the second would not necessarily give the first. But we are now supposing that numerical selection is only *numerically* definite: definite as to the number, not as to the instances which make up that number. When therefore we say " 10 Xs are among 15 Ys " we say neither more nor less than when we say " 10 Xs are among the Ys." It is in fact " 10 of the Xs are 10 of the Ys " and the converse " 10 of the Ys are 10 of the Xs " is the same proposition.

Now let us take a negative proposition, " 10 of the Xs are not to be found, *any one of them*, among some 15 Ys," abbreviated into " 10 Xs are not in 15 Ys." If there be 25 Ys in existence this proposition must be true; mean X and Y what they may. It is as true as that the X which is one Y is not any other Y. Say there are 25 or more Ys: take any 10 Xs you choose, and put them down on any 10 Ys you choose. Then certainly there are 15 Ys left, no one of which is any of those 10 Xs. Again, if there be 25 Xs in existence, still the proposition must be true. For if the 15 Ys were all there are, and they were all Xs, there still remain 10 Xs which are not any one in the 15 Ys. Accordingly, the proposition " *m* Xs are all clear of *n* Ys," whenever either the whole number of Xs, or the whole number of Ys, exceeds $m+n$, says no more than is conveyed in our permanent understanding that no object of thought can be more than one X or one Y. But let it be otherwise; let neither Xs nor Ys be as many as $m+n$ in number. Say there are 20 Xs and 23 Ys and let 10 Xs be clear of 15 Ys. There must now be at least $15+10-20$, or 5 Ys which are *no* Xs *at all*, and at least $15+10-23$, or 2 Xs which are *no* Ys *at all*. First, it is **[151]** plain that there are no 10 Xs among those Ys which are clear of 15 Ys: for there are but 23 Ys in all. Therefore, 2 at least of these 10 Xs must be Xs which are not Ys: which with 8 Xs that may be Ys, will be clear of the remaining 15 Ys. Therefore 2 Xs at least are not Ys. Again, there are no 15 Ys among those Xs which are clear of 10 Xs, for there are but 20 Xs in all. Five Ys which are not Xs must exist, which with 10 that may be Xs, will be clear of the remaining 10 Xs. Accordingly, if the whole number of Xs be ξ, and the whole number of Ys be η, the proposition "there are m Xs which are no one to be found among n Ys" is essentially true of every case of that universe, whenever $m+n$ is less than either ξ or η. But when $m+n$ is greater than both ξ and η, there are two propositions, necessarily involved, which are not essentials of all cases of that universe: namely, that there are $m+n-\xi$ Ys which are not any Xs, and $m+n-\eta$ Xs which are not any Ys.

But, it may be asked, if η should be less than ξ, and $m+n$ greater than η, but still less than ξ, may we not affirm that $m+n-\eta$ Xs are not Ys? Undoubtedly we may, but then we do not affirm so much as already belongs to every case of the universe. For if ξ be greater than η, no more than η Xs can be Ys, and there are left $\xi-\eta$ Xs which cannot be Ys: and $\xi-\eta$ is, in the case supposed, more than $m+n-\eta$.

Let v be the number of instances in the universe, ξ and η being the number of Xs and of Ys. The following uses of the notation will be readily seen to express preceding results, or others immediately deducible.

ξ greater than η $\quad(\xi-\eta)X{:}\eta Y \quad$ or $(\xi-\eta)Xy$
η greater than ξ $\quad(\eta-\xi)Y{:}\xi X \quad$ or $(\eta-\xi)Yx$

$m+n$ greater than ξ and than η gives

$$mX{:}nY=(m+n-\eta)X{:}\eta Y=(m+n-\xi)Y{:}\xi X$$

$A_,$ | $X)Y=\xi XY \quad =(v-\eta)yx \quad =\xi Y{:}(v-\xi)x$
$O_,$ | $X{:}Y=mX{:}\eta Y \quad =mXy \quad =my{:}(v-\xi)x$
A' | $Y)X=\eta XY \quad =(v-\xi)xy \quad =\eta X{:}(v-\eta)y$
O' | $Y{:}X=mY{:}\xi X \quad =mYx \quad =mx{:}(v-\eta)y$
$E_,$ | $Y.X=\xi Xy \quad =\eta Yx \quad =\xi y{:}(v-\xi)x=\eta x{:}(v-\eta)y$
$I_,$ | $XY=mXY \quad =mX{:}(v-\eta)y=mY{:}(v-\xi)x$
E' | $x.y=(v-\xi)xY=(v-\eta)yX=(v-\xi)Y{:}\xi X=(v-\eta)X{:}\eta Y$
I' | $xy=mxy \quad =mx{:}\eta Y \quad =my{:}\xi X$

[152] I now examine the modes of contradicting mXY and $mX{:}nY$. As to the first, it is obvious that (m always meaning that m are, but that more may be) either m or more Xs are Ys, or else $\xi-m+1$ or more Xs are not Ys. The contradiction then is either of the equivalents

$$(\xi-m+1)X{:}\eta Y \text{ and } (\eta-m+1)Y{:}\xi X$$

It will be satisfactory to evolve the contradiction of $mX{:}nY$ by a method which will again demonstrate the cases in which no contradiction exists; or in which the proposition is always true. Let us put the two names in the least favourable position for making $mX{:}nY$ true. Let p then be the number of Xs which are not Ys, all the rest being Ys. Take the p Xs which are not Ys (p must not be so great as m, for then the proposition is made good by the Xs which are not any Ys) and $m-p$ from those which are Ys. All the m Xs thus obtained are clear of $\eta-(m-p)$ or $\eta-m+p$ Ys. Let this just be n: that is, let $p=m+n-\eta$. Then $\xi-p$, the number of Xs which *are* Ys, is $\xi-(m+n-\eta)$ or $\xi+\eta-m-n$. Let but one more X be Y, and the proposition begins to be contradicted: for now $m+n-\eta-1$ Xs are not Ys, we must take up $\eta+1-n$ of those which are Ys to make m Xs, and there only remain $\eta-(\eta+1-n)$ or

$n-1$ Ys clear of the m Xs. And it is plain that if we cannot do it by using first all the Xs which are not Ys at all, still less can it be done by using those which are. Accordingly the contradiction of mX:nY is

$$(\xi+\eta-m-n+1)\mathrm{XY}$$

Then, in order to have a proposition which can be contradicted, $m+n$ must be greater than ξ, or equal to $\xi+1$ at least, for otherwise $\xi+\eta-m-n+1$ would be greater than η, or more Ys than η must be Xs, which is absurd : and similarly $m+n$ must be greater than η. Otherwise, all contradiction is absurd, or mX:nY is always true.

Assuming these last conditions, however, the contradiction of mX:nY is made easier. To be capable of contradiction, it must amount to $(m+n-\eta)$X:ηY. Now when $m+n-\eta$ Xs are not Ys, and no more, $\xi+\eta-m-n$ Xs *are* Ys. One or more **[153]** above this, or let $(\xi+\eta-m-n+1)$XY, and mX:nY cannot be true.

Thus much for contradictory or contrary propositions. I shall presently consider the *contranominal* propositions.

We must guard ourselves from prescribing the use of any premise which necessarily belongs to all cases in the universe (of propositions). Let P be a proposition which may or may not be true, laid down as a premise, and Q a proposition which is true in every case. Let R be their necessary consequence, or legitimate inference : then it is not " whenever P and Q are true, R is true," but " whenever P is true, R is true." So far as R is a consequence of Q, so far it is a consequence of every thing which necessarily gives Q ; and thus it is a consequence of the supposed constitution of the universe from which the propositions are taken. Now this constitution is always understood ; it may be a convenience that R should be deduced by first deducing Q, but it cannot be a necessity. And R is a consequence of P and this constitution, not of P and Q.

For example, let the universe of propositions be all that can be formed out of the suppositions of the existence of

20 Xs, and 30 Ys, and 40 Zs, in one universe of names. Let us join together 15XY and 10Z : 20Y. Our rules of inference have shown us on page 145 that 5X : 10Z is the necessary consequence of these premises: but this result is not only true when 15XY is true, without anything else, but even without that; because 5 + 10 falls short of 40.

Again, we must guard ourselves from adopting the conclusion which follows from premises, when that conclusion is true in all cases by the constitution of the universe: it is then a sort of *spurious* * conclusion, legitimate enough as an inference, but of a perfectly distinct character from inferences which would bear **[154]** doubt but for the premises, or would bear contradiction under other premises. Say that in the above universe we join the propositions 15XY and 30Z : 20Y. Both these propositions are capable of contradiction: the second is 20Z : ηY (η means 30, but the symbol reminds the reader that 30 is *all*) or 10Y : ζZ (ζ being 40). Now, by laws of inference, 15XY + 30Z : 20Y yields 5X : 30Z, which is always true in that universe.

Here is a case in which premises capable of contradiction give a conclusion which is not.

The rule of inference is obviously as follows. We cannot show that Xs are Zs by comparison of both with a third name, unless we can assign a number of instances of that third name, *more than filled up* by Xs and Zs: that is to say, such that the very least number of Xs and Zs which it can contain are together more in number than there are separate places to put them in. If our premises, for example, separate some 30 Ys, and dictate that among those 30 Ys there must be 20 Xs and 15 Zs, it is clear that there must be at least 5 Zs which are Xs. For if we put down the 20 Xs which are to go in, and try to put the Zs into separate places, we are stopped as soon as we have filled up the 10 remaining out of the 30 Ys, and must put the other 5 Zs among the Ys which have been made Xs. Accordingly, so many Xs at least must be Zs as there are units in the number by which the Xs and Zs to be placed, together exceed the number of places for them. All the other rules of inference are modifications of this. For example, to prove that 10 Xs are not Zs, we must show some number of instances (be they Ys or ys, or part one and part the other) *overfull* (in the above sense) of Xs and zs, to the amount of 10 at least; so that 10 Xs are zs, or are not Zs. To prove that some xs are zs, we must show a number of instances in which the *least* numbers of xs and zs [155] which it can contain, *overfill* it, or in which the *greatest* number of Xs and Zs which it can contain *underfill* it, or do not fill it, though made completely separate.

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* To this word, as here used, I have heard much objection; and when I first took it, it was unwillingly, and for want of a better. But on further consideration I am well satisfied with it. The objection arises from the idea of false or worthless being generally attached to the word. But, though it may be usual for spurious things to be worthless, it is not necessary. If a London maker of razors should put the name of a great Sheffield house upon them, those razors would be spurious. Suppose them as good as those of the Sheffield maker, or better, they are still spurious: though it may be true enough that the chances are rather in favour of their resembling the ware of Peter Pindar's hero. In this work, a spurious inference is that which passes for the consequence of certain premises, but does not in reality follow from those premises any more than from an infinity of others: being true by the constitution of the universe. It is made to have the mark of those premises, when in truth we cannot know whether those premises be possible or not, until we have first examined a constitution which virtually contains our conclusion.

In examining the fundamental laws of syllogistic inference, it is not necessary to consider any thing but the positive forms. For $m\mathrm{X}{:}n\mathrm{Y}$, when not spurious (and we shall see that the spurious cases may be rejected) is $(m+n-\eta)\mathrm{X}{:}\eta\mathrm{Y}$, which is $(m+n-\eta)\mathrm{Xy}$ or $(m+n-\xi)\mathrm{xY}$. There are, then, but two fundamental cases: one in which the predicates are the same, one in which they are contraries. We shall accordingly have to consider

$$m\mathrm{XY}+n\mathrm{ZY} \quad \text{and} \quad m\mathrm{XY}+n\mathrm{Zy}:$$

and it will presently appear that not more than one, even of these, is absolutely necessary. In each case we must ask, what collective instances of Y or of y, or partly of one and partly of the other, receive any dictation as to how they are to be filled with Xs, with xs, with Zs, or with zs: and what is the least number of each which can be allowed to every such collection. But there is yet something to do, suggested by the preceding remarks. Let us take one proposition, a type of all we shall have to consider, say mXY. This means that XY is true to at least m instances. Now, this proposition may involve Xy, or xY, or xy. First, as to Xy. To get the least number of Xs among the ys, we must put the greatest number among the Ys. If all the Xs will go among the Ys (or if η be greater than or equal to ξ) there need be no Xs among the ys: but if not (or if η be less than ξ) then $\xi-\eta$ Xs must be among the ys, in every case. Accordingly

$$m\mathrm{XY} \text{ gives } (\xi-\eta)\mathrm{Xy}$$

where by $\xi-\eta$ understand o, not only when ξ is equal to η, but when it is less. This result is spurious, since it is true or false, by the mere constitution of the universe, independently of mXY. Secondly, as to xY. Since mXY is equally mYX, the same reasoning shows that

$$m\mathrm{XY} \text{ gives } (\eta-\xi)\mathrm{xY}$$

where $\eta-\xi$ is to be understood in the same way. This result is also spurious for a like reason.

[156] Thirdly, as to xy. Since there must be m Xs among the Ys, the greatest possible number of xs is $\eta-m$. If this be as great as $v-\xi$, the whole number of xs, there need be no xs among the ys: but if $\eta-m$ be less than $v-\xi$, there must then be at least $v-\xi-(\eta-m)$ xs among the ys, or $v+m-\xi-\eta$. Consequently

$$m\mathrm{XY}=(v+m-\eta-\xi)\ \mathrm{xy}.$$

I here put the sign = because these propositions are really equivalents. Treat the second in the same way as that which deduced it from the first, and we have

$$\begin{aligned}(v+m-\eta-\xi)\mathrm{xy}&=(v+\overline{v+m-\eta-\xi}-\overline{v-\eta}-\overline{v-\xi})\mathrm{XY}\\&=m\mathrm{XY}\end{aligned}$$

If $v+m$ be not greater than $\eta+\xi$, the equivalent does not exist. We are already well acquainted with one case of this proposition. Let $m=\xi$: then mXY is X)Y and the equivalent becomes $(v-\eta)$xy, which, as $v-\eta$ is the whole number of ys, is y)x.

The rule is, if two names have a certain number of instances at least in common, to the whole number in the universe add that number of instances, and see if the sum exceed the whole number of instances of both names together. If it do so, the excess shows the least number of instances which the contraries of these two names must have in common. Follow this rule, and we have

$$\begin{aligned} m\mathrm{XY} &= (v+m-\eta-\xi)\mathrm{xy} \\ m\mathrm{xY} &= (\xi+m-\eta)\mathrm{Xy} \\ m\mathrm{Xy} &= (\eta+m-\xi)\mathrm{xY} \\ m\mathrm{xy} &= (\xi+\eta+m-v)\mathrm{XY} \end{aligned}$$

Here are exhibited the equivalent *contranominal* forms. The following results may now be deduced.

First, these contranominals being formed in the same way, each from the other, in any one pair, whatever we prove of the first from the second, we also prove of the second from the first. The mathematician would call them *conjugate* pairs. Next, since all the four pairs are but versions of the first, with difference of names, whatever we prove universally of the first pair, we prove of all. Now, taking the first of any pair and making it possible, which is done by allowing m not to exceed the number of either of the names mentioned, the second may be possible or impossi**[157]**ble, according as the subtraction indicated can be done or not. *But whenever the second is impossible, the first is spurious.* Take mXY, and let $(v+m-\xi-\eta)$xy be impossible, or $v+m$ (and still more v) less than $\xi+\eta$. Now as all the ξ Xs and η Ys must find place in the v instances of the universe, and $\xi+\eta$ exceeds v, we must, in every case of the universe of propositions, have at least $(\xi+\eta-v)$XY. But $v+m$ is less than $\xi+\eta$ or $\xi+\eta-v$ greater than m:

consequently, mXY is spurious, a larger proposition being always true.

As we are not to admit spurious propositions among our premises, we had better write all premises double, putting down each of the forms, and making double forms of inference. The presence of the symbols of all necessary subtractions will remind the reader of the suppositions which must be made, to insure a legitimate syllogism. I now take the several forms.

$$\begin{array}{c} m \quad \mathrm{XY} \\ (v+m-\xi-\eta)\ \mathrm{xy} \end{array} + \begin{array}{c} n \quad \mathrm{ZY} \\ (v+\eta-\zeta-\eta)\ \mathrm{zy} \end{array} = \begin{array}{c} (m+n-\eta)\mathrm{XZ} \\ (v+m+n-\eta-\xi-\zeta)\mathrm{xz} \end{array}$$

The law of inference here tells us (page 154), that $m+n$ being greater than η, $(m+n-\eta)$XZ, be it spurious or not, follows from the upper premises. The lower premises also give their inference if

$$(v+m-\xi-\eta)+(v+n-\zeta-\eta) \text{ be greater than } v-\eta$$

$v-\eta$ being the number of the ys. This last is equivalent to saying that $v+m+n$ is greater than $\xi+\eta+\zeta$. First, remark that one spurious premise necessarily gives a spurious conclusion. Say that $v+m$ is less than $\xi+\eta$, or that mXY is spurious. Then, since $v+m$ is less than $\xi+\eta$, and n does not exceed ζ, it follows that $v+m+n$ is less than $\xi+\eta+\zeta$; whence the contranominal of the conclusion does not exist, or the conclusion is spurious, as asserted.

Next, observe that the conclusion may be spurious, though neither of the premises be so. For though $v+m$ be greater than $\xi+\eta$, and $v+n$ than $\zeta+\eta$, and therefore $2v+m+n$ greater than $\xi+\zeta+2\eta$, or $v+m+n+(v-\eta)$ greater than $\eta+\xi+\zeta$, it by no means follows that $v+m+n$ alone is greater than $\eta+\xi+\zeta$. It is also visible in the mode of formation of the second inference, that to say $v+m$ exceeds $\xi+\eta$, and $v+n$ exceeds $\zeta+\eta$, only gives [158] *existence* to the premises: to give them conclusion, the sum of the two excesses must itself exceed $v-\eta$, the whole number of ys.

Thirdly, we must not omit to examine the possible case in which a premise is *partially spurious*. For example, there are 10 Xs and 20 Ys in a universe of 25 instances; accordingly, $10+20-25$, or 5, of the Xs *must* be Ys. Let one of the premises be 8XY: this is not then all contingent, and capable of contradiction; we only learn something about 3 out of the 8 Xs. And I call this proposition partially spurious. But it will give no trouble: for we must deal with the premises and their contranominal equivalents before we can pronounce for a conclusion; and of two propositions which are contranominal equivalents of each other, one *must* be partially spurious. To show this, observe that if mXY be *not* partially spurious, it is because v is greater than $\xi+\eta$; or $2v$ than $\xi+\eta+v$; or $(v-\xi)+(v-\eta)$ than v. But then the numbers of xs and ys together exceed the whole number of instances in the universe; whence some xs must be ys, or the contranominal equivalent of mXY *is* partially spurious.

Now, to write down the various forms of inference. There are sixteen ways of trying for an inference: we may combine a proposition in XY, or xy, or xY, or Xy, with one in XZ, or xz, or xZ, or Xz. But these sixteen cases really combine four and four into only four distinct cases. Thus the one we have been considering, really contains the combinations of XY and YZ, XY and yz, xy and YZ, and xy and yz. It is in our power to make either pair the principal pair, and to give the other pair as contranominals of the first pair.

Thus, we may write the case of inference we have been considering, as in the first of the following list, the others being obtained from the first, by changing X into x, or Z into z, or both. The sign + placed in the middle implies the coexistence of the four propositions: and independent numeral letters are introduced as seen, which will presently be connected with the others by equations, instead of being expressed in terms of them.

$$\begin{matrix} m\mathrm{XY} \\ m'\mathrm{xy}\end{matrix} + \left.\begin{matrix} n\mathrm{YZ} \\ n'\mathrm{yz}\end{matrix}\right\} = \left\{\begin{matrix} p\mathrm{XZ} \\ p'\mathrm{xz}\end{matrix}\right.$$

The equations presently given for this case apply with certain changes to the other cases.

[159]

$$\begin{matrix} m\mathrm{xY} \\ m'\mathrm{Xy}\end{matrix} + \left.\begin{matrix} n\mathrm{YZ} \\ n'\mathrm{yz}\end{matrix}\right\} = \left\{\begin{matrix} p\mathrm{xZ} \\ p'\mathrm{Xz}\end{matrix}\right.$$

Here X and x are made to change their former places: in the equations, ξ and ξ' must change places.

$$\begin{matrix} m\mathrm{XY} \\ m'\mathrm{xy}\end{matrix} + \left.\begin{matrix} n\mathrm{Yz} \\ n'\mathrm{yZ}\end{matrix}\right\} = \left\{\begin{matrix} p\mathrm{Xz} \\ p'\mathrm{xZ}\end{matrix}\right.$$

Here Z and z change places: as must ζ and ζ' in the equations.

$$\begin{matrix} m\mathrm{xY} \\ m'\mathrm{Xy}\end{matrix} + \left.\begin{matrix} n\mathrm{Yz} \\ n'\mathrm{yZ}\end{matrix}\right\} = \left\{\begin{matrix} p\mathrm{xz} \\ p'\mathrm{XZ}\end{matrix}\right.$$

Here X and x, and also Z and z, change places; as must ξ and ξ', and ζ and ζ', in the equations.

In the new manner of writing the form we have already considered, being the first of the four, we have just written

$$\begin{matrix} m' \text{ for } v+m-\xi-\eta & . & p \text{ for } m+n-\eta \\ n' \text{ for } v+n-\zeta-\eta & . & p' \text{ for } v+m+n-\eta-\xi-\zeta \end{matrix}$$

Let us write ξ', η', ζ', for $v-\xi$, $v-\eta$, $v-\zeta$, the numbers of xs, ys, and zs: and then, $\xi+\xi'$, $\eta+\eta'$, $\zeta+\zeta'$, being all the same, (for each is v) we may write $\eta-\xi'$ for $\xi-\eta'$, $\zeta'-\xi$ for $\xi'-\zeta$, and so on. That is, in the difference of two, one of which is accented, we may interchange the letters if we please. The equations of connection for the first or standard case, are then

$$\begin{array}{l|l} m'=m+\xi'-\eta=m+\eta'-\xi & m=m'+\xi-\eta'=m'+\eta-\xi' \\ n'=n+\zeta'-\eta=n+\eta'-\zeta & n=n'+\zeta-\eta'=n'+\eta-\zeta' \end{array}$$

$$\left.\begin{array}{r} p=m+n-\eta=m'+n'+\eta-\zeta'-\xi' \\ \text{or } m'+n'+\xi-\eta'-\zeta' \\ \text{or } m'+n'+\zeta-\xi-\eta' \end{array}\right\} \begin{array}{l} =m+n'-\zeta' \\ =n+m'-\xi' \end{array}$$

$$\left.\begin{array}{r} p'=m'+n'-\eta'=m+n+\eta'-\zeta-\xi \\ \text{or } m+n+\xi'-\eta-\zeta \\ \text{or } m+n+\zeta'-\xi-\eta \end{array}\right\} \begin{array}{l} =m'+n-\zeta \\ =n'+m-\xi \end{array}$$

For the second case we must write $m' = m + \xi - \eta = m + \eta' - \xi'$, and so on. I now proceed to the several divisions into which our usual modes of thinking make it convenient to separate the cases of this most general form.

[160] First, when every thing is numerically definite. In this case, as seen, every form requires an examination of the premises and conclusion, as to whether they are or are not spurious.

Secondly, when v, the number of instances in the whole universe of names, is wholly unknown. In this case ξ' is indefinite when ξ is definite, and *vice versâ* ; and similarly one at least of each two, η or η', ζ or ζ', is indefinite. There are then no spurious conclusions ; or, which is the same thing, none which are known to be such : for the spuriousness of a premise or conclusion consists in our *knowing* that it must be true of its two terms, independently of all comparison of those terms with a third.

Thirdly, when ξ, η, ζ, are all indefinite, as well as v. In this case, as here stated, there is no possibility of inference. We cannot tell whether $m+n$ be or be not greater than η, if we do not know what η is, in any manner, or to any extent.

But here we introduce that degree of definiteness by which we distinguish the *universal* from the *particular* (or *possible* particular, see page 56) proposition. If we can know that either of the two, m and n, is the same as η (greater neither can be) then we know that $m+n$ is greater than η. And at the same time we make Y universal, in one or the other of the premises. And the same if we can know that either m' or n' is η'.

The following are the forms which may all be derived from the first, by using all the varieties of contrary names and contranominal equivalents. If we want, for instance, to show the connection of the fourteenth with the first, we throw the first into the form

$$(m + \eta' - \xi)\mathrm{xy} + n\mathrm{YZ} = (m + n + \eta' - \xi - \zeta)\mathrm{xz}$$

We then change x into X, and Z into z, changing at the same time ξ into ξ' and ζ into ζ': and thus we get

$$(m+\eta'-\xi')\mathrm{Xy}+n\mathrm{Yz}=(m+n+\eta'-\xi'-\zeta')\mathrm{XZ}$$

Now, for $m+\eta'-\xi'$ write m', that is, for m write $m'-\eta'+\xi'$ and we have

$$m'\mathrm{Xy}+n\mathrm{Yz}=(m'+n-\zeta')\mathrm{XZ}$$

which is one of the forms of the fourteenth. And $(n+m'-\xi)$ xz is only the contranominal of $(m'+n-\zeta')$XZ.

[161]

1. $m\mathrm{XY}\ +n\mathrm{YZ}=(m+n-\eta)\mathrm{XZ}=(m+n+\eta'-\xi-\zeta)\mathrm{xz}$
2. $m'\mathrm{xy}\ +n\mathrm{YZ}=(n+m'-\xi')\mathrm{XZ}=(m'+n-\zeta)\mathrm{xz}$
3. $m\mathrm{XY}\ +n'\mathrm{yz}=(m+n'-\zeta')\mathrm{XZ}=(n'+m-\xi)\mathrm{xz}$
4. $m'\mathrm{xy}\ +n'\mathrm{yz}=(m'+n'+\eta-\zeta'-\xi')\mathrm{XZ}=(m'+n'-\eta')\mathrm{xz}$
5. $m\mathrm{xY}\ +n\mathrm{YZ}=(m+n-\eta)\mathrm{xZ}=(m+n+\eta'-\xi'-\zeta)\mathrm{Xz}$
6. $m'\mathrm{Xy}\ +n\mathrm{YZ}=(n+m'-\xi)\mathrm{xZ}=(m'+n-\zeta)\mathrm{Xz}$
7. $m\mathrm{xY}\ +n'\mathrm{yz}=(m+n'-\zeta')\mathrm{xZ}=(n'+m-\xi')\mathrm{Xz}$
8. $m'\mathrm{Xy}\ +n'\mathrm{yz}=(m'+n'+\eta-\zeta'-\xi)\mathrm{xZ}=(m'+n'-\eta')\mathrm{Xz}$
9. $m\mathrm{XY}\ +n\mathrm{Yz}=(m+n-\eta)\mathrm{Xz}=(m+n+\eta'-\xi-\zeta')\mathrm{xZ}$
10. $m'\mathrm{xy}\ +n\mathrm{Yz}=(n+m'-\xi')\mathrm{Xz}=(m'+n-\zeta')\mathrm{xZ}$
11. $m\mathrm{XY}\ +n'\mathrm{yZ}=(m+n'-\zeta)\mathrm{Xz}=(n'+m-\xi)\mathrm{xZ}$
12. $m'\mathrm{xy}\ +n'\mathrm{yZ}=(m'+n'+\eta-\zeta-\xi')\mathrm{Xz}=(m'+n'-\eta')\mathrm{xZ}$
13. $m\mathrm{xY}\ +n\mathrm{Yz}=(m+n-\eta)\mathrm{xz}=(m+n+\eta'-\xi'-\zeta')\mathrm{XZ}$
14. $m'\mathrm{Xy}\ +n\mathrm{Yz}=(n+m'-\xi)\mathrm{xz}=(m'+n-\zeta')\mathrm{XZ}$
15. $m\mathrm{xY}\ +n'\mathrm{yZ}=(m+n'-\zeta)\mathrm{xz}=(n'+m-\xi')\mathrm{XZ}$
16. $m'\mathrm{Xy}\ +n'\mathrm{yZ}=(m'+n'+\eta-\zeta-\xi)\mathrm{xz}=(m'+n'-\eta')\mathrm{XZ}$

The syllogisms of chapter V are all particular cases of the above list, obtained as follows:—

1.	$m=\eta$	$\mathrm{A'I_{,}I_{,}}$	9.	$m=\eta$	$\mathrm{A'O_{,}O_{,}}$
	$m=\eta, n=\zeta$	$\mathrm{A'A'A'}$		$m=\eta, n=\zeta'$	$\mathrm{A'E'E'}$
	$n=\eta$	$\mathrm{I_{,}A_{,}I_{,}}$		$n=\eta$	$\mathrm{I_{,}E_{,}O_{,}}$
	$n=\eta, m=\xi$	$\mathrm{A_{,}A_{,}A_{,}}$		$n=\eta, m=\xi$	$\mathrm{A_{,}E_{,}E_{,}}$
	$m=\eta, n=\eta$	$\mathrm{A'A_{,}I_{,}}$		$m=\eta, n=\eta$	$\mathrm{A'E_{,}O_{,}}$
	$m=\xi, n=\zeta$	$\mathrm{A_{,}A'I'}$		$m=\xi, n=\zeta'$	$\mathrm{A_{,}E'O'}$
2.	$m'=\xi'$	$\mathrm{A'I_{,}I_{,}}$	10.	$m'=\xi'$	$\mathrm{A'O_{,}O_{,}}$
	$n=\zeta$	$\mathrm{I'A'I'}$		$n=\zeta'$	$\mathrm{I'E'O'}$
	$m'=\xi', n=\zeta$	$\mathrm{A'A'A'}$		$m'=\xi', n=\zeta'$	$\mathrm{A'E'E'}$
3.	$m=\xi$	$\mathrm{A_{,}I'I'}$	11.	$m=\xi$	$\mathrm{A_{,}O'O'}$
	$n'=\zeta'$	$\mathrm{I_{,}A_{,}I_{,}}$		$n'=\zeta$	$\mathrm{I_{,}E_{,}O_{,}}$
	$m=\xi\ n'=\zeta'$	$\mathrm{A_{,}A_{,}A_{,}}$		$m=\xi, n'=\zeta$	$\mathrm{A_{,}E_{,}E_{,}}$
4.	$m'=\eta'$	$\mathrm{A_{,}I'I'}$	12.	$m'=\eta'$	$\mathrm{A_{,}O'O'}$
	$m'=\eta', n'=\zeta'$	$\mathrm{A_{,}A_{,}A_{,}}$		$m'=\eta', n'=\zeta$	$\mathrm{A_{,}E_{,}E_{,}}$
	$n'=\eta'$	$\mathrm{I'A'I'}$		$n'=\eta'$	$\mathrm{I'E'O'}$
	$n'=\eta', m'=\xi'$	$\mathrm{A'A'A'}$		$n'=\eta', m'=\xi'$	$\mathrm{A'E'E'}$
	$m'=\eta', n'=\eta'$	$\mathrm{A'A_{,}I'}$		$m'=\eta', n'=\eta'$	$\mathrm{A_{,}E'O'}$
	$m'=\xi', n'=\zeta'$	$\mathrm{A'A_{,}I_{,}}$		$m'=\xi', n'=\zeta$	$\mathrm{A'E_{,}O_{,}}$
[162] 5.	$m=\eta$	$\mathrm{E_{,}I_{,}O'}$	13.	$m=\eta$	$\mathrm{E_{,}O_{,}I'}$
	$m=\eta, n=\zeta$	$\mathrm{E_{,}A'E_{,}}$		$m=\eta, n=\zeta'$	$\mathrm{E_{,}E'A_{,}}$
	$n=\eta$	$\mathrm{O'A_{,}O'}$		$n=\eta$	$\mathrm{O'E_{,}I'}$
	$n=\eta, m=\xi'$	$\mathrm{E'A_{,}E'}$		$n=\eta, m=\xi'$	$\mathrm{E'E_{,}A'}$
	$m=\eta, n=\eta$	$\mathrm{E_{,}A_{,}O'}$		$m=\eta, n=\eta$	$\mathrm{E_{,}E_{,}I'}$
	$m=\xi', n=\zeta$	$\mathrm{E'A'O_{,}}$		$m=\xi', n=\zeta'$	$\mathrm{E'E'I_{,}}$
6.	$m'=\xi$	$\mathrm{E_{,}I_{,}O'}$	14.	$m'=\xi$	$\mathrm{E_{,}O_{,}I'}$
	$n=\zeta$	$\mathrm{O_{,}A'O_{,}}$		$n=\zeta'$	$\mathrm{O_{,}E'I_{,}}$
	$m'=\xi, n=\zeta$	$\mathrm{E_{,}A'E_{,}}$		$m'=\xi, n=\zeta'$	$\mathrm{E_{,}E'A_{,}}$
7.	$m=\xi'$	$\mathrm{E'I'O_{,}}$	15.	$m=\xi'$	$\mathrm{E'O'I_{,}}$
	$n'=\zeta'$	$\mathrm{O'A_{,}O'}$		$n'=\zeta$	$\mathrm{O'E_{,}I'}$
	$m=\xi', n'=\zeta'$	$\mathrm{E'A_{,}E'}$		$m=\xi', n=\zeta$	$\mathrm{E'E_{,}A'}$
8.	$m'=\eta'$	$\mathrm{E'I'O_{,}}$	16.	$m'=\eta'$	$\mathrm{E'O'I_{,}}$
	$m'=\eta', n'=\zeta'$	$\mathrm{E'A_{,}E'}$		$m'=\eta', n'=\zeta$	$\mathrm{E'E_{,}A'}$
	$n'=\eta'$	$\mathrm{O_{,}A'O_{,}}$		$n'=\eta'$	$\mathrm{O_{,}E'I_{,}}$
	$n'=\eta', m'=\xi$	$\mathrm{E_{,}A'E_{,}}$		$n'=\eta', m'=\xi$	$\mathrm{E_{,}E'A_{,}}$
	$m'=\eta', n'=\eta'$	$\mathrm{E'A'O_{,}}$		$m'=\eta', n'=\eta'$	$\mathrm{E'E'I_{,}}$
	$m'=\xi, n'=\zeta'$	$\mathrm{E_{,}A_{,}O'}$		$m'=\xi, n'=\zeta$	$\mathrm{E_{,}E_{,}I'}$

We have thus another mode of establishing the completeness of the system of syllogism, laid down in Chapter V: that is, of the system in which there is only the common universal and particular quantity. These syllogisms of numerical quantity, in which conditions of inference belonging to every imaginable case are represented by the general forms which numerical symbols take in algebra, must of necessity be the most general of their kind. And examination makes it clear that, except the preceding, there can be no syllogism existing between X, Y, Z, and their contraries. Many subordinate laws of connexion might be noticed between the general forms and their particular cases. Thus, each universal occurs three times, each fundamental particular twice, and each strengthened particular twice. The first form in pages 158, 159, gives only affirmative, the fourth only negative, premises: the second and third one of each kind, commencing with a negative in the second, and with an affirmative in the third.

There are two remarkable species of syllogism (or rather, which ought to have been remarkable): which I shall now proceed to notice.

The distinction of larger and smaller part, when division into **[163]** two parts is made, is as much received into the common idiom of language as the distinction of whole and part itself. "Most of the Xs are Ys," is nearly as common as "All the Xs are Ys:" though "fewest of the Xs are Ys," is only seen as "most of the Xs are not Ys." The syllogisms which can be made legitimate by the use of this language will do equally well for any fraction, provided we couple with it the fraction complemental to unity (which in the case of one half is one half itself). Let α and β stand for two fractions which have unity for their sum, as $\frac{3}{7}$ and $\frac{4}{7}$. Let ${}_{\alpha}XY$ and ${}_{\alpha}X{:}Y$ indicate that less than the fraction α of the Xs are or are not Ys. Let ${}^{\alpha}XY$ and ${}^{\alpha}X{:}Y$ indicate that more than the fraction α of the Xs are or are not Ys.

Then the following syllogisms arise from the cases with the numbers prefixed.

1. ${}^{\alpha}YX + {}^{\beta}YX = XZ$
4. ${}^{\alpha}y{:}X + {}^{\beta}y{:}Z = xz$
5. ${}^{\alpha}Y{:}X + {}^{\beta}YZ = Z{:}X$
8. ${}^{\alpha}yX + {}^{\beta}y{:}Z = X{:}Z$
9. ${}^{\alpha}YX + {}^{\beta}Y{:}Z = X{:}Z$
12. ${}^{\alpha}y{:}X + {}^{\beta}yZ = Z{:}X$
13. ${}^{\alpha}Y{:}X + {}^{\beta}Y{:}Z = xz$
16. ${}^{\alpha}yX + {}^{\beta}yZ = XZ$

It will be seen that here are but three really distinct forms; of which the simplest examples are as follows,

Most Ys are Xs; Most Ys are Zs; therefore some Xs are Zs.
Most Ys are Xs; Most Ys are not Zs; therefore some Xs are not Zs.
Most Ys are not Xs; Most Ys are not Zs; therefore some things are neither Zs nor Xs.

It is hardly necessary to observe that in one of the premises "more than" may be reduced to "as much as:" but not in both. Thus, if two-sevenths exactly of the Ys be Xs, and more than five-sevenths of the Ys be Zs, it follows that some Xs are Zs.

The above syllogisms admit a change of premise, as follows: If we say that more than $\frac{2}{7}$ths of the Ys are Xs, we thereby say that less than $\frac{5}{7}$ths of the Ys are xs: or ${}^{\alpha}YX$ and ${}_{\beta}Y{:}X$ are the same propositions. Thus, "most are" is equivalent to "a minority (none included) are not." Hence we have

$${}_{\beta}Y{:}X + {}_{\alpha}Y{:}Z = XZ$$

and so on. Or we may combine the two forms, as in

$${}^{\alpha}YX + {}_{\alpha}Y{:}Z = XZ$$

[164] The above are the only syllogisms in which indefinite particulars give conclusions, by reason of that approach to definiteness which consists in describing what fractions of *the middle term* are spoken of, at least, or at most. But they are not the only syllogisms of the same general species. In every case inference follows when there is a certain preponderance; and the largeness of the inference depends upon the extent of that preponderance. Thus in (12) there is an Xz inference when $m' + n' + \eta$ exceeds $\zeta + \xi'$: so many

units as there are in this excess, so many Xs (at least) are zs. Now in every case, a pair of universal premises give inference: and in every case there must be a degree of approach to universality at which inference begins. The ordinary syllogisms, I suspect, are, and are meant to be, not such as "*Every* X is Y, *every* Y is Z, therefore *every* X is Z," but "generally speaking X is Y, and generally speaking Y is Z, therefore generally speaking X is Z." And by "generally speaking" is meant the assertion that an enormous majority of instances make the assertion true. A syllogism of this sort is the opposite of the *à fortiori* syllogism; and might be said to be true *ab infirmiori*. If we have X)Y with p exceptions, and Y)Z with q exceptions; then, in form (I.) we have $m=\xi-p, n=\eta-q$, and $m+n-\eta=\xi-p-q$. As long, then, as the number of exceptions altogether fall short of the number of Xs, there is inference: if the total number of exceptions be very small, compared with the number of Xs, there is the "generally speaking" kind of inference. Examine all the universal cases, and it will be found that the same law prevails; namely, that there is inference when the numbers of exceptional instances in both premises together do not amount to the number of instances in the universal term of the conclusion; and that there is *exceptional* universality (as we may call it) in the conclusion, whenever the whole amount of exception is very small, compared with that number of instances.

This leads us to what I will call the theory of *exceptional* particular syllogisms. We have seen that the eight complex affirmatory syllogisms, which are all *à fortiori* in their conclusions, afford each two particular syllogisms. We have denoted coexistence by +; and the coexistence of two propositions gives more than either. Let us denote exceptive coexistence by —: thus, P—Q means that the proposition P is true except in the **[165]** instances contained in Q. Thus, X)Y—X:Y means that every X (with some exceptions) is Y. This is, of course, A_1-O_1, and only differs from I_1 in the mode of expression not being "some more than none at all" but "some less than all." In the expression

$$(A_{\prime}-O_{\prime})(A_{\prime}-O_{\prime})(A_{\prime}-O_{\prime})$$

we have the symbol of the *ab infirmiori* syllogism stated above, subject to the possibility of nonexistence if the number of exceptions in the two premises should exceed the number of instances in the universal term of the conclusion. If we look at $A_{\prime}O_{\prime}$, as a symbol descriptive of premises, we see one of the inconclusive forms; that is, a form from which we cannot draw an inference. But this is only because our inferences are all positive, and imply assertion of *sufficiency* in the premises. There is no use (except to show the manner in which the parts of a system hang together) in declarations of *insufficiency*: for we know that all collections of premises, whatever they may be sufficient for, will be insufficient for an infinite number of different things. And it is important to remember that while sufficiency is accompanied by *must be*, insufficiency only allows *may be*. From $A_{\prime}A_{\prime}$ the conclusion $A_{\prime}$ *must* be true: from $A_{\prime}O_{\prime}$ (and as far as these are concerned) it may be false. Accordingly $A_{\prime}O_{\prime}O_{\prime}$ and $O_{\prime}A_{\prime}O_{\prime}$ may serve to express the two defects of $(A_{\prime}-O_{\prime})$ $(A_{\prime}-O_{\prime})$ $(A_{\prime}-O_{\prime})$ from $A_{\prime}A_{\prime}A_{\prime}$, existing in the *ab infirmiori* syllogism, and possibly preventing conclusion altogether: just as $A_{\prime}O'O'$ and $O'A_{\prime}O'$ show the additional conditions by the fulfilment of which $A_{\prime}A_{\prime}A_{\prime}$ is elevated into the *à fortiori* syllogism $D_{\prime}D_{\prime}D_{\prime}$. It is worth while to dwell upon the varieties of this case. The *ab infirmiori* syllogisms of the strengthened particulars were previously considered.

In all the cases yet treated, we have had, *more or less*, the power of giving instances in common language, without recourse to numerical relation expressed in unusual terms. This of course, is always the case in the syllogisms of chapter V.; and we have given one *common instance* (though never met with in books of logic) from each set of *ab infirmiori* syllogisms. But there are still cases of the same sort to be considered. Though in our definite relation (page 56) of *all*, we usually (in books of logic at **[166]** least) make the relation exist, for each proposition, between the terms of the proposition itself, yet it may be asked whether we cannot sometimes infer such a species of universal as this, "for every Z there is an X which is Y;" Z being one of the names of the second premise. If we examine the first two cases, which will be guide enough, we shall find the following results from the new suppositions now made.

1. $m=\zeta$, $n=\xi$, gives $\zeta XY+\xi YZ=\eta' xz$: or if for every Z there be an X which is Y, and for every X a Z which is Y, then, so many ys as there are, so many things which are neither X nor Z. This syllogism has little new meaning, and no new application: it requires $\xi=\zeta$, and therefore X)Y and Z)Y.

2. $m'=\zeta$, gives $\zeta\, xy+n\, YZ=nxz$, or if for every Z there be that which is neither X nor Y, and if some Ys be Zs, there are as many instances which are neither X nor Z. This is a new and effective form.

3. $n=\xi'$, gives $m'xy+\xi' YZ=m'XZ$, a new form.

These two cases will be presently further considered. Now, observe that if $m+n$ in the first form, or $m'+n$ in the second, be v, that is, if the pair m and n be ξ and ξ', or η and η', or ζ and ζ', we have inference of the kind required. The first form gives no new syllogism: since v is more than η, Ys which are Xs, and Ys which are Zs, to the number of v, give the form (1.) by the main law of inference (page 154). In the second form, if $m'+n=v$, we distribute among the Ys and ys, Zs and xs to the full number of both, so that wherever there are not xs (that is, wherever there are Xs) there are Zs: or X)Z as obtained from the form.

But every way of constructing $m'xy+nYZ=(m'+n-\xi')XZ$ which gives $m'+n=v$, is only a case of $A_1A_1A_1$. For m' cannot exceed η', and n cannot exceed η: and $m'+n$ being v or $\eta+\eta'$, we must have $m'=\eta'$ and $n=\eta$ [27]; whence the assertion made. The forms we are now in search of, so far as quite new, are all contained in the two new ones above noted ; and of these, the second is but a transformation of the first. The eight varieties derived from use of contraries, or from the forms in page 161, beginning with the simplest, are

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[27] [In Edition 1 the *n* was misprinted as *u*.—Ed.]

$$\begin{array}{rllll|llll}
 & \zeta XY & +nZ{:}\eta Y & = & nX{:}\zeta Z & \zeta' XY & +nyz & = & nXZ \\
 & \zeta X{:}\eta Y & +nZY & = & nX{:}\zeta Z & \zeta' X{:}\eta Y & +nY{:}\zeta Z & = & nXZ \\
\text{[167]} & \zeta Y{:}\xi X & +nZ{:}\eta Y & = & nxz & \zeta' Y{:}\xi X & +nyz & = & nZ{:}\xi X \\
 & \zeta xy & +nZY & = & nxz & \xi' xy & +nY{:}\zeta Z & = & nZ{:}\xi X
\end{array}$$

These are syllogisms, which exhibit a curious kind of antagonism to the particular syllogisms. Take the syllogism $A_{\prime}O'O'$, the terms being M,Y,Z ; we have then M)Y +Z:Y=Z:M. Of course the conclusion M:Z is not legitimate from these premises alone : but if M have as many instances as Z, then M:Z *is* legitimate. For if Ms, as many as there are Zs, be among the Ys, and some of the Zs be not among the Ys, though all the rest were, there would not be enough to match all the Ms, or some Ms are not Zs. Now, let M be a name given to an X which is Y, and let such Xs have as many instances as Z, and the above becomes the first of the syllogisms in the last list. Thus, $I_{\prime}O'O_{\prime}$ is legitimate, if the quantity of the subject mentioned in $I_{\prime}$ be taken from the Zs. The second syllogism is $E_{\prime}I_{\prime}O'$, altered into $O_{\prime}I_{\prime}O_{\prime}$ in the same manner.

The reader may find all the results of the above case in the following rule, in which it is understood that all the super-propositions are to be written either way : thus, A' is written x)y, or Y)X, and O' is $nx{:}\eta y$, or $nY{:}\xi X$ (page 62). Write down any pair of particulars, followed by I if the pair be of the same sign, and O if the pair be of different signs : as in OOI or IOO. Accent the pair in contradiction to either the direct rule (page 92) as far as the words affirmative and negative are concerned : that is, let a *negative* beginning isolate nothing, and an *affirmative* beginning isolate the middle proposition : or else, accent the pair according to the inverse rule. Thus, $O_{\prime}O_{\prime}I_{\prime}$ and O'O'I' contradict the direct rule, and $O'O'I_,$ and $O_,O_,I'$ preserve the inverse rule. To make these syllogisms good (in the particular way in question) proceed thus :—When the *direct* rule is contradicted, take the quantity of the *first* concluding term from the total of the *second*, if the second premise be affirmative, and from its contrary, if negative. When the inverse rule is preserved, take the quantity of the *second* from the total of the *first*. Thus, in $O'O'I'$ the direct rule is contradicted: and it stands $m'\mathrm{x}{:}\eta'\mathrm{y}+n'\mathrm{y}{:}\zeta'\mathrm{z}=p'\mathrm{xz}$. The second premise is negative, the total of its predicate ζ' instances, that of the contrary ζ. Accordingly, $\zeta\mathrm{x}{:}\eta'\mathrm{y}+n'\mathrm{y}{:}\zeta'\mathrm{z}=n'\mathrm{xz}$, or $\zeta\mathrm{Y}{:}\xi\mathrm{X}+$[168]$n'\mathrm{Z}{:}\eta\mathrm{Y}=n'\mathrm{xz}$, which is one of the forms already obtained. Again, $O'O'I_,$ preserves the inverse rule, and is $m'\mathrm{x}{:}\eta'\mathrm{y}+n\mathrm{Z}{:}\eta\mathrm{Y}=p\mathrm{XZ}$. The total of the first term is ξ' instances. Hence, $m'\mathrm{x}{:}\eta'\mathrm{y}+\xi'\mathrm{Z}{:}\eta\mathrm{Y}=m'\mathrm{XZ}$, or $m'\mathrm{Y}{:}\xi\mathrm{X}+\xi'\mathrm{Z}{:}\eta\mathrm{Y}=m'\mathrm{XZ}$, which is derived from one of the forms given, by interchanging X and Z.

This class of syllogisms *with transposed quantity* naturally leads to the question, Is it used? Do such syllogisms occur in ordinary or in literary life? If not, there is no reason for selecting them from the infinite number of cases which the numerically definite system affords. To try this, suppose a person, on reviewing his purchases for the day, finds, by his countercheques, that he has certainly drawn as many cheques on his banker (and may be more) as he has made purchases. But he knows that he paid some of his purchases in money, or otherwise than by cheques. He infers then that he has drawn cheques for something else except that day's purchases. He infers rightly enough; but his inference cannot be reduced to a common syllogism, with the names in question for terms. It is really a syllogism of transposed quantity, as follows :—

For every "memorandum of a purchase" a "countercheque" is a "transaction involving the drawing of a cheque."

Some "purchases" are not "transactions involving, &c."

Therefore some "countercheques" are not "memoranda of purchases."

It may be worth while to give one instance of the verification of the contradictory form. By page 152 it appears that the contradiction of mXY is $(\xi-m+1)$Xy, or $(\eta-m+1)$xY, and that of m'Xy is $(\xi-m'+1)$XY, or $(\eta'-m'+1)$xy.

To mXY join the contrary of $(m+n-\eta)$XZ, or $(\xi+\eta-m-n+1)$Xz: we have then

$$m\mathrm{YX}+(\xi+\eta-m-n+1)\mathrm{zX};$$

the inference of which is $(m+\xi+\eta-m-n+1-\xi)$Yz, that is, $(\eta-n+1)$Yz, the contrary of nYZ.

Returning to the forms in page 161, it will be observed that we have no double inferences. In every case we have made use of one form of inference: if v be known, the other is a real equivalent; or else it is impossible, and as we have seen, then the **[169]** first is spurious. If v be not known then the second is either perfectly indefinite, or else identical with the one chosen. Examination will show that in every one of the cases cited in page 161, the neglected form of inference is only saved from perfect indefiniteness when we are able to apply the word *all* to one or other of the terms: the number being as indefinite as before; the *relation* thus obtained being definite. Take the first form, and make $n=\eta$; by the first inference we then get the syllogism $\mathrm{I}_1\mathrm{A}_1\mathrm{I}_1$: by the second, we get $(m+v-\xi-\zeta)$xz, indefinite both in number and relation. We do not know what v, ξ, and ζ are. If we knew as much as that $m+v$ is less than $\xi+\zeta$, we should know our inference to be spurious,* it being not the less an inference. Now, add the condition $m=\xi$: the first inference gives the syllogism $\mathrm{A}_1\mathrm{A}_1\mathrm{A}_1$, the second inference now becomes $(v-\zeta)$xz : definite relation enters, and we have z)x, or X)Z, or A_1, as before. And the same of the other forms.

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* I must again remind the reader, of the distinction between *spurious* and *illegitimate*, which exists in my language. The spurious inference follows from the premises, and is perfectly good and true: but from the constitution of the universe, it will always be true, whatever premises in that universe are taken. The illegitimate inference is that which does not follow from the premises. A conclusion not *known* to be spurious, that is, there not being the means of knowledge, *is not* spurious: but an illegitimate conclusion cannot be made legitimate, that is, following from the premises, by any further knowledge.

The reader may perhaps suppose that I ought to have commenced this chapter with the complex numerical syllogism, in imitation of the method which I followed in treating the ordinary syllogism. But in truth there is no system of complex syllogism of perfect numerical definiteness both in premises and conclusion. To show this, let m,XY with the comma, mean that there are exactly m Xs which are Ys, neither more nor fewer. Accordingly m,XY is a synonyme for $m\mathrm{XY}+(\eta-m)\mathrm{xY}$. Now combine m,XY and n,ZY, or

$$(m\mathrm{XY}+\overline{\eta-m}\,\mathrm{xY}),^{28}(n\mathrm{ZY}+\overline{\eta-n}\,\mathrm{zY})$$

We then have

$$\begin{aligned} m\mathrm{XY}+n\mathrm{ZY} &= (m+n-\eta)\mathrm{XZ} \\ (\eta-m)\mathrm{xY}+(\eta-n)\mathrm{zY} &= (\eta-m-n)\mathrm{xz} \\ m\mathrm{XY}+(\eta-n)\mathrm{zY} &= (m-n)\mathrm{Xz} \\ (\eta-m)\mathrm{xY}+n\mathrm{ZY} &= (n-m)\mathrm{xZ} \end{aligned}$$

[170] Two only of these have meaning : let them be the two upper ones. We can assign then Z or z to $(m+n-\eta)+(m-n)$, or to $2m-\eta$ of the Xs. But there are not all of the Xs here : for m is less than η, and than ξ, whence $2m$ is less than $\eta+\xi$, or $2m-\eta$ less than ξ. The rest of the Xs, $\xi+\eta-2m$ in number, may, for aught these premises declare, be either Zs or zs.

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[28] [The comma is wanting in the original edition, but seems indispensable to the sense as otherwise, with De Morgan's usual symbolism, the proposition would be I.—Ed.]

CHAPTER IX.

ON PROBABILITY.

THE most difficult inquiry which any one can propose to himself is to find out what any thing *is* : in all probability we do not know what we are talking about when we ask such a question. The philosophers of the middle ages were much concerned with the *is*, or *essence*, of things : they argued to their own minds, with great justice, that if they could only find out what a thing is, they should find out all about it : they tried, and failed. Their successors, taking warning by their example, have inverted the proposition ; and have satisfied themselves that the only way of finding what a thing is, lies in finding what we can about it ; that modes of relation and connexion are all we can know of the essence of any thing ; in short, that the proverb "tell me who you are with, and I will tell you what you are," applies as much to the nature of things as to the characters of men. We are apt to think that we know more of the essence of objects than of ideas ; or rather, of ideas which have an objective source, than of those which are the consequence of the mind's action upon them. I doubt whether the reverse be not the case : at any rate, when we content ourselves with inquiry into properties and relations, we have certain knowledge upon our most abstract ideas. The object of this chapter is the consideration of the degrees of knowledge itself. That which we know, of which we are certain, of which we are well assured nothing could persuade us to the contrary, is the existence of our own minds, thoughts, and perceptions, the two last when actually present. This highest knowledge, this absolute certainty, admits of no imagination of the possibility of falsehood.

We cannot, by stopping to consider, **[171]** make ourselves more sure than we are already, that we exist, think, see, &c. Next to this, come the things of which we cannot but say *at last* we are as certain of them as of our own existence; but of which, nevertheless, we are obliged to say that we arrive at them by process, by reflection. These we call *necessary truths* (page 33). The *necessity* of admitting these things causes some to imagine that they are merely identities, that they amount to saying that when a thing is, it is: but this is not correct. To say that two and two make four (which must be), and that a certain man wears a black coat (when he does so) both involve the pure identity that whatever is, is; and not one more than the other. Nor is two and two identically four, though necessarily so. Our definitions of number arise in the process of simple counting. Throw a pebble into a basket, and we say *one*: throw in another, and we say *two*: yet one more, and we say *three*, and so on. The full definitions of the successive numbers are seen in

$$1 \quad (1+1) \quad \{(1+1)+1\} \quad [\{(1+1)+1\}+1], \text{ \&c.}$$

That three and one are four is *definition*: it is our pleasure to give the name *four* to $3+1$. But that $3+1$ is $2+2$ is neither definition nor pure identity. It is not even true that "two and two" *is* four; that

$$[\{(1+1)+1\}+1] \textit{ is } (1+1)+(1+1)$$

It is true, no doubt, that "two and two" is four, in amount, value, &c. but not in form, construction, definition, &c.

There is no further use in drawing distinction between the knowledge which we have of our own existence, and that of two and two amounting to four. This absolute and inassailable feeling we shall call *certainty*. We have lower grades of knowledge, which we usually call *degrees of belief*, but they are really *degrees of knowledge*. A man knows at this moment that two and two make four: did he know it yesterday? He feels perfectly certain that he knew it yesterday. But he may have been seized with a fit yesterday, which kept him in unconsciousness all day: and those about him may have been warned by the medical man not to give him the least hint of what has taken place. He could swear, as oaths are usually understood, that it was not so: if he **[172]** could not swear to this, no man could swear to anything except necessary truths. But he could not regard the assertion that it was not so, as incapable of contradiction: he knows it well, but, as long as it may possibly be contradicted, he cannot but say that he might know it better.

It may seem a strange thing to treat *knowledge* as a magnitude, in the same manner as length, or weight, or surface. This is what all writers do who treat of probability, and what all their readers have done, long before they ever saw a book on the subject. But it is not customary to make the statement so openly as I now do: and I consider that some justification of it is necessary.

By degree of probability we really mean, or ought to mean, degree of belief. It is true that we may, if we like, divide probability into ideal and objective, and that we must do so, in order to represent common language. It is perfectly correct to say "It is much more likely than not, *whether you know it or not*, that rain will soon follow the fall of the barometer." We mean that rain does soon follow much more often than not, and that there do exist the means of arriving at this knowledge. The thing is so, every one will say, and can be known. It is not remembered, perhaps, that there is an *ideal probability*, a pure state of the mind, involved in this assertion: namely, that the things which have been are correct representatives of the things which are to be. That up to this 21st of June, 1847, the above statement has been true, ever since the barometer was used as a weather-glass, is not denied by any who have examined it: that the connexion of natural phenomena will, for some time to come, be what it has been, cannot be settled by examination: we all have strong reason to believe it, but our knowledge is *ideal*, as distinguished from *objective*. And it will be found that, frame what circumstances we may, we cannot invent a case of purely objective probability. I put ten white balls and ten black ones into an urn, and lock the door of the room. I may feel well assured that, when I unlock the room again, and draw a ball, I am justified in saying it is an even chance that it will be a white one. If all the metaphysicians who ever wrote on probability were to witness the trial, they would, each in his own sense and manner, hold me right in my assertion. But how many things there are to be taken for granted! Do my eyes still distinguish colours as be-[173]fore? Some persons never do, and eyes alter with age. Has the black paint melted, and blackened the white balls? Has any one else possessed a key of the room, or got in at the window, and changed the balls? We may be *very sure*, as those words are commonly used, that none of these things have happened, and it may turn out (and I have no doubt will do so, if the reader try the circumstances) that the ten white and ten black balls will be found, as distinguishable as ever, and unchanged. But for all that, there is much to be assumed in reckoning upon such a result, which is not so objective (in the sense in which I have used the word) as the knowledge of what the balls were when they were put into the urn. We have to assume all that is requisite to make our experience of the past the means of judging the future.

Having made this illustration to draw a distinction, I now premise that I throw away objective *probability* altogether, and consider the word as meaning the state of the mind with respect to an assertion, a coming event, or any other matter on which absolute knowledge does not exist. "It is more probable than improbable" means in this chapter "I believe that it will happen more than I believe that it will not happen." Or rather "I *ought* to believe &c.:" for it may happen that the state of mind which *is*, is not the state of mind which should be. D'Alembert believed that it was *two* to *one* that the first head which the throw of a halfpenny was to give would occur before the third throw: a juster view of the mode of applying the theory would have taught him it was *three* to *one*. But he *believed* it, and thought he could show reason for his belief: to him the probability *was* two to one. But I shall say, for all that, that the probability *is* three to one: meaning, that in the universal opinion of those who examine the subject, the state of mind to which a person *ought* to be able to bring himself is to look three times as confidently upon the arrival as upon the non-arrival.

Probability then, refers to and implies belief, more or less, and belief is but another name for imperfect knowledge, or it may be, expresses the mind in a state of imperfect knowledge. There is accurate meaning in the phrase "to the best of his *knowledge* and *belief*," the first word applying to the state of his circumstances with respect to external objects, the second to the state of his [174] mind with respect to the circumstances. But we cannot make any use of the distinction here: what we know is to regulate what we believe; nor can we make any effective use of what we know, except in obtaining and describing what we believe, or ought to believe. According to common idiom, belief is often a lower degree of knowledge: but it is imperative upon us to drop all the *quantitative* distinctions of common life, or rather to remodel them, when we come to the construction of a science of quantity.

I have said that we treat knowledge and belief as magnitudes: I will now put a broad illustration of what I mean. We know, (suppose it *known*) that an urn contains nothing but two balls, one white and one black, undistinguishable by feeling: and we know (suppose this also) that a ball is to be drawn. Disjunctively then we know "white will be drawn: black will be drawn," one or the other must be. How do we stand as to "white will be drawn," and "black will be drawn," separately? Clearly in no preponderance with respect to either. May we then properly and reasonably say that we divide our knowledge and belief of the event "one or the other" into two halves, and give one half to each. I can conceive much objection to this supposition: but, whether they formally make it or not, I am sure writers on probability act upon it, and are accepted by their readers.

Let us consider what magnitude is, that is to say, how we know we are talking about a magnitude. We know that whenever we can attach a distinct conception of more and less to different instances, so as to say this has more than that, we are talking of comparable magnitudes. We speak of a quantity of talent, or of prudence: we say one man has more talent than another, and one man more prudence than another: but we never say that one man has more talent than another has prudence. If we occasionally say he (the same one man) has more talent than prudence, it is only as an abbreviation: we mean that he has not prudence enough to guide his talent. Just as we might say (though we do not) that there is more cart than horse, when the horse cannot draw the cart: just as, speaking very loosely, we *do* say, the *pressure* of the atmosphere is not fifty *inches;* meaning that it is not enough to balance the pressure of fifty inches of mercury in the barometer. And thus, both up to, and beyond our means [175] of measurement, we form to ourselves distinct notions of comparable magnitudes, and incomparable magnitudes, as well as of the meaning of the somewhat incorrect, but easily amended, figures of speech by which we sometimes talk of comparing the latter.

But the object of all quantitative science is not merely magnitude, but the *measurement* of magnitude. And when are we entitled to say that we can measure magnitude? As soon as we know how, from the greater, to take off a part equal to the less: a process which necessarily involves the test of which is the greater, and which is the less, and, in certain cases, as it may happen, of neither being the greater nor the less. As to some magnitudes, the clear idea of measurement comes soon: in the case of length, for example. But let us take a more difficult one, and trace the steps by which we acquire and fix the idea: say *weight.* What weight is, we need not know: the Newtonian, who makes it depend on the earth's attraction, and the Aristotelian, who referred it to an impulse which all bodies possess to seek their *natural places,* are quite at one on their notions of the measurable magnitude which their several philosophies discuss. We know it as a magnitude before we give it a name: any child can discover the *more* that there is in a bullet, and the *less* that there is in a cork of twice its size. Had it not been for the simple contrivance of the balance, which we are well assured (how, it matters not here) enables us to poise equal weights against one another, that is, to detect equality and inequality, and thence to ascertain how many times the greater contains the less, we might not to this day have had much clearer ideas on the subject of weight, as a magnitude, than we have on those of talent, prudence, or self-denial, looked at in the same light. All who are ever so little of geometers will remember the time when their notions of an angle, as a magnitude, were as vague as, perhaps more so than, those of a moral quality: and they will also remember the steps by which this vagueness became clearness and precision.

Now a very little consideration will show us that, the moment we begin to talk of our belief (the mind's measure of our knowledge) of propositions set before us, we recognize the relations called more and less. Does the child feel that the bullet has [176] more something than the cork one bit better than an educated man feels that his belief in the story of the death of Cæsar is more than his belief in that of the death of Remus. Let any one try whether he have not in his mind the means of arranging the following set in order of magnitude of belief, including within that term all the range which comes between certain knowledge of the falsehood, and certain knowledge of the truth, of an assertion. Let them be 1. Cæsar invaded Britain with the sole view of benefiting the natives. 2. Two and two make five. 3. Two and two make four. 4. Cæsar invaded Britain. 5. Romulus founded Rome. He will probably discover the gradations of necessary truth, moral certainty, reasonable presumption, utter incredibility, and necessary falsehood. These are but names given to different states of the mind with respect to knowledge of propositions asserted; and I say they express different states of quantity.

The only difficulty, and a serious one it can be made, may be stated in the following question;—Are we to consider the sort of belief which we have of a necessary proposition (as two and two make four), that is, absolute knowledge, to which contradiction is glaring absurdity as only a strengthened or augmented specimen of the sort of knowledge which we have of any contingent proposition (such as Cæsar invaded Britain) which may have been, or might have been, false, and can be contradicted without absurdity. I answer, we can easily show that the difference of the two cases is connected with the difference between finite and infinite, not between two magnitudes of different kinds. The mathematician will easily apprehend this, and will look upon the various difficulties which surround even the explanation as upon things to which he is well accustomed, and which he understands by many parallel instances. We can invent circumstances under which a contingent proposition shall make any degree of approach to necessity which we please, but so that no actual attainment shall be arrived at. If an urn contain balls, and if one ball must be drawn, then, the balls being all white, it is necessary that a white ball must be drawn, as necessary as that two and two being in any place, there are four in that place: for there are no degrees of necessity. But let it be that there are black balls also, at the rate of one to a thousand [177] white ones: the drawing of a white ball is no longer necessary; but there is still a strong degree of assurance that a white ball will be drawn. We do not readily see how much: because the urn has no visible relation to our usual cases of judgment. But let it be made to represent the life of a youth of twenty: and let the drawing of a white ball represent his living to come of age, and of a black one his death in the interval. There ought to be *seven* black balls to the thousand white ones to make the cases parallel. And yet we know that our assurance of his survival is generally very strong: be it wise assurance or not, it exists, and we act upon it. Now suppose the rate to be one black to a million of white: the assurance is much increased, but still there is no necessity; the black ball may be drawn. Take one black to a million of million of white, or a million of million of million, &c.: long before we have arrived at such a point, we have lost all conception of the quantitative difference between our belief in drawing a white ball, and our belief that two and two are four. We say it is *almost impossible* that one trial should give a black ball: and this very phrase is a recognition of the sameness for which I am contending. Except on the supposition of such sameness, there is no *almost impossible*, nor *nearly certain*. Between the impossible and the possible, the certain and the not certain, there must be every imaginable difference, if we do not admit unlimited approach. For it will clearly not be contended that, representing certainty, say by 100, we can make an approach to it by an uncertainty counting as, say 90, but nothing higher. Representing the state of absolute knowledge by 100, any one, with a little consideration, will say that the laws of thought fix no numerical limit to our approach towards this state: but that things short of certainty are capable of being brought within any degree of nearness to certainty. On such considerations, I shall assume that necessity on the one hand, a certainty for, and impossibility on the other, a certainty against, are extreme limits, which being represented by quantities, may allow our knowledge of all contingent propositions to be represented by intermediate quantities.

It must be fully allowed, nay, imperatively insisted on, that nothing in the numerical view, tending to connect necessary and contingent propositions, can at all lessen the distinction between [178] them: nor give the latter any resemblance to the former, except only in the quantities by which they are indicated. Though there be only one black ball to as many white ones as would fill the visible universe, yet between that case and the one of no black balls must always exist the essential difference, that in the former a black ball *may* be drawn, and in the latter it *cannot*. But this very great distinction between the necessarily certain and the contingent, is it compatible with their being represented by numerical quantities as near to one another as we please? I answer that all who are acquainted with the relations of quantity are aware that nearness of value is no bar to any amount of difference of properties. A common fraction, for instance, may be made as near as we please in value to an integer: but there do not exist, even among propositions, more essential, or more striking, differences, than those which exist between the properties of integers and of fractions. There are crowds of theorems (I should rather say unlimited crowds of classes of theorems) which are always true when integers are used, and never true when fractions are used. Let any quantities be named, integer or fractional, and it is easy to make classes of theorems which are true for those quantities, and not for any others, however near to them. The reader who is not a mathematician must rely upon the knowledge of the one who is, that the difference between two quantities, no matter how nearly equal, may be connected with other differences as complete, and by practice as easily recognized, as the difference between necessary and contingent truth.

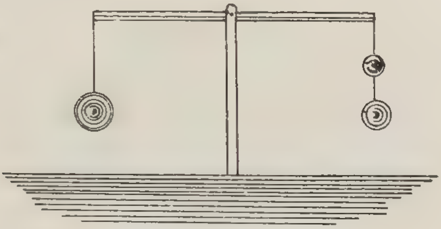
I will take it then that all the grades of knowledge, from knowledge of impossibility to knowledge of necessity, are capable of being quantitatively conceived. The next question is, are these quantities capable, *in any case*, of measurement, or of comparison with one another. At present, we stand as the child stands with respect to the bullet and the cork: perceptive of more and less, but without a balance by which to make comparisons. To show the postulate on which our balance depends, let us suppose an urn, which, to our knowledge, contains white, black, red, green, and blue balls, one of each colour. It is within our knowledge that a ball must be drawn: accordingly we have full knowledge (and of course *entire belief*) that the result "no ball" is impossible, and that "white, or black, or red, or green, or [179] blue" is necessary. To the result "white" we accord a certain probability, that is, a certain amount of belief. If a man tell us that white will be drawn, we may hold him rash, but we do not pronounce his communication incredible: let another tell us that "black, or red, or green, or blue" will be drawn, and we hold him not so rash, and his communication more credible. We may hold with either, if he will describe his knowledge and belief as partial and give them their proper amounts. Now, whether we shall proceed, or stop short at this point, depends upon our acceptance or non-acceptance of the following POSTULATE:—

When any number of events are disjunctively possible, so that one of them may happen, but not more than one, the measure of our belief that one out of any some of them will happen, ought to be the amount of the measures of our separate beliefs in each one of those some.

I mean that any one should say, A, B, C, being things of which not more than one can happen, "my belief that one of the three will happen is the sum of my separate beliefs in A, and in B, and in C." This is the postulate on which the balance depends; and there is a similar postulate before we can use the physical balance. The only difference (and that but apparent) is that we are to speak of weights collectively, and of events disjunctively. The weight of the (conjunctive) mass is the sum of the weights of its parts: the credibility of the (disjunctive) event is the sum of the credibilities of its components. There are several *may-bes*, any one of which may become a *has-been*: when we speak *disjunctively*, it is of the *will-be*, which cannot be said of more than one: the *may-be* of an event described as contained in "A, B, C," is to be represented as in quantity the sum of those in "A," in "B," and in "C."

Is it matter of mere necessity that, talking of physical weight, the weight of the whole is equal to the sum of the weights of the parts? We have learnt to admit this postulate, of which no man ever doubted: but no one can say that it was necessary. The laws of matter and mind being both what they are, the connexion between physical collection and mental summation is, I grant, necessary: the simplest of manual, and the simplest of mental, operations, are and, with us, must be, concomitants.

But, in the first place, it is *not true* that the weight of the [180] whole is equal to the sum of the weights of the parts, in the manner in which the reader probably imagines it to be true. Let the first part we hang on the balance be the weight which is correctly measured by W. Then if we hang under it another weight, as correctly represented by V, we think we are quite sure when we say that the collective mass must have a weight W+V because its parts have the weights W and V. But its parts have *not* the weights W and V. The weight of V is diminished by the upward attraction of W, and is, say, V−M: the weight of W is as much increased by the downward attraction of V, and is W+M. And though V−M and W+M added together do give V+W, yet it was not in this way that the reader made out his necessary truth. The universal equality of action and reaction did not exist in the thoughts of the first person who formed a distinct conception of the weight of the whole as composed of the sum of that of the parts: and he



was only right by the (so far as he was concerned) accidental circumstance, that two things of which he knew nothing, counterbalanced each other's effects. Nor do we know at this moment, as of necessity, that the proposition is correct. We have much reason to think that the law of equality of action and reaction is mathematically true: but, let it fail to the amount of only one grain in a thousand million of tons, and the proposition is not true, but only nearly true.

Again, the co-existence of those laws of mind and matter which best, so to speak, fit each other, and which make the phenomena of the external world, after due consideration, appear to be almost what they *must* have been, is not, to our apprehensions, a necessary coexistence. We can imagine the following result, though we cannot trace what the full consequences of it would be on the expression of the laws of thought. Conceive sentient beings, to whom the simplest mode of arithmetical succession is not 0, 1, 2, 3, &c. but 1, 10, 100, 1000, &c. *their* powers of numeration being so constructed that the second of these successions has that character of fundamental simplicity which we attach to the first. Of course, their primary symbols would be **[181]** significative of 1, 10, 100, &c. It would be impossible for us to conceive any mode by which *ten* or any other number could be thus fundamentally attached to unity, in a manner shared by no third number: but, I am not saying, " Imagine how this could be," but, " Imagine that it is." There is no contradiction in the supposition, either to itself, or, till we know much more of the mind than we now do, to anything else. Beings so constituted would have *logarithmic* brains; and if, thus constituted, they were placed among our material laws of existence, the manner in which the weight of the whole is to be inferred from those of the parts, would be a profound mystery for ages, only to be solved in an advanced stage of mathematical science. A recent mode of constructing mathematical tables, which generally carries with it the name of its eminent inventor, Gauss, would constitute one of their principal necessities: they would have to use it as their only mode (except actual experiment) of finding out that what we represent by 156 and 200, together make (and this making would be a complicated process) 356.

Instead, then, of trying to establish it as perfectly natural and necessary to say that our belief of " one of the two A or B, when both cannot happen," is, quantitatively speaking, the sum of our belief in A, and our belief in B, I have rather endeavoured to show that the analogous cases with which we first think of comparing this proposition, other kinds of composition, are not so natural and necessary as is supposed. There are two ways of levelling; by bringing up the lower, or bringing down the higher. And I particularly wish in this chapter to prevent the reader from accepting the arithmetical doctrine of probability quite so rapidly as is usually done. In furtherance of this object, I proceed to the following possible objection.

It may be said, you have, by thus formally identifying probability with belief, and stating a postulate which, in express terms, has not the most axiomatic degree of evidence, rendered somewhat difficult that which in the ordinary view of simple chances, is very easy. This charge, I hope, is true: such was my intention, at least. And my reason is, that in the ordinary view of the subject, one of two things occurs: either probability is separated by definition from state of belief, though it be known that the two words will afterwards be confounded without any per-[182]mission; or else the postulate is tacitly assumed, and the difficulty which I suppose myself charged with introducing, is slurred over. Take a common question;—An urn has two white balls and five black ones: there are seven equally likely drawings, two white; therefore the chance or probability of drawing a white ball is called two-sevenths. But the chance of either particular white ball is one-seventh. Now first, if any one should say that this is mere definition, I can, of course, allow it: but it then remains to show what connexion this defined probability has with any ordinary acceptation of the word. But if, probability meaning belief, or sentiment of probability actually existing in the mind, or index of the proper degree of belief, &c. &c—the above statement be made as fundamentally evident, I should then ask how it is known that the probability of "one or the other white ball being drawn" is properly set down as the *sum* of the probabilities of the separate white balls. And I cannot conceive any answer except that it is by an assumption of the postulate. That such assumption will finally be knowingly made, on the fullest conviction, by every one who studies the theory, I have no doubt whatever: nor that it has been made, no matter in what words, nor with what clearness of avowal, by every one who has studied that theory. And therefore I hold it desirable that the beginner should know what I have here told him.

It is indifferent, as far as the theory is concerned, what numerical scale of belief we take. We might, if we pleased, copy Fahrenheit's thermometer, set down knowledge of impossibility as 32°, perfect certainty as 212°, and other states of mind accordingly. Thus, 122° would represent perfect indecision, belief inclining neither way, an even chance. But this would complicate our formulæ: the usual and preferable plan is to assume 0 as the index of knowledge of impossibility, 1 as that of certainty, and intermediate fractions for the intermediate states. This mode of estimation makes formulæ and processes so much more easy than any other, that it must be adopted; but there is a strong objection to it in one point of view: as follows.

When we speak of belief in common life, we always mean that we consider the object of belief more likely than not: the state of mind in which we rather reject than admit, we call *un*belief. When the mind is quite unbalanced either way, we [183] have no word to express it, because the state is not a popular * one. The quantitative theory calls by the name of belief every admission of possibility. When there is only one black ball to a million of white ones, there is some belief that a black ball will be drawn ; a much larger belief in a white one. It would be advantageous in some respects that 0 should represent the state of indifference, $+1$, that of knowledge of certainty, and -1, that of knowledge of impossibility. But this would complicate formulæ too much. I consider it therefore desirable to use the common measures and formulæ, but to associate them with the one just proposed, in the following manner.

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* Many minds, and almost all uneducated ones, can hardly retain an intermediate state. Put it to the first comer, what he thinks on the question whether there be volcanoes on the unseen side of the moon larger than those on our side. The odds are, that though he has never thought of the question, he has a pretty stiff opinion in three seconds.

When a person tells us that his belief in an assertion is, say $\frac{3}{10}$, meaning that he considers it 3 for and 7 against, or 7 to 3 against, we should say in common talk that he disbelieves, but not very strongly. In the language of this theory, we say that he both believes and disbelieves, the latter more strongly than the former. Let us add that his *authority* is against the conclusion. If he say that it is in his mind an even chance, or that he has no opinion one way or the other, let us say that he gives no authority either way. If we adapt this definition to the supposition that $+1$ and -1 represent the extremes of authority for and against, we have the following rules. The measure of authority is twice the measure of belief diminished by unity, for, when positive, against, when negative : the measure of belief is half of unity increased (algebraically) by the measure of authority. If a represent the measure of belief, and A that of authority, then

$$A=2a-1 \quad , \quad a=\tfrac{1}{2}(1+A)$$

It is also advisable to have a term to represent what are usually called the *odds*. Some might think it desirable to rid the subject as much as possible of words derived from gambling : astronomers have done the same thing with the phrases of astrology, and chemists with those of alchemy. When it is 7 for and 3 against, [184] it might be said that the *relative testimony* for, is $\frac{7}{3}$, and that against, $\frac{3}{7}$. But the brevity of the first phrase will insure its continuance, let who will try to change it.

The ordinary rule is a consequence of the notions hereinbefore laid down, and of the particular mode of measurement adopted. It is as follows;—When all the things that can happen can be resolved into a number of equally probable (or credible) cases, some favourable and some unfavourable to the event under consideration, then the fraction which the favourable cases are of all the cases, measures the probability (or credibility) of the arrival of the event: and the fraction which the unfavourable cases are of all the cases, measures the probability (or credibility) of the non-arrival. There are, for instance, in an urn, 5 white, 4 black, and 3 red balls, 12 in all. It is assumed that we know them to be equally likely to be drawn; which here means no more than that we know nothing to the contrary. That one ball must be drawn, is supposed certainly known. Accordingly, our belief in "one or another" is represented by 1: which is, by the postulate, the sum of the several credibilities of the balls; which last are all equal. Therefore each ball has $\frac{1}{12}$: and by the same postulate, the event "one or other of the white balls" or the drawing of a white ball, has $\frac{5}{12}$; of a black ball $\frac{4}{12}$; of a red ball, $\frac{3}{12}$.

Instances like the above, in which we invent all the cases and have arbitrary power over their number, are the only ones on which we can employ *à priori* numerical reasoning. They are also the only ones on which we can try experiments. It is important to know whether, as a matter of fact, our belief, numerically formed, will be approximately justified by the results of trial. And this justification is found to exist, in the following way. It is a remote, but certain, conclusion from the theory, requiring mathematical reasoning too complicated to introduce here, that events will, in the long run, happen in numbers proportional to the objective probabilities under which the trials are made. For instance;—if a die be correctly formed, so that no one face has more tendency than another to fall upwards, the probability of throwing an ace is $\frac{1}{6}$; that of not throwing an ace is $\frac{5}{6}$. The theory tells us its own worthlessness, if in the long run, *not-ace* do not occur five times as often as *ace*. If 60,000 trials were made, the theory would tell us to expect *about* 10,000 [185] aces and about 50,000 *not-aces*. Practice confirms the theory: not, that I know of, in the actual case just cited, but in similar ones. I will state an instance.

Throw a half-penny up, and if it give *tail*, repeat the throw, and so on, till *head* arrives: and let this succession be called a *set*. The probability that a set shall consist of one throw, is shewn by the theory to be ½; that it shall have two throws, ¼; three throws, ⅛; and so on. If a very large number of sets be tried, we are to expect that about half will be of one throw, about a quarter of two throws, about an eighth of three throws; and so on, as long as the number is large enough to give any prospect of something like an average. This experiment has been tried twice: once by the celebrated Buffon, and once by a young pupil of mine, for his own satisfaction; both in 2,048 sets. The results were as follows; the third column showing the number of each kind which the theory asserts to be most probable.

	B	H	
Head at the first throw	1061	1048	1024
No head till the 2nd throw	494	507	512
3rd —	232	248	256
4th —	137	99	128
5th —	56	71	64
6th —	29	38	32
7th —	25	17	16
8th —	8	9	8
9th —	6	5	4
10th —	0	3	2
11th —	0	1	1
12th —	0	0	1
13th —	0	0	
14th —	0	1	
15th —	0	0	
16th —	0	1	
&c —	0	0	
	2048	2048	2048

In Buffon's trials, there were altogether 1992 tails to 2048 heads, and in Mr. H's there were 2044 tails to 2048 heads.

Instances in which we can command all the cases are to the [186] mind, in this theory, what accessible lengths are to the eye. We can measure the latter by a rule, and so train the organ to judge of lengths which cannot be approached, or cases in which the rule is not at hand.

I shall now refer the reader to other works on the subject, for further details on the operative part, and proceed to just as much as is necessary for the particular purpose of the next chapter, namely, the application of the hypothesis of measure of belief to questions of argument and testimony. Two theorems will be enough: the first relating to independent events, the second to the probability of events which are neither wholly independent, nor wholly consequent, either upon the other. The word *event* is used in the widest possible sense: it does not even necessarily mean future event. Unless our knowledge, either of the circumstances, or of the event itself, thereby undergo some alteration, it is nothing to us now whether it has happened, or is to happen.

Let there be two events, P and Q, of which the probabilities are the fractions a and b; and let them be wholly independent of one another, the arrival or non-arrival of either being perfectly independent of that of the other. The probability that both shall happen is the product of a and b: and similarly for more events than two. Suppose, to take an instance, that a is $\frac{4}{7}$ and b is $\frac{3}{5}$. We must then consider P as an event which has 3 ways of failing to 4 of happening: if we would have an urn from which the credibility of drawing a white ball should be that of the happening of P, we must put in 4 white balls and 3 not white (say black) balls. Similarly to represent Q, we must have an urn of 3 white and 2 black balls. Now to ascertain the prospect of drawing white from both urns, we must count all the cases. A ball from the urn of 7 may be combined with one from the urn of 5, in 7×5 or 35 ways. But a *white* ball from the first urn may be combined with a *white* ball from the second, in 4×3 or 12 different ways. There are then 35 cases in all, 12 of which are favourable: hence the probability in favour of white from both (which is that of the two events both happening) is

$$\frac{12}{35} \quad\text{or}\quad \frac{4\times3}{7\times5} \quad\text{or}\quad \frac{4}{7}\times\frac{3}{5} \quad\text{or}\quad ab.$$

[187] Similar reasoning may be applied to more events than two. This theorem has a large number of consequences, some of which we may notice.

When a is the probability for, $1-a$ is the probability against. This I shall always denote by a': similarly b' will stand for $1-b$; and so on.

Required the probability that of a number of independent events, P,Q,R, &c. one or more shall happen. Let a,b,c, &c. be the several probabilities, then that of their all failing is the product $a'b'c'$ .... and that of their *not* all failing (or of one or more happening) is $1-a'b'c'$ .... Accordingly, if there be only two events, for "one or both" we have $1-(1-a)(1-b)$ which is $a+b-ab$. If the number of events be n, and all equally probable (so that $a=b=c$, &c.) for "one or more" we have $1-a'^n$ or $1-(1-a)^n$.

It is a consequence of this last that, however unlikely an event may be, it is sure (in the common sense of the word) to happen, if the trial can be repeated as often as we please. However small a may be, or however near to unity $1-a$, n may be taken so great that $(1-a)^n$ shall be as small as we please, or $1-(1-a)^n$ as near to unity as we please, or the probability that the unlikely event will happen once or more in n times, as great as we please. Let $a=1:(k+1)$, which means that the odds are k to 1 against the event on any one trial: the following rough deductions will show what kind of results the formula gives, true within an instance or two when k is considerable. In $\frac{7}{10}k$ instances it is an even chance that the event happens once or more; in $2{\cdot}3k$, it is 9 to 1; in $4{\cdot}6k$, 99 to 1; $6{\cdot}9k$, 999 to 1; $9{\cdot}2k$, 9999 to 1: and in $23k$, it is ten thousand millions to 1. Thus, suppose at each trial it is a hundred to one against success. Then of those who try 70 efforts, as many will succeed once or more as will altogether fail, in the long run. Of those who try 6900 times, only one of a thousand will always fail. A person who will not examine an assertion that comes to him with ten to one against it, must count it an even chance that he throws away one or more truths, if he follow his plan seven times.

Let us now suppose that there are reasons why the several instances which can arrive are not equally credible. Suppose the urn to contain a white, a black, and a red ball, and ourselves to [188] have reasons to think the balls not equally probable or credible, but that 6, 5, and 2 are the proportions of the degrees of belief we should accord to them severally. If then $6x$ represent the probability of a black ball, $5x$ and $2x$ will represent those of the other two severally. By the postulate, $13x$ represents that of one or the other. But this is certainty; whence x must be $\frac{1}{13}$, and $\frac{6}{13}$, $\frac{5}{13}$, and $\frac{2}{13}$ are the probabilities of the white, black, and red balls. That is to say, when the several instances are unequally probable, we must count each instance as though it occurred a number of times proportioned to its probability, and then proceed as in the case of equally probable instances. Thus, in the above, instead of saying (as we should do if the balls were equally probable) that the probability of the white ball is

$$\frac{1}{1+1+1}, \quad \text{we say it is} \quad \frac{6}{6+5+2}; \quad \text{or} \quad \frac{6m}{6m+5m+2m}$$

would do, m being any number or fraction whatsoever.

Now suppose two urns, one of all white balls, and the other of all black ones. If we actually draw a ball, and find it white, we know that the urn chosen to draw from must have been the first: the second *could not* have given that drawing. But suppose the first urn to have 99 white balls to one black, and the second one white to 1000 black. If we now draw again, and draw a white one, not knowing from which we drew, we feel almost certain, from the drawing, that we have chosen the first urn. We still feel *almost* certain that the second urn would have given a black ball. This inversion of circumstances, this conclusion that the circumstances under which the event did happen, are most probably those which would have been most likely to bring about the event, is of the utmost evidence to our minds: but the question now before us is, are we to call it a second postulate, or is it deducible from the other one? It is so deducible, and is not a second postulate; but it has not been usual to give a very distinct account of the deduction.* If it could not be made, [189] the following process would, no doubt, be sufficient: it has often been held so. Let the urns have 6 white balls to 1 black, and 2 white balls to 9 black. Then the probabilities of drawing a white ball from the two are $\frac{6}{7}$ and $\frac{2}{11}$, which are in the proportion of 33 to 7. If, because when we choose the first urn, we have nearly five times as much chance of a white ball as the second one would give, we conclude that a known white ball from an unknown urn is in that proportion more likely to have come from the first urn; we shall have $\frac{33}{40}$ and $\frac{7}{40}$ for the proper degrees of belief in the two urns. For if $33x$ be that for the first urn, then $7x$ must, by the assumption, be that for the second: and for one or the other, we have $40x$. But this is certainty; whence x must be $\frac{1}{40}$.

To reduce this result to dependence upon the first postulate, proceed as follows. The probability that two events are *connected*, our belief, that is, in the *connexion*, must be the same whether the two events, or either of them, have happened, or whether they be yet to happen: unless there be something in the happening which alters our knowledge, and puts us in a different state for forming a judgment.

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* So well established is this species of inversion in the mind, that both Laplace and Poisson, the two most eminent mathematical writers on the subject, of the present century, have in a certain case assumed that an equation which gives the most probable value of x in terms of y, is therefore the one which gives the most probable value of y in terms of x. This is carrying the principle too far.

Suppose I make up my mind, rightly or wrongly, as to how far I will believe that a white ball, *if* drawn, *will have been* drawn from the first urn. An instant after, I am told that the trial I anticipated has been made, and the contingency which I supposed has occurred ; a white ball has been drawn. I know no more than I took myself to know in my hypothesis ; and cannot therefore have any means of altering my opinion. Now, without altering the proportions in the urns, change the numbers of the balls, so that there may be the same total number in each : let them be

{66 white, 11 black} {14 white, 63 black}

Now put each ball in an urn by itself, 154 urns in all. This gives $\frac{1}{154}$ to any one ball, if I choose an urn at hazard. But it was so before : as to the first of the two urns for instance, $\frac{1}{2}$ was the probability of choosing that urn, and $\frac{1}{77}$ that of choosing one particular ball from it : and $\frac{1}{2}\times\frac{1}{77}$ is $\frac{1}{154}$. If we then remove all the urns with black balls, so that a white ball must be drawn, the chance of its being one of the 66 is $\frac{66}{80}$ or $\frac{33}{40}$. If without removing the black balls, we think of the probability of a white **[190]** ball, if drawn, being of the 66, or of the 14, the credibilities of those suppositions are as 66 to 14. If, having chosen an urn, we find it contains a white ball, the same probabilities are still in that proportion.

The rules derived from similar reasoning, whether for judging of the probabilities of precedents from an observed consequent, or for judging of the probabilities of events which restrict each other, are precisely the same, as follows. If the probability of the observed event, supposed still future, from the several possible precedents, severally supposed actually to exist, be $a,b,c,$ &c. : then, when the event is known to have happened, the probabilities that it happened from the several precedents are

$$\frac{a}{a+b+c+\ldots} \text{ for the first, } \frac{b}{a+b+c+\ldots} \text{ for the second, \&c.}$$

Again, if there be several events, which are not all that could have happened; and if, by a new arrangement (or by additional knowledge of old ones) we find that these several events are now made all that can happen, without alteration of their relative credibilities: their probabilities are found by the same rule. If a, b, c, &c. be the probabilities of the several events, when not restricted to be the only ones: then, after the restriction, the probability of the first is $a \div (a+b+\ldots)$, of the second, $b \div (a+b+\ldots)$ and so on.

We may obtain a very distinct notion of this last theorem, as follows. Suppose two events, which are *among* those that can happen, and let one, say, be twice as probable as the other. This means, that among all the independent, and equally likely, cases, there are twice as many favourable to the first as to the second. Now, suppose by some alteration of suppositions, the introduction of new knowledge, for instance, it is found, all the cases remaining as before, that all are prevented from happening except these two events. This new state of things does not alter the cases in number: accordingly, the *proportion* of the probabilities of the two events is as before, two to one. But now one of them must happen: or the sum of these probabilities must be unity. It follows then that one of them is $\frac{2}{3}$, and the other $\frac{1}{3}$. The same reasoning may be applied to more complicated cases.

It frequently happens, when different problems are solved by **[191]** the same formula, that they may be considered as the same problem in two different points of view: and also that one and the same problem may be considered as belonging to either class. For instance;—Let there be two witnesses, whose credibilities (or the probabilities that in any given instance they are correct) are a and b. As long as we do not know that they are talking about the same thing, the probability that both will tell truth is ab. But the moment we know that they both assert the same thing, the problem is changed: they must now be either both right or both wrong; before, one might have been right and the other wrong. To take the first view of the problem, we have now an observed event, both state that the circumstance did happen. There are two precedents; the event did, or did not, happen. If it did, the probability of the observed event (which is then that both are right) would be ab; if it did not, it would then be $(1-a)(1-b)$. Accordingly, the probability that the observed event did happen, will be, by the rule above, ab divided by $ab+(1-a)(1-b)$.

If we take the second view, we have, before the restriction, four possible cases, the probabilities of which are ab, $a(1-b)$, $b(1-a)$ and $(1-a)(1-b)$. After the restriction, only the first and fourth are possible: whence the conclusion is as just given. Full exemplifications of these methods will be found in the next chapter.

CHAPTER X.

ON PROBABLE INFERENCE.

THERE are two sources of conviction, *argument* and *testimony*, reason why the thing should be, statement that the thing is. When the argument is necessarily good, we call it *demonstration*: when the statement can be absolutely relied on, we call it *authority*. Both words are used in lower than their absolute senses; thus, very cogent arguments are often called demonstration, and very good evidence, authority.

I shall suppose all the arguments I speak of to be logically **[192]** valid; that is, having conclusions which certainly follow from the premises. If then the premises be all true, the conclusion is certainly true. If a, b, c, &c. be the probabilities of the independent premises, or the independent propositions from which premises are deduced, then the product $abc\ldots.$ is the probability that the argument is every way good.

Argument being offer of proof, its failure is only failure of proof: and the conclusion may yet be true. But testimony is an assertion of the truth of the conclusion; and its failure can only be failure of truth. If a proposition of Euclid turn out to be badly demonstrated, the enunciation need not therefore be false. An argument may prove, disprove, or neither prove nor disprove: a testimony cannot be true, false, or neither true nor false. This distinction generally gains no more than a one-sided admission: persons begin to see it when some over-zealous brother writes weakly on their own side of a question; but they are very apt to think, with respect to the other side, that answering the arguments is disproving the conclusion.

Testimony is, for the above reason, more easily understood than argument. It is the most effective mode of conveying knowledge to the uneducated. But it must not be supposed that, in any stage of reason, argument can be the only vehicle of information, even on subjects called argumentative. This point is one of great importance.

When argument is demonstration, it establishes its conclusion against all testimony. The idea of an infallible witness bearing evidence against a demonstrated conclusion, is a contradiction. That n consecutive numbers have a sum which is divisible by n, whenever n is odd, is demonstrated. If a thousand of the best qualified witnesses that ever lived, both for honesty and arithmetic, were to swear that they had discovered 101 very high consecutive numbers, the sum of which is not divisible by 101, any mere beginner in mathematics would be more sure that a thousand good witnesses had lost their wits or their characters, than any one else can be of anything not admitting of demonstration.

But when argument does not amount to demonstration, not only is the truth or falsehood of the conclusion matter of credibility, but the issue of the argument is not that mere truth or [193] falsehood. It does not stand thus: "According as this argument is good or bad, so is the conclusion true or false," but "According as this argument is good or bad, so is the conclusion *true in this way*, or *not true in this way*, (that is, either false, or true in some other way)." If we were to say "men are trees, and trees have reason, therefore men have reason," we have a perfectly logical argument, false in the matter of both premises: but we cannot deny the conclusion.

Suppose now that an argument is presented to us of which we are satisfied that the like will prove their conclusions to be true in the particular modes asserted, in nine cases out of ten. What are we to say of the truth or falsehood of the conclusion? We have $\frac{9}{10}$ of belief to its being true in one particular way: how much shall we add for other possible ways? Are we to rest in the conclusion as having 9 to 1 for it, or are we to allow more? We cannot say, let us confine ourselves to the grounds we have got, and believe or disbelieve, not in the conclusion, but in the conclusion as obtained in that one way.

I take it for granted that the mind must have a state with respect to every assertion presented to it, with reason, or without reason. Every proposition, the terms of which convey any meaning, at once, when brought forward, puts the hearer into some degree of belief, or, if we use the common phrase, of belief or unbelief: including, of course, the intermediate state, which is as clearly marked upon our scale as any other. Men who are accustomed to suspend their opinion, as it is called, that is, to throw themselves into the intermediate state when they have no definite reason to think either way, are interested in this question as much as any others. If there be some state, though not numerically appreciable, in which their belief must be, there is some state, which they would rather know numerically than not, in which it ought to be. In the preceding case, suppose it known that 9 to 1, or $\frac{9}{10}$, is granted to the conclusion from the argument alone, and any one wishes to suspend his opinion as to the remaining $\frac{1}{10}$. Is he to grant half of that $\frac{1}{10}$, and say that $\frac{9}{10}+\frac{1}{20}$ or $\frac{19}{20}$ is what he would wish to make the measure of his belief, if he knew how? The consideration of this question will enter among others.

The manner in which he deals with the result of the argument [194] must depend upon *testimony*, using the word in its widest sense. First, every man has, as just noticed, a testimony in his own mind as to every proposition. He may set out with the intermediate state: he may have no reason to lean either way, *and may know it*; that is to say, he may have to apply an argument of $\frac{9}{10}$ to an existing probability of $\frac{1}{2}$. Or he may have previous good reason, or bad reason, which makes him lean to the assertion or denial; and the measure of this leaning must then be combined with $\frac{9}{10}$. Or he may have other testimony to combine with that of his own previous state. Any way, he cannot have a definite opinion on the bare truth or falsehood of the conclusion of the argument, without appeal to the previous state of his own mind at least, if not to that of others.

It is generally said that we throw away authority, and judge by argument alone; that our reason is to be convinced, and not biassed by the opinion of others; that no conclusions are worth anything, except those which a man forms for himself. All the forms in which this frequent caution is expressed, I take to be distortions of the very needful warning not to allow authority more weight than is properly due to it: a warning, by the way, which is just as much wanted with respect to argument as to authority. For every mistake which has been made by taking authorities *on trust* (that is, taking bad witnesses to prove the goodness of asserted good ones), one mistake at least has been made by taking arguments *on preponderance:* that is, treating them as proving their conclusion, as soon as they show it to be more likely than its contradiction.

To form the habit of allowing authority *no more* weight than is due to it, and the same of argument, is undoubtedly one great object of mental cultivation: but it ought not to be forgotten that it is another and just as great an object to form the habit of allowing them *no less*. Suppose an argument of value $\frac{9}{10}$ is presented, and that at the same time we have the testimony of a witness against the conclusion, of whom we *know* that he leads us right 1000 times for each once that he misleads us. Is there any sense in reducing this witness to one of no authority, or of an even chance, upon the principle of depending on argument only? Except the argument be demonstration, we must be prepared to admit that a witness may be as good as an argument, or better. **[195]** I shall now proceed to the several problems which this subject requires, considering first testimony alone, next argument alone, and then the two in combination.

Problem 1. There are independent testimonies to the truth of an assertion, of the value, μ, ν, ϱ, &c. (one of them being the initial testimony of the mind itself which is to form the judgment): required the value of the united testimony.

Let μ' be $1-\mu$, &c. as in page 187. Here is a problem of the same class as in page 190; the restrictions are, that all the testimonies are right, or all wrong, the independent chances of which are $\mu\nu\varrho\ldots$ and $\mu'\nu'\varrho'\ldots$ Hence the probabilities are

$$\frac{\mu\nu\varrho\ldots}{\mu\nu\varrho\ldots+\mu'\nu'\varrho'\ldots}\text{ for; }\quad\frac{\mu'\nu'\varrho'\ldots}{\mu\nu\varrho\ldots+\mu'\nu'\varrho'\ldots}\text{ against.}$$

Observe, first, that any numbers proportional to μ, μ' &c. will do as well: and if the products have a common denominator, (as generally they have) the numerators only need be used. Secondly, the easiest way of expressing the result is by saying that it is $\mu\nu\varrho\ldots$ to $\mu'\nu'\varrho'\ldots$ for, or $\mu'\nu'\varrho'\ldots$ to $\mu\nu\varrho\ldots$ against.

For instance, let it be in my mind 99 to one against an assertion, that is, I bear only the testimony $\frac{1}{100}$ *in favour* of it. Let four witnesses, for whose accuracy it is 2 to 1, 3 to 1, 4 to 1, 5 to 1, depose in favour of it: I want to know how it ought to stand in my mind. The testimonies for and against, are

$$\frac{1}{100},\ \frac{2}{3},\ \frac{3}{4},\ \frac{4}{5},\ \frac{5}{6};\ \text{and}\ \frac{99}{100},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6};$$

Hence, neglecting the common denominator, it ought to be $1\times2\times3\times4\times5$ to $99\times1\times1\times1\times1$, or 120 to 99, or 40 to 33, for the assertion.

Observe that in saying the witness gives testimony, say $\frac{2}{3}$, it is of no consequence whether it be a question of judgment, or of veracity, or of both together. I mean that, come how it may, I am satisfied that when he says anything, it is 2 to 1 he says what is correct.

An easy rule for the more common modes of expression presents itself thus. The combined relative testimony is the product of the separate relative testimonies. Thus, two witnesses of 6 truths to one error, and of 7 truths to one error, are equivalent [196] to one witness of 42 (or 6×7) truths to one error. Three witnesses of 8, 6, 5 truths to

7, 3, 11 errors are equivalent to one witness of 80 truths to 77 errors.

A jury of twelve equally trustworthy persons, after conferring together, agree to an assertion on which previously I had no leaning. Supposing me fully satisfied that such agreement gives 100 to 1 for their result, what am I to think of the *deliberate* opinion of any one among them, that is, of his opinion after he has had the advantage of discussion with others.

Let μ be the value of such testimony from any one; then by the question

$$\mu^{12} : (1-\mu)^{12} : : 100 : 1, \quad \text{or } \mu : 1-\mu : : \sqrt[12]{100} : 1$$

say as 1·468 to 1. That is, I think inconsistently if I rely on the united verdict as upon 100 to 1, unless I am prepared to think it 1468 to 1000, or about 3 to 2, for each juror alone.

Of $m+n$ equally trustworthy jurors, a majority m are for, and n against, a conclusion. If μ be the value of the testimony of each, then the odds are to be taken as being $\mu^m(1-\mu)^n$ for, and $\mu^m(1-\mu)^n$ against. But

$$\mu^m(1-\mu)^n : \mu^n(1-\mu)^m : : \mu^{m-n} : (1-\mu)^{m\quad n}$$

which are exactly as if the majority $m-n$ had been all, and unanimous. From the original formula it will appear that two equally good testimonies on opposite sides produce no effect on the result.

If then, the unanimity of the jury box in this country could be considered as that of deliberate conviction, we might say that a larger jury, with the condition that the majority should exceed the minority by 12 at least, would be always as good, and often better. But there are various considerations which prevent the above result from being applicable. The necessity of being unanimous, as our law stands, may lower the value of the verdict. On the other hand, a jury of 30, required to find by a majority of 12, would generally proceed to a vote before they had put the matter to each other with the real desire to gain opinion which the present practice produces: consequently, the value of their verdict would perhaps be lower than that of the majority only, required to be unanimous.

[197] The theory thus appears to confirm the notion on which we often act, that a given excess of majority over minority, is of the same value whatever the numbers in the two may be. And this might be the case, if the thing called deliberation in a large body, were as well adapted to the discovery of truth as the same thing in a smaller one. The reader must remember that this test does not compare the one witness on his own judgment with a number after common deliberation; but the first, after common deliberation with others, is compared with the whole.

But in this, and all the problems of this chapter, the distinction must be carefully drawn between the credibility of a circumstance at one time and at another. For example, a witness enters with 10 to 1 in his favour, and owing to combination with others, the result comes out that it is 100 to 1 he is in error in the particular matter on which he gives evidence. We cannot believe both that it is 10 to 1 he is right, and 100 to 1 that he is wrong. What we believe is the latter, for the case in question.

As another instance, suppose m independent witnesses of equal goodness (μ) unite in affirming that a certain ball was drawn from a lottery of n balls: collusion being supposed impossible. My knowledge of the circumstances of the affirmation here alters the problem. If n be considerable, it is almost impossible that the witnesses, by independent falsehood or error, should all pitch on the same wrong ball. To find the bias this ought to give me to the conclusion that they have told the truth, I must observe that there being $n-1$ balls not drawn, whichever of these any one chooses, by error, the chance of any one of the rest choosing the same is $1 \div (n-1)$, the probability that all the $m-1$ shall choose the same is $1 \div (n-1)^{m-1}$. Hence, the odds are as μ^m to $(1-\mu)^m$ multiplied by the last-named expression, or as $(n-1)^{m-1}\mu^m$ to $(1-\mu)^m$. If n be very great, the odds may be enormous for the assertion, even though μ, the credibility of each witness, may be small. In cases of ordinary evidence, the thing asserted is usually one out of almost an infinite number of equally possible assertions, and the agreement of even two witnesses (for when m is *two* or upwards, n appears in the formula) is certain conviction, if, as assumed, we know the two witnesses to be not in collusion. If $\mu=1\div n$, which is as much as to say that the evidence of each witness makes a ball no more likely to have been the one drawn, **[198]** because he says it, that it was on our mere knowledge that a ball *had* been drawn, it turns out 1 to $n-1$ for the truth of the assertion, just as it was before the evidence. But let $\mu=(1+a)\div n$, a being any fraction, however small, that is, let each witness make the assertion more probable than at first, however little: then the odds for its truth become

$$(1+a)^m \text{ to } \left(1-\frac{a}{n-1}\right)^{m-1}(n-1-a)$$

which odds may be made as great as we please, by sufficiently increasing m. That is to say, however little each witness may be good for, in real support of the assertion, or in making it more probable than it is of itself, a sufficient number of witnesses, certainly independent, will give it any degree of credibility whatever.

The student of this subject is always struck by the frequency of the problems in which the science confirms an ordinary notion of common life, or is confirmed by it, according to his state of mind with respect to the whole doctrine. It is impossible to say that we have a theory *made to explain* common phenomena, and hence affording no reason for surprise that it does explain them. The first principles are too few and too simple, the train of deduction ends in conclusions far too remote. I believe hundreds of cases might be cited in which the results of this theory are found already established by the common sense of mankind: in many of them, the mathematical sciences were not powerful enough to give the modes of calculation, when the principles of the theory were first digested.

There are problems, however, in which we cannot easily come into possession of data on which many will agree. The simple question of independent witnesses is not one of them: but the question of collusion is. One of the difficulties is as follows. We cannot institute independent hypotheses upon the goodness of the witnesses and the probability of their having conferred upon their evidence. They declare, expressly or by implication, that they have not done so: if they have, there is falsehood in one part of their evidence; or, which makes the difficulty still greater, there may have been general, but (as they assert or imply) not particular conference: they may have been biassed by **[199]** each other, without knowing how or to what extent. The first step in one view of the problem is easily made, as follows.

Let μ be the value of the evidence of each witness, m their number, n the number of assertions they have power to choose from, all as before. Let λ be the probability that there has been particular conference between them. There are then four cases to which the problem is restricted:—(1) they have conferred and agreed to speak truth; (2) they have not conferred and all speak truth; (3) they have conferred and agreed on a falsehood; (4) they have not conferred and have all lighted upon the same falsehood. The a priori probabilities of these four cases are

$$\lambda\mu^m, \quad (1-\lambda)\mu^m, \quad \lambda(1-\mu)^m, \quad \frac{(1-\lambda)(1-\mu)^m}{(n-1)^{m-1}}:$$

and the odds that they speak the truth (supposing n so great that we may reject the fourth case) are μ^m to $\lambda(1-\mu)^m$. Now comes the practical difficulty of this question;—How are λ and μ to be connected? Every case which is worth examining supposes that the greater the chance of there having been particular conference, the less is the witness worth from that very circumstance. For it is to be remembered that we are not generally able to give the witness a character wholly independent of his evidence in the case before us; in historical questions, for instance, it frequently happens that we have nothing but the witnesses to try * the case by, and nothing but the case to try the witnesses by. A very common occurrence is this;—that a case is one in which no one would throw any doubt upon the witnesses, except for suspicion of conference, and just as much doubt as there is suspicion of conference. This makes $\mu = 1 - \lambda$, and gives $(1 - \lambda^m) : \lambda^{m+1}$ for the odds in favour of the assertion. On this supposition, it follows that whenever the chances are against all the witnesses having conferred particularly, their number, if great enough, ought to give any degree of credibility to the assertion.

[200] *Problem* 2. Let there be any number of different assertions, of which one must be true, and only one: or of which one may be true, and not more than one: or of which any given number may be true, but not more: required the probability of any one possible case.

The solution of all these varieties depends on one principle, explained in page 190; requiring the previous probabilities of all the consistent cases to be compared. As an instance, suppose four assertions, A,B,C,D, and suppose μ,ν,ϱ,σ, to be the probabilities from testimony, for each of them. If either of them have several testimonies, their united force must be ascertained by the last problem. First, let it be that one of them must be true, and one only. The probabilities in favour of A,B,C,D, are in the proportion of $\mu\nu'\varrho'\sigma'$, $\nu\mu'\varrho'\sigma'$, $\varrho\mu'\nu'\sigma'$, and $\sigma\mu'\nu'\varrho'$. Either of these, divided by the sum of all, represents the probability of its case. Secondly, let it be that one of them only can be true, and all may be false. Put on the fifth quantity $\mu'\nu'\varrho'\sigma'$, for the case in which all are false. For example, there are four distinct assertions, not more than one of which can be true. The separate evidences for these four assertions give them the probabilities $\frac{2}{7}$, $\frac{3}{11}$, $\frac{1}{8}$ and $\frac{4}{5}$. There is a certain assertion which is true if either of the first three be true: required the probability of that assertion. Here, neglecting the common denominator, which is $7 \times 11 \times 8 \times 5$ in every case, the probabilities of the several assertions, and that of all being false, are as 2.8.7.1, 3.5.7.1, 1.5.8.1, 4.5.8.7, and 5.8.7.1, or as 112, 105, 40, 1120, and 280. The odds for one of the first three cases against one of the other two are $112+105+40$ to $1120+280$ or as 257 to 1400; or it is 1400 to 257 against the truth of the assertion.

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* This gives rise to two great tendencies, which very nearly divide the world among them. Some settle the case in their own minds, and then try the witnesses: some settle the witnesses and then try the case: not a few bring their second result back again to justify their first assumption. When there are two unknown quantities with only one equation, it is easy for those who will assume either to find the other. But the difficulty is to find the most probable value of both.

Suppose the condition were that two of the assertions, but not more, *may* be true, and that one *must* be true. Then the possible cases, meaning by an accent that the assertion is not true, are AB'C'D, BA'C'D', CA'B'D', DA'B'C', ABC'D', ACB'D', ADB'C', BCA'D', BDA'C', CDA'B'. Consequently, the probabilities of these cases are in the proportion of $\mu\nu'\varrho'\sigma'$, $\nu\mu'\varrho'\sigma'$, $\varrho\mu'\nu'\sigma'$, &c. And the odds in favour of, say A, being true, are as the sum of all the terms which contain μ, to the sum of those which contain μ'.

When we wish to signify that no evidence is offered either for **[201]** or against one of the assertions, we must put it down as having the testimony $\frac{1}{2}$. To put down 0 in the place of $\frac{1}{2}$ would be to make an infallible witness declare that it is not true. Suppose there are four assertions, one of which must be true and one only: evidence of goodness $\frac{4}{7}$ is offered for the first, and none either way for the others. Required the probability of the first. The probabilities of the four assertions are in the proportion of 4.1.1.1, 1.3.1.1, 1.3.1.1, and 1.3.1.1, and it is 4 to 9 for the first, or 9 to 4 against it.

Problem 3. Arguments being supposed logically good, and the probabilities of their proving their conclusions (that is, of all their premises being *true*) being called their validities, let there be a conclusion for which a number of arguments are presented, of validities a, b, c, &c. Required the probability that the conclusion is proved.

This problem differs from those which precede in a material point. Testimonies are all true together or all false together: but one of the arguments may be perfectly sound, though all the rest be preposterous. The question then is, what is the chance that one or more of the arguments proves its conclusion. That all shall fail, the probability is $a'b'c'$ ....that all shall not fail, the probability is $1-a'b'c'$.... Accordingly, if we suppose n equal arguments, each of validity a, the probability that the conclusion is proved is $1-(1-a)^n$. And, as in page 187, if the odds against each argument be k to 1, then, the number of such arguments being as much as $\frac{7}{10}\,k$, the conclusion is rendered as likely as not.

But are we really to believe, having arguments against the validity of each of which it is 10 to 1, that seven such arguments make the conclusion about as likely to be true as not. If such be the case, the theory, usually so accordant with common notions, is strangely at variance with them. This point will require some further consideration.

In this problem I consider only argument, and not testimony, which, nevertheless, cannot be finally excluded (see page 194). If the conclusion be one on which our minds are wholly unbiassed to begin with, it may seem that we have no escape from the preceding result. And to it we must oppose, for consideration at least, the common opinion of mankind that strong arguments are the presumption of truth, weak arguments of falsehood. **[202]** If a controversialist were to bring forward a hundred arguments, and if his opponent were so far to answer them as to make it ten to one against each, there can be no doubt that the latter would be considered as having fairly contradicted the former.

We must not forget that argument, in a great many cases, involves and produces the effect of testimony, and this in an easily explicable and perfectly justifiable manner. If I were to pick up a bit of paper in the streets, on which an argument is written, for a conclusion on which I have no previous opinion, and by an unknown writer, and if I could say that that argument left on my mind the impression of ten to one against its validity, I might be prepared to allow it to stand as giving $\frac{1}{11}$ of probability, and upon that supposition to combine it with my previous opinion, $\frac{1}{2}$, as in the next problem. But suppose it is on a question of physics, and Newton is the proposer of it, and that it is his only argument, and therefore, I conclude, his best. The case is now entirely altered: possibly the conclusion is one on which the following argument would have great probability: "If this conclusion were true, it could be proved; if it could be proved, Newton could have proved it; therefore if it were true, Newton could have proved it: but Newton cannot prove it; therefore it is not true." If the case be such that the two premises of this last argument have each 9 to 1 for it, or $\frac{9}{10}$; then, though the original argument give $\frac{1}{11}$ for the conclusion, the mere circumstance of Newton bringing this argument as his best is $\frac{81}{100}$ against it. If Newton at the same time declare his belief in the conclusion, we have on one side his argument and his authority, on the other side the argument arising from his being reduced to such an argument.

That such considerations have weight, we know: and that they ought to have weight, we may easily see. It is of course, dependent upon the particular conclusion what weight shall be attached to the assertion, "if this conclusion were true it could be proved." The courts of law constantly act upon this principle. They consider (very justly I think) that evidence, however good it may be, is much lowered by not being the best evidence that could be brought forward. If a man be alive, and capable of being produced with sufficient ease, they will not take any number of good witnesses to the fact of his having been very **[203]** recently alive. In enumerating the arguments, then, for or against a proposition, those must be included, if any, which arise out of the nature, mode of production, or producers, of any among them. And until this has been properly done, we are not in a condition to apply the methods of the present chapter.

Problem 4. A conclusion and its contradiction being produced, one or the other of which must be true, and arguments being produced on both sides, required the probability that the conclusion is proved, disproved (*i.e.* the contradiction proved), or left neither proved nor disproved.

Collect all the arguments for the conclusion, as in the last problem, and let a be the probability that one or more of them prove the conclusion. Similarly, let b be the probability that one or more of the opposite arguments prove the contradiction. Both these cases cannot be true, though both may be false. The probabilities of the different cases are thus derived. Either the conclusion is proved, and the contradiction not proved, or the conclusion not proved and the contradiction proved, or both are left unproved. The probabilities for these cases are as $a(1-b)$, $b(1-a)$ and $(1-a)(1-b)$, and the probability that the conclusion is *proved* is $a(1-b)$ divided by the sum of the three, and so on. The fraction $(1-a)(1-b)$ divided by this sum may be called the *inconclusiveness* of the combined arguments. The manner in which this inconclusiveness is to be distributed between the hypothesis of the truth and falsehood of the conclusion *must* depend upon testimony, in the complete sense of the word.

The predominance of one side or the other, as far as arguments only are concerned, depends on which is the greatest, $a(1-b)$ or $b(1-a)$, or simply on which is the greatest, a or b. If the arguments on both sides be very strong, or a and b both very near to unity, then, though $a(1-b)$ and $b(1-a)$ are both small, yet $(1-a)(1-b)$ is very small compared with either. The ratio of $a(1-b)$ to $b(1-a)$ on which the degree of predominance depends, may, consistently with this supposition, be anything whatever. But we cannot pretend that, when opposite sides are thus both nearly demonstrated, the mind can take cognizance of the predominance which depends upon the ratio of the small and imperceptible defects from absolute certainty. The necessary consequence is, that the arguments are evenly balanced, and are [204] as if they were equal: there is no sensible notion of predominance. This is the state to which most well conducted oppositions of argument bring a good many of their followers. They are fairly outwitted by both sides, and unable to answer either, and the conclusion to which they come is determined by their own previous impressions, and by the authorities to which they attach most weight; and these are, of course, those which favour their own previously adopted side of the question.

When no argument is produced on one side of the question, the case is very different from the case of the preceding problems, in which no testimony is produced. Here the question is, "Has the conclusion been proved or not proved;" and when no argument is produced, we are certain it has not been proved. Accordingly, if no argument were urged for the contradiction, we should have $1-b=1$, or $b=0$.

If, in the preceding problem, the two sides of the question be not contradictions, but subcontradictions, of which neither need be true, but both cannot be, the problem is solved in the same way, for the cases are just the same. But we may introduce a distinction which the former case would not admit. When one must be true, every argument against one is of equal force for the other; which is not the case when neither need be true. Let there, then, be arguments for the first conclusion and against it, and let a and p be the probabilities that one or more of the arguments for, prove it, or against, disprove it. Let b and q be the similar probabilities for the second conclusion. Then, there are these cases:—1. The arguments (or some of them) for the first are valid, against it invalid, and those *for* the second are invalid (it matters nothing whether those *against* the second be valid or invalid). 2. The arguments for the first are invalid, those for the second valid, and against it invalid. 3. The arguments against the first are valid, and those for it invalid. 4. The arguments against the second are valid, and those for it invalid. 5. All the arguments are invalid. Accordingly, the probabilities that the first is proved, that it is disproved, that the second is proved, that it is disproved, and that neither of the two is proved nor disproved, are in the proportion of $a(1-p)(1-b)$, $(1-a)p$, $b(1-q)(1-a)$, $(1-b)q$, and $(1-a)(1-b)(1-p)(1-q)$.

Problem 5. Given both testimony and argument to both sides **[205]** of a contradiction, one side of which must be true, required the probability of the truth of each side.

This is the most important of our cases, as representing all ordinary controversy. Collect all the testimonies, and let their united force for the first side be μ, and, from the nature of this case, $1-\mu$ for the other side. Let a and b be the probabilities that the first side and the second side are proved by one or more of the arguments in their favour. Now, observe that, for the *truth* of either side, it is not essential that the argument for it should be valid, but only that the argument against it should be invalid. Accordingly, the probabilities of the two sides are in the proportion of $\mu(1-b)$ and $(1-\mu)(1-a)$, and the probabilities of the two sides are represented by

$$\frac{\mu(1-b)\ )}{\mu(1-b)+(1-\mu)(1-a)} \qquad \frac{(1-\mu)(1-a)}{\mu(1-b)+(1-\mu)(1-a)}$$

First, let there be no testimony either way: we must then have $\mu=\frac{1}{2}=1-\mu$; consequently, these probabilities are as $1-b$ to $1-a$. Let no argument have been offered for the second side, or let $b=0$. Then we have 1 to $1-a$, for the odds, or $1\div(2-a)$ for the probability of the first side being true. It has been usual to say that if an argument be presented of which the probability is a, the truth of the conclusion has also the probability a. Probably the above was the case intended as to testimony, &c., and the probability should then have been

$$\frac{1}{2-a} \quad \text{or} \quad a+\frac{(1-a)^2}{2-a}$$

which is always greater than a. Or, as we might expect, the possibility of the conclusion being true, though the argument should be invalid, always adds something to the probability of its being true. Moreover, $1 \div (2-a)$ is always greater than $\frac{1}{2}$: or any argument, however weak, adds something to the force of the previous probability. The same thing is true in every case. Suppose a new argument to be produced for the first side, of the force k. The effect upon the formula is to change $1-a$ into $(1-a)(1-k)$, and the odds in favour of the conclusion are increased in the proportion of 1 to $1-k$. But this is to be under-**[206]**stood strictly in the sense described in page 202, namely, we are to suppose that the newly produced argument *is* single, that is, does not by the circumstances of its production cause itself to be accompanied by an argument for the second side, or against the first. If this last should happen, and the argument thus created for the second side have the force l, the odds are altered in the proportion of $1-l$ to $1-k$.

From the above it appears that opposite arguments of the force a and b are exactly equivalent to a testimony the odds for the truth of which are as $1-b$ to $1-a$. Thus, suppose we have for a conclusion witnesses whose testimonies are worth $\frac{2}{3}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{9}{10}$; arguments for of the several forces, $\frac{1}{5}$, $\frac{11}{12}$, $\frac{5}{8}$; and arguments against of the forces $\frac{2}{9}$, $\frac{2}{11}$, $\frac{6}{7}$. Writing numerators only, we put down

For, 2, 2, 4, 9; 7, 9, 1:
Against, 1, 1, 3, 1; 4, 1, 3.

Hence it is, 2. 2. 4. 9. 7. 9. 1 to 1. 1. 3. 1. 4. 1. 3, or 252 to 1 for the conclusion.

An argument, we should infer beforehand, is better than a testimony of the same force; for the failure of the argument is nothing against the conclusion, but the failure of the testimony is its overthrow. So says the formula also: the introduction of a testimony of the value k, not before received, alters the existing odds in the proportion of k to $1-k$: but the introduction of an argument of the same force alters them in the greater proportion of 1 to $1-k$. Thus, the introduction of the testimony of a person who is as often wrong as right ($\frac{1}{2}$) alters the odds in the proportion of 1 to 1, or does not alter them at all: but the introduction of an argument which is as likely as not to prove the conclusion, alters them in the proportion of 1 to $1-\frac{1}{2}$, or of 2 to 1.

Are we not in the habit, unconsciously, of recognizing some such distinction? Do we not give much more weight to argument than to testimony? I suspect the answer should be in the affirmative: that an argument of 3 to 1 does convince us much more than a testimony of 3 to 1. I suspect we show it, not in numerical appreciation, of course, but in listening to and allow-**[207]**ing weight to arguments, when we should refuse testimony of the same character.

It may be doubted, however, whether we have much scope for experiment on the lower degrees either of testimony or argument. Perhaps it is not often we meet a witness, whether as bearing testimony of veracity to a fact, or of judgment to a conclusion, whose evidence is as low as $\frac{1}{2}$; and the same perhaps of an argument.

I have spoken, in the previous part of this chapter, of the rejection of authority, that is, of testimony, authority being only high testimony. Let us now examine by the formula and see what it amounts to. Let a be the probability that the argument proves its conclusion: and let us therefore persist in saying that a is the probability for the truth of the conclusion. In the formula, b being $=0$, let μ be made $a \div (1+a)$, it will be found that the probability for the conclusion, μ divided by $\mu+(1-\mu)(1-a)$, comes out a, as required. Consequently, in the case of a single argument, the total rejection, as it would be thought, of all testimony, is really equivalent to accompanying every argument by a testimony less than $\frac{1}{2}$, depending upon its own force. It is to declare that, by the laws of thought, an argument of $\frac{7}{10}$ is of its own nature accompanied by a witness of $\frac{7}{17}$, one of $\frac{3}{4}$ by a witness of $\frac{3}{7}$, and so on; this is clearly not what was meant. Nor, I suppose, can it be meant that we are arbitrarily to start with the testimony $\frac{1}{2}$, and to reduce our own evidence, and that of all others, to the same. If there be any sense in which the rejection of authority is defensible, it must be when we are required to proceed as if we were in perfect ignorance what the value of the authority is. We cannot suppose it to be as likely to have one value as another. Suppose, for instance, that the arguments have unknown proposers: we cannot treat their authorities as if they were just as likely to be excessively high or low as to be very near to none at all. The more rational supposition is that the authority should be more likely to be small than great, as likely to be against as for, and very unlikely to be excessively great either for or against. I cannot here enter into the mode in which such an hypothesis can be expressed or used: but the result of the simplest formula which satisfies the above conditions, is as follows:—Let $r=(1-b)\div(1-a)$, b and a meaning as above; then the **[208]** probability that the conclusion is true, which has a for the validity of its argument, &c. is

$$r(r^3-6r^2+3r+6r\log r+2)\div(r-1)^4$$

where $\log r$ means the *Naperian* logarithm (99-43rds of the common logarithm will be near enough for the present purpose). If, for instance, $r=2$, which, on the supposition of no previous balance of testimony, would give 2 to 1 for the conclusion, the formula just written gives ·636, or 636 to 364, something less than 2 to 1.

In the case first discussed in page 202, it may be thought that the weakness of a proposed argument, from one who should have brought a better, if there had been one, may be considered as a *testimony* against the conclusion rather than an *argument*. Suppose his argument, for instance, to have only the probability $\frac{1}{10}$. He tells us then, that after he has done his best, it is 9 to 1 against the proposition being proved. If we are very confident that it could be proved, if true, and that he could do it, if any one, he comes before us as a testimony of 9 to 1 against the *truth* of the conclusion, or very nearly so. If we take, then, all that his argument wants of demonstration, as so much evidence from him against the conclusion, this amounts to supposing that, a being the validity of his argument, a is also his testimony for the conclusion (and $1-a$ that against it). If there be only argument for, and none against, and if our minds be previously unbiassed, we represent this case by putting a for μ in the formula, and the odds for the conclusion are then as a to $(1-a)^2$. On this supposition, which I incline to think well worthy of attention, we should not consider an unopposed argument from an acute reasoner as giving the conclusion to be as likely as not, unless $a=(1-a)^2$ or $a=\cdot 382$, a little more than $\frac{1}{3}$. Were it not for our peculiar introduction of testimony, then, the conclusion being as likely as not to begin with, an argument which has any probability of proving it, would have made it more likely than not, as before seen.

But that the introduced testimony should be exactly as above, is a mere supposition. If it were a mathematical proposition, for instance, and Euler were to declare himself unable to give more than a probability of proof, I, for one, should consider him **[209]** as giving a much higher rate of testimony against the truth of the assertion than is supposed in the preceding. But all this has reference to the question how to measure testimonies and validities in particular cases, which is quite a distinct thing from the investigation of the way to use them when measured.

In cases in which the number of arguments is multiplied, it generally happens that they stand or fall together, in parcels: namely, that the same failure which makes one invalid, necessarily makes others invalid. In this case, independent arguments must be selected, and the probabilities for them alone employed.

We see in this problem an illustration of the commonly observed result, that the same argument produces very different final conclusions in two different minds; and this when, so far as can be judged, both are disposed to give the same probabilities to the several premises of the argument. The initial odds, come how they may, or μ to $1-\mu$, should be altered by the arguments in the proportion of $1-b$ to $1-a$. Accordingly, b and a being the same to both parties, their belief in the conclusion may have any kind of difference, if μ be not the same thing to both.

Problem 6. Given an assertion, A, which has the probability a; what does that probability become, when it is made known that there is the probability m that B is a necessary consequence of A, B having the probability b? And what does the probability of B then become?

First, let A and B not be inconsistent. The cases are now as follows, with respect to A. Either A is true, and it is not true that both the connexion exists and B is false: or A is false. This is much too concise a statement for the beginner, except when it is supposed left to him to verify by collecting all the cases. The odds for the truth of A, either as above or by the collection, are $a\{1-m(1-b)\}$ to $1-a$. As to B, either B is true, or B is false and it is not true that A and the connexion are both true. Accordingly, the odds for B are as b to $(1-b)(1-ma)$.

The reader must remember that when B necessarily follows from A, B must be true when A is true, but may be true when A is false; while A must be false when B is false. And now we see that a proposition is not necessarily unlikely, because it is very likely to lead to an incredibility, or even to an absolute impossibility. Let $b=0$, or let B be impossible: then the odds for A **[210]** are as $a(1-m)$ to $1-a$. Say that it is 9 to 1 that the connection exists; then these odds are as a to $10(1-a)$. If a be greater than $\frac{10}{11}$, still A remains more likely than not, even when it is 9 to 1 that it leads to the absurdity B.

Secondly, let A and B be inconsistent, so that both cannot be true. Either then A is true, B false, and the connexion does not exist; or A is false. The odds for A are then as $a(1-b)\ (1-m)$ to $1-a$. With respect to B, either B is true and A is false, or B is false, and A and the connexion are not both true. The odds for B are then as $b(1-a)$ to $(1-b)\ (1-ma)$.

Among the early sophisms with which the Greeks tried the power of logic, as a formal mode of detecting fallacies, was the construction of what we may call *suicidal* propositions, assertions the truth of which would be their own falsehood. If a man should say " I lie," he speaks neither truth nor falsehood; for if he say true, he lies, and if he lie, he speaks truth. Such a speech cannot be interpreted. Again, the Cretan, Epimenides, said that all the Cretans were incredible liars; is he to be believed or not? If we believe him, we must, he being a Cretan, disbelieve him. Some stated it thus;—" If we believe him, then the Cretans are liars, and we should not believe him; then there is no evidence against the Cretans, or we may believe him, so that the evidence against the Cretans revives," &c. &c. &c. Refer such a proposition to the theory of probabilities, and the difficulty immediately disappears. Whatever the credit of Epimenides as a witness may be, that is, whatever, upon his word, the odds may be for his proposition, the same odds are there against him from the proposition itself. These equal conflicting testimonies balance one another (problem 1) and leave the effect of other testimonies to the same point unaltered. The sophism of Epimenides, as stated, is but an extreme case of the second of the problems before us. The proposition B is inconsistent with A, and the connexion is certain ($m=1$): the odds for B must then be as $b(1-a)$ to $(1-b)(1-a)$, or as b to $1-b$, exactly what they are independently of the previous assertion.

[211]

CHAPTER XI.

ON INDUCTION.

THE theory of what is now called *induction* must occupy a large space in every work which professes to treat of the matter of arguments; but there is not much to say upon the genuine meaning of the word, in any system of formal logic. And that little would be less, if it were not for the mistaken opposition which it has long been customary to consider as existing between the inductive process and the rest of our subject.

By induction (ἐπαγωγή) is meant the inference of a universal proposition by the separate inference of all the particulars of which it is composed: whether these particulars descend so low as single instances or not. Thus if X be a name which includes P,Q,R, so that every thing which is X must be one of the three: then if it be shown separately that every P is Y, and that every Q is Y, and that every R is Y; it follows that every X is Y. And this last is said to be proved by induction. Thus (Chapter VI)

$$X)P,Q,R+P)Y+Q)Y+R)Y=X)Y$$

is an inductive process. In form, it may be reduced as in page 123, to one ordinary syllogism.

Complete induction is demonstration, and strictly syllogistic in its character. In the preceding process we have y)p, y)q, y)r, which give y)pqr: and X)P,Q,R is pqr)x; whence y)x, or X)Y. It is a question of names, that is, it depends upon the existence or nonexistence of names, whether a complete induction shall preserve that form, or lose it in the appearance of a *Barbara* syllogism, formed by help of the conjunctive postulate of Chapter VI.

But when the number of species or instances contained under a name X is above enumeration, and it is therefore practically impossible to collect and examine all the cases, the final induction, that is, the statement of a universal from its particulars, becomes impossible, except as a *probable* statement: unless it should happen that we can detect some law connecting the species or instances, by which the result, when obtained as to a certain number, may be inferred as to the rest.

[212] This last named kind of *induction by connexion,* is common enough in mathematics, but can hardly occur in any other kind of knowledge. In an innumerable series of propositions, represented by P_1,P_2,P_3,P_4, &c., it may and does happen that means will exist of showing that when any consecutive number, suppose three, of them are true, the next must be true. When this happens, a formal induction may be made, as soon as the three first are established. For by the law of connexion, P_1,P_2, and P_3, establish P_4; but P_2,P_3, and P_4, establish P_5; and then P_3,P_4, and P_5, establish P_6; and so on *ad infinitum.* It is to be observed that this is really *induction*: there is no way, in this process, of compelling an opponent to admit the truth of P_{100} without forcing him, if he decline to admit it otherwise, through all the previous cases.

As an easy instance, observe the proof that the *square* of any number is equal to the sum of as many consecutive odd numbers, beginning with unity, as there are units in that number: as seen in

$$6 \times 6 = 1 + 3 + 5 + 7 + 9 + 11$$

Take any number, n; and write n ns (representing a unit by a dot) in rank and file. To enlarge this figure into $(n+1)$ $(n+1)$s, we must place n more dots at each of two adjacent sides, and one more at the corner. So that the square of n is turned into the square of $n+1$ by adding $2n+1$, which is the $(n+1)$th odd number. Thus 100×100 is turned into 101×101 by adding the 101st odd number, or 201. If then the theorem alleged be true of $n \times n$, it is therefore true of $(n+1) \times (n+1)$. But it is true of the first number, 1×1 being 1; therefore it is true of the second, or $2 \times 2 = 1 + 3$; therefore it is true of the third, or $3 \times 3 = 1 + 3 + 5$; and so on.

But when we can neither examine every case, nor frame a method of connecting one case with another, no absolutely demonstrative induction can exist. That which is usually called by the name is the declaration of a universal truth from the enumeration of some particulars, being the assumption that the unexamined particulars will agree with those which have been examined, in every point in which those which have been examined agree with one another. The result thus obtained is one of **[213]** probability; and though a moral certainty, or an unimpeachably high degree of probability, can easily be obtained, and actually is obtained, and though most of our conclusions with respect to the external world are really thus obtained, yet it is an error to put the result of such an induction in the same class with that of a demonstration. There is no objection whatever to any one saying that the former results are *to his mind* more certain than those of the latter: the fact may be that they are so. The difference between necessary and contingent propositions lies in the qualities from which they receive those adjectives, more than in difference of credibility. I know that a stone *will* fall to the ground, when let go: and I know that a square number *must* be equal to the sum of the odd numbers, as above: and though, when I stop to think, I do become sensible of more assurance for the second than for the first, yet it is only on reflection that I can distinguish the certainty from that which is so near to it.

The rule of probability of a *pure induction* is easily given. Supposing the simple question to be whether X *is* or *is not* Y, there being no previous circumstances whatsoever to make us think that any one X is more likely than not to be Y, or less likely than not. These are the circumstances of what I call a *pure* induction. To begin with, it is 1 to 1 that the first X examined shall be a Y: if this be done, and X_1 be a Y, then it is 2 to 1 that X_2 shall be a Y: should it so happen, then it is 3 to 1 that X_3 shall be a Y. Generally, when the first m Xs have all been examined, and all turn out to be Ys, it is $m+1$ to 1 that the $(m+1)$th X shall be a Y.

The simplicity of this rule must not lead the student to suppose he can find a simple reason for it. Let 10 Xs have been examined and found to be Ys: what do we assert when we say it is 11 to 1 that the 11th X shall be a Y? We assert that if an *infinite* number of urns were collected, each having white balls and black balls in *infinite number* but in a definite ratio, and so that every possible ratio of white balls to black ones occurs once; and if every possible way of drawing eleven balls, the first ten of which are white, were selected and put aside: then, of those put aside, there are eleven in which the eleventh ball is white, for one in which the eleventh ball is black. The reader will find some difficulty in forming a distinct conception of this, and of **[214]** course will find it impossible to have any axiomatic perception of the truth or falsehood of the result.

It may be worth while to show that a supposition making some degree of approach to the preceding circumstances will give some approach to the result. First, in lieu of an infinite number of balls in each box, which is supposed only that withdrawal of a definite number may not alter the ratio, let each ball drawn be put back again, which will answer the same purpose. Let there be only ten urns with ten balls in each, of which let the first have one white, the second two white, &c. and the last all white. The number of ways of drawing eleven white balls successively out of any one urn is the eleventh power of the number of white balls in the urn: that of drawing ten white balls followed by one black one is the tenth power of the number of white balls multiplied by the number of black ones. If we were to put together all the first, and then all the second, we should find about 21 times as many ways of arriving at the first result (ten white, followed by a white) as the second (ten white followed by a black). But if we now increased the number of urns, and took a hundred, having one, two, &c. white balls, we should find instead of 21, a number much nearer to 11; and so on.

Accordingly, when without any previously formed bias, we find that m Xs, successively examined, are each of them a Y, we ought then to believe it to be $m+1$ to 1 that the next, or $(m+1)$th X, will be a Y. And further, a being a fraction less than unity, we have a right to say there is the probability $1-a^{m+1}$ that the Xs make up the fraction a or more, of the Ys. Or thus;—if the fraction a be, say $\frac{6}{7}$, and if m be 10: then if the 10 first Xs be all Ys, the probability that $\frac{6}{7}$ or more of the Xs are Ys is just that of drawing one or more black balls in 11 drawings, from an urn in which $\frac{6}{7}$ of the balls are always white.

If, for example, the first 100 Xs were all Ys, it would be found to be 1000 to 1 that $93\frac{4}{10}$ per cent, at least, of all the Xs are Ys.

If as before, the first m Xs observed have all been Ys, and we ask what probability thence, and thence only, arises that the next n Xs examined shall all be Ys, the answer is that the odds in favour of it are $m+1$ to n, and against it n to $m+1$. No induction then, however extensive, can by itself, afford much probability **[215]** to a universal conclusion if the number of instances to be examined be very great compared with those which have been examined. If 100 instances have been examined, and 1000 remain, it is 1000 to 101 against all the thousand being as the hundred.

This result is at variance with all our notions; and yet it is demonstrably as rational as any other result of the theory. The truth is, that our notions are not wholly formed on what I have called the *pure induction*. In this it is supposed that we know no reason to judge, except the mere mode of occurrence of the induced instances. Accordingly, the probabilities shown by the above rules are merely *minima*, which may be augmented by other sources of knowledge. For instance, the strong belief, founded upon the most extensive previous induction, that phenomena are regulated by uniform laws, makes the first instance *of a new case,* by itself, furnish as strong a presumption as many instances would do, independently of such belief and reason for it.

With this however I have nothing farther to do, except to observe that, in the language of many, induction is used in a sense very different from its original and logical one. It is made to mean, not the collection of a universal from particulars, but the mode of arrival at a common cause for varied, but similar, phenomena. A great part of what is thus called induction consists in discovery of *differences,* not *resemblances.* Under this confused use of language, the usual theory is introduced, namely, that Aristotle was opposed to all induction, that Bacon was opposed to every thing else, that the whole world up to the time of Bacon followed Aristotle, that the former was the first who showed the way to oppose the latter, that each had a logic of his own, &c. &c. The whole of this account abounds with misstatements.[29] The admitted and sufficiently striking difference between the philosophy of modern and ancient times, in all natural and material branches of inquiry, is not so easily explained as by choosing two men, one to bear all the blame, the other all the credit: nor are Copernicus, Gilbert, Tycho Brahé, Galileo,[30] and the other predecessors of the *Novum Organum,* destined to be always deprived of their proper rank.

What is now called induction, meaning the discovery of laws from instances, and higher laws from lower ones, is beyond the province of formal logic. Its instruments are induction properly **[216]** so called, separation of apparently related, but really distinct particulars (the neglect of which was far more hurtful to the old philosophy than a neglect of induction proper would have been, even had it existed) mathematical deduction, ordinary logic, &c. &c. &c. It is the use of the whole box of tools: and it would be as absurd to attempt it here, as to append a chapter on carpentry to a description of the mode of cutting the teeth of a saw.

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[29] [Misstatements in Edition 1—query a misprint or usual contemporary spelling.—Ed.]

[30] [Galileo should hardly be reckoned as a "predecessor" of Bacon; the *Novum Organum* appeared in 1620; Galileo's *Dialogue* was not published until 1632 (six years after Bacon's death) and his *Discourses on Two New Sciences* not until 1638.—Ed.]

The processes of Aristotle and of Bacon are equally those which we are in the habit of performing every day of our lives. But some perform them well, and some ill. It is extraordinary that there should be such division of opinion on the question whether a careful analysis of them, and study of the parts into which they decompose, is of any use towards performing them well. On this point, and on the character of Bacon's office in philosophy, a living writer, to whom I should think it likely that many yet unborn would owe their first notions of Bacon's writings, expresses himself in a manner which I quote, and comment on at length, as the best exposition I can find, of a class of opinions which is very prevalent, and, I fully believe, to the prejudice of sober thought and accurate knowledge.

> The vulgar notion about Bacon we take to be this, that he invented a new method of arriving at truth, which method is called Induction, and that he detected some fallacy in the syllogistic reasoning which had been in vogue before his time. This notion is about as well founded as that of the people who, in the middle ages, imagined that Virgil was a great conjuror. Many who are far too well informed to talk such extravagant nonsense, entertain what we think incorrect notions as to what Bacon really effected in this matter.
>
> The inductive method has been practised ever since the beginning of the world, by every human being. It is constantly practised by the most ignorant clown, by the most thoughtless schoolboy, by the very child at the breast. That method leads the clown to the conclusion that if he sows barley, he shall not reap wheat. By that method a schoolboy learns that a cloudy day is the best for catching trout. The very infant, we imagine, is led by induction to expect milk from his mother or nurse, and none from his father.
>
> Not only is it not true that Bacon invented the inductive method; but it is not true that he was the first person who correctly analysed that method and explained its uses. Aristotle had long before pointed out the absurdity of supposing that syllogistic reasoning could ever conduct men to the discovery of any new principle, had

shown that such discoveries must be made by induction, and by [217] induction alone, and had given the history of the inductive process, concisely indeed, but with great perspicuity and precision.

Again, we are not inclined to ascribe much practical value to that analysis of the inductive method which Bacon has given in the second book of the *Novum Organum.* It is indeed an elaborate and correct analysis. But it is an analysis of that which we are all doing from morning to night, and which we continue to do even in our dreams. A plain man finds his stomach out of order. He never heard Lord Bacon's name. But he proceeds in the strictest conformity with the rules laid down in the second book of the *Novum Organum,* and satisfies himself that minced pies have done the mischief. "I eat minced pies on Monday and Wednesday, and I was kept awake by indigestion all night." This is the *comparentia ad intellectum instantiarum convenientium.* "I did not eat any on Tuesday and Friday, and I was quite well." This is the *comparentia instantiarum in proximo quæ natura data privantur.* "I ate very sparingly of them on Sunday, and was very slightly indisposed in the evening. But on Christmas-day I almost dined on them, and was so ill that I was in great danger." This is the *comparentia instantiarum secundum magis et minus.* "It cannot have been the brandy which I took with them; for I have drunk brandy daily for years without being the worse for it." This is the *rejectio naturarum.* Our invalid then proceeds to what is termed by Bacon the *Vindemiatio,* and pronounces that minced pies do not agree with him.

We repeat that we dispute neither the ingenuity nor the accuracy of the theory contained in the second book of the *Novum Organum*; but we think that Bacon greatly overrated its utility. We conceive that the inductive process, like many other processes, is not likely to be better performed merely because men know how they perform it. William Tell would not have been one whit more likely to cleave the apple if he had known that his arrow would describe a parabola under the influence of the attraction of the earth. Captain Barclay would not have been more likely to walk a thousand miles in a thousand hours, if he had known the place and name of every muscle in his legs. Monsieur Jourdain probably did not pronounce D and F more correctly after he had been apprised that D is pronounced by touching the teeth with the end of the tongue, and F by putting the upper teeth on the lower lip. We cannot perceive that the study of grammar makes the smallest difference in the speech of people who have always lived in good society. Not one Londoner in ten thousand can lay down the proper rules for the use of *will* and *shall.* Yet not one Londoner in a million ever misplaces his *will* and *shall.* Dr. Robertson could, undoubtedly, have written a luminous dissertation on the use of these words. Yet, even in his latest work he sometimes misplaced them ludicrously. No man uses figures of speech with more propriety because he knows that one figure of speech is called a metonymy, and another a synecdoche. A drayman in a passion calls out " You are a pretty fellow," without suspecting that he is uttering irony, and that irony is one of the four primary tropes. The old systems of rhetoric were never regarded by the most experienced and discerning judges as of any use for the purpose of forming an orator. " Ego [218] hanc vim intelligo " said Cicero " esse in præceptis omnibus, non ut ea secuti oratores eloquentiæ laudem sint adepti, sed quæ suâ sponte homines eloquentes facerent, ea quosdam observasse, atque id egisse ; sic esse non eloquentiam ex artificio, sed artificium ex eloquentia natum." We must own that we entertain the same opinion concerning the study of Logic, which Cicero entertained concerning the study of Rhetoric. A man of sense syllogizes in *celarent* and *cesare* all day long without suspecting it : and though he may not know what an *ignoratio elenchi* is, has no difficulty in exposing it whenever he falls in with it.—(" *Lord Bacon*" in *Critical and Historical Essays contributed to the Edinburgh Review*. By Thomas Babington Macaulay.)

This brilliant passage has, I have no doubt, appeared to many completely decisive of the question which it affirms : and, as so often happens in like cases, there is a certain exaggeration against which it is of truth. It is good against those who confound analysis and recombination of existing materials with introduction of them : and who might profess to see in agriculture something which would have benefited mankind, though plants and animals had not been natural products of the soil. But I now proceed to examine it, against those who affirm that Aristotle and Bacon are of *no* use, and who very frequently fall into the common logical fallacy of supposing that their case is proved, as soon as it is made out that they are not of *all the* use : which Mr. Macaulay himself has done, except as against the exaggerators aforesaid.

We reason inductively from morning till night, and even in our dreams. True : and how badly we often do it, particularly in sleep. A plain man is then produced, to reason on Bacon's principles : and Mr. Macaulay has imitated a plain man better than he intended, by making him do it wrongly. Look over the induction, and it will appear that the case is not made out ; an exclusion is wanting : it may have been the *mixture* of minced pies and brandy which did the mischief. The plain man should have tried minced pies without brandy; but he had drunk the latter daily for years, and it never struck him. This is precisely one of the points in which we are most apt to deceive ourselves, and for which we most need to have recourse to the completeness of a system of rules; something is left taken for granted. The things of course, our daily habits, are neglected in the consideration of anything of a less usual character: the plain man left off the minced pies upon trial; but not the brandy: Christ-**[219]**mas mischief must be referred, he thinks, entirely to Christmas fare, if at all.

But even if this omission had been supplied, and the result found to confirm the conclusion, yet the plain man has stopped where the plain man frequently does stop, at what Bacon calls the *Vindemiatio prima*, the rudiments of interpretation. Completeness is seldom anything but study and system. Philosophy ought to bring him to the result that daily brandy has made that spirit cease to give the stimulus which, were its use only occasional, would enable his stomach to bear an unusually rich diet for a short time. Our plain friend is precisely in the position of a bankrupt who curses the times, on reasoning strictly Baconian as far as it goes, and forgets that a casual tightness in the money market would never have upset him, if it had not been for the previous years of extravagant living and rash speculation.

But there are many processes which are not better performed because men know "how they perform them." Mr. Macaulay here means "because men know the laws of that part of the process which nature does for them." That men should not know better how to perform for knowing how *they* perform is almost a contradiction in terms. William Tell knew how to shoot all the better for knowing which end of the arrow *he* was accustomed to fit to the string: had he wanted this knowledge, his chance of cleaving the apple would have been much diminished. But he would not have been improved by knowing that his arrow described a parabola. True, because it did not do so. The *centre of gravity* of the arrow would describe a parabola, if it were not for the resistance of the air ; or something so near it as to be undistinguishable. But, taking the description as roughly correct, William Tell did know, inductively, that the arrow describes a curve, concave to the earth : and had made thousands of experiments in connexion of the *two ends* of that curve, which were all that he was concerned with. It is no argument against the study, as a study, of *induction*, that the amount of useful result which it had recorded in the mind of William Tell in the shape of habit, would not have been augmented by *deductive* knowledge of an intermediate *status* with which *he* had nothing to do. But let knowledge advance, under both modes of progress, and Tell becomes an artillery officer, the rude arrow **[220]** a truly shaped and balanced ball, means of measurement are applied, the true curve is more correctly represented than by the parabola, and thirty pounds of iron are thrown to four times the distance which an arrow ever reached, and with a certainty almost equal to that of the legend.

But if Captain Barclay had known the places and names of the muscles, he would not have been more likely to walk a thousand miles in a thousand hours. The instance is far fetched : because the feat consisted in the exhibition of power of endurance acquired by practice. If my denial seem as far fetched, it is the fault of the proposer. Captain Barclay must, by habit, by induction, have acquired facility in varying his pace and gesture so as to ease the muscles. Had he been well acquainted with the *disposition and uses* of these organs to begin with (towards which knowledge of their *places and names* would have contributed) he would have learnt this art more easily. Though not altogether *ad elenchum*, yet I may say that in this case the effect of such knowledge would have been that he would have been *less* likely to have performed the feat. Had he directed his attention to some science of observation, he would not have needed to have sought fame, or exhaustion of remarkable energy, in such a trifling pursuit. And further, in a very common case, mechanics has taught what few ever learn by induction, though they have constant opportunities of doing it: namely, that in walking, the ordinary practice of swinging the arms is injurious and tiring; that a very trifling amount of it tells seriously in a long journey. Here is one useful result, which natural induction does not commonly teach, and there may be many more of the same kind: the question between it and regular study requires the consideration, not only of what is done, and whether it might be done better, but of what is not done.

Next, M. Jourdain did not pronounce D and F more correctly after his attention had been called to the details of the act of pronunciation. None but Molière ever knew whether he did or not: but all who have watched the progress of instruction know that the bad habits or natural imperfections of children are removed or alleviated by making them practice mechanical pronunciation, with perceptive adoption of rules. In every one of a few detached instances in which I have seen children at their **[221]** reading lessons in France, I have noticed that a return upon the habits of pronunciation is always a part of the exercise: and that the letters are pronounced with that distinct effort which makes the pupil sensible of the action required. I have always attributed to this practice the more uniform standard of pronunciation which prevails among the educated French, as compared with ourselves.

But the study of grammar makes no difference in the speech of people who have always lived in good society. If Mr. Macaulay mean merely as to the use of *shall* and *will*, and the like, it may certainly be said that the perpetual use of speech (which is not reasoning) does enable every one to form the habits of those about him. But that grammar, as a whole, produces no effect upon the speech of good society, is one side of a balanced matter of opinion. Many contend that it has produced, in our generation and the one above it, a very unfortunate effect: they aver that the purity and character of our English has been deteriorated by Lindley Murray and his school, and that we much want better grammar teaching. On the subject of *shall* and *will* it is remarkable that Mr. Macaulay, whom a vigorous faculty of illustration, combined with immense reading, enables to strew his path with instances, has to invent his case, and to refer to a treatise which Robertson could have written. But it is not enough: if we grant that such a treatise would have been *luminous*, we may be safe; but would it have been *correct?* And further, knowledge must abdicate at once, if we pronounce useless all that has been clearly explained by those who have not rightly practised. Bacon himself might have taken *exsors ipsa secandi*[31] for his motto.

Next, it is said that no man uses figures of speech more correctly because he knows that one is *metonymy* and another *synecdoche*. True; and in like manner no man consults his books more easily because he has a bookcase. But, having the bookcase, he arranges his books in it, and then he knows where to find them. Mr. Macaulay dwells throughout upon nomenclature. I might insist upon its superstructure: but even mere naming is useful, when the meaning of the name is clearly understood. A mind well stocked with understood names cannot keep itself from being constantly in the act of classification, **[222]** which contains induction. The mere involuntary reference of instance number two to instance number one, which is made when we remember that the second must have the same name as the first, is comparison and induction, leads to reflection, cultivates taste, and gives power. The drayman, who calls out in a passion, "You are a pretty fellow!" without knowing that he is uttering irony, is an incomplete picture: there is omitted a wish relative to the eyes of his opponent, and an adjective which is (in such quarrels) sometimes prophetically, but seldom descriptively, true.

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[31] [Horace, *Ars Poetica*, 304, ergo fungar vice cotis, acutum reddere quae ferrum valet, exsors ipsa secandi.—Ed.]

The value of the difference between this savage irony and the more elegant form of it which is so pleasing in the description of the plain man's induction quoted above, is not within the comprehension of the drayman: the foundation of a better mode of expression than undisciplined rhetoric furnishes, so far as its adoption is matter of taste, was laid by those who placed irony among the primary tropes. Good taste is a result of comparisons, which could not have been made without nomenclature.

Did Cicero declare that systems of rhetoric are not of any use? The very quotation appears to mean that these systems, *præcepta*, have their power; that men get them by observation, and put them into practice. The *ea secuti oratores* refers to what was done in the first instance, by the first eloquent men, *suâ sponte*. Most truly does he say that the art of rhetoric is derived from eloquence, and not *vice versâ*: most falsely, as far as can be judged, does he seem to insinuate that it was all done at one step; first, some one or more consummate orators, secondly, a finished system, drawn from observation of their methods. Perhaps he intended a particular reference to a certain orator then nameless: the sentence, thus construed, contains nothing but matter which Tully is likely enough to have whispered to Cicero.

A system is a tool, and it must be employed upon materials which different men furnish from their different means. But the coat must be cut according to the cloth, both in size and quality: no reproach to the scissors, nor prejudice to their superiority over the sharpened wood of the savage, even though practice will enable him to use the latter better than any civilized man who is not a tailor can use the former. The formation of tools, mental or material, is a cyclical process. The first iron **[223]** was obtained by help of wood; one of the first uses of it was to make better tools, to get more iron, with which better tools still were made, and so on. And in this way we may trace back any art to natural tools, and to materials which are to be had for the gathering. The assertion made by Mr. Macaulay, and many others, that in logic only, of all the abstract sciences, our natural means are as good as those which result from diligent analysis, is one which terminates in an issue of fact. The instances given are contained in the assertion that a *man of sense* syllogizes in *cesare* and *celarent* all day long without *suspecting it*, and though he does not know what an *ignoratio elenchi* is, can always detect it when he meets with it.

Mr. Macaulay begins with an indefinite term, a man *of sense:* and the clause is deficient in logical perspicuity. First, what is a man *of sense?* I grant that I should doubt the sense of a man who could not make the inferences described by *cesare* and *celarent.* But do men become men of sense by nature, without education? if yes, I deny the assertion that men of sense reason (correctly) in *cesare*, &c. The man of sense who is not educated is as likely to assert that *cesaro* is all that can be obtained, or to invent the form *fesape*, as the plain man to forget to try the mince pies without brandy before he concludes. If no, then the assertion is itself *ignoratio elenchi:* for the very question is how to make men of sense; can they not be, *ceteris paribus*, formed better and faster with study of logic than without: it being agreed on all hands that *this* man of sense is always a practical logician.

Next, a man of sense reasons, &c. without suspecting it. Suspecting what? that he is reasoning, or that he is reasoning in *cesare?* I suppose the latter: that is to say, I take it to be meant that a man of sense *may* (not *must*, for some Aristotelians are men of sense) not know that the logicians call the form of reasoning he uses *cesare.* This is easily granted: but what is it but the celebrated *ignoratio elenchi* of Locke,[32] who fancied that he raised an objection against the pretensions of the logicians, when he declared he never could believe that God had made men only two-legged, and left it to Aristotle to make them rational. No one ever denied that men reasoned before Aristotle, and would have reasoned still if he had never lived.

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[32] [*Essay*, Bk. IV, c. 17, § 4, "But God has not been so sparing to men to make them barely two-legged creatures, and left it to Aristotle to make them rational."—Ed.]

Mr. Macaulay, probably without so much as a new application **[224]** to the inkstand, after falling into the *ignoratio elenchi*, singles out this very fallacy as the one which a man of sense is sure to detect. But if there be a fallacy which is the staple of paralogism, it is this one. *Delectat domi,*[33] for ordinary discussion (especially after dinner) is little else; *impedit foris*, for three fourths of public debate, from the Houses of Parliament downwards, is made up of it. A man who exposes it in conversation is considered a tiresome, and if he do it often, an uncourteous person: he "has no conversation," he "harps upon one subject," he "won't let you speak."

I have made the above comments upon a very marked passage of an eminent writer, in preference to introducing their substance as a dissertation of my own, that I might have the advantage of the reader seeing that I meet real arguments, instead of my own version or selection. It would probably be difficult to find a better concentration of the substance of the antagonist views, with respect to the formal study of reasoning, than is contained in my quotation from Mr. Macaulay: and I may safely take his adoption of them as proof that these views yet require the notice of a writer on logic.

* * *

There is one result of the theory of probabilities, closely connected with induction proper, which it will be advisable to notice here.

When the syllogism is declared illegitimate, on account of both premises being particular, a probable conclusion of great strength may be admitted in many cases. This must be the more insisted on, because it is too common to attend to nothing but the demonstrative syllogism, leaving all of which the conclusions are only probable, however probable, entirely out of view.

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[33] [Cicero, *pro Archiva*, VII, 16, haec studia delectant domi, non impediunt foris.—Ed.]

I take as the instance the syllogism, or imperfect syllogism, "Some Xs are Ys, some Zs are Ys, therefore there is some probability that some Xs are Zs." If the number of Xs and Zs together exceed the number of Ys (as in Chapter VIII) there is a certainty that some Xs are Zs. Let us then suppose this is not the case.

Let the whole number of Ys in existence be η, and let m and **[225]** n be the numbers of Xs and Zs which are among them. I shall consider two distinct cases:—First, when the distribution of the Xs and Zs among the Ys is utterly unknown; secondly, when their distribution is that of *contiguity*, that is, when the Ys being for some reason arranged in a particular order, the Xs which are Ys are successive Ys, and the same of the Zs which are Ys.

For the first case a very rough notion will do, confined to the supposition that few Xs and Zs are mentioned, compared with the whole number of Ys. When the Xs and Zs [34]

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[34] [The exact solution of the problem is as follows:

Given that there are 1000 things of which 100 are xs and 100 are zs; required the odds that at least one x is a z. The odds required are those in favour of the truth of at least one of the 100 propositions, "x_1 is a z," "x_2 is a z," . . . "x_{100} is a z," or, what comes to the same thing, the odds *against* the falsity of *all* the 100 propositions. If we call the probability that at least one x is a z, k, and the probability that all the 100 propositions are false h, we have $k=1-h$, and the odds we required to find are $\frac{k}{h}=\frac{1-h}{h}$, which we will call m.

Now, since there are 100 zs among the 1000 things, the chance that x_1 (any one of the 100 xs taken at random) is a $z=100/1000$, the chance that it is not a z (i.e. that "x_1 is a z" is false) are $900/1000$. Again, if "x_1 is a z" is false (and the probability that it is false is, as we have seen, $900/1000$), the probability that "x_2 is a z" is false will obviously be $899/999$ (as, on the hypothesis just 100 of the remaining 999 things are zs). Hence the probability that "x_1 is a z" and "x_2 is a z" are both false is $\frac{900}{1000}\times\frac{899}{999}$.

By reasoning repeatedly in the same way, we find that h (the probability that all the propositions "x_1 is a z" . . . "x_{100} is a z" are false)

$$=\frac{900\times 899\times\ ..\ \times 801}{1000\times 999\times\ ..\ \times 901}.$$

Hence, since $m=(1-h)/h$, we have

$$m+1=\frac{1000}{900}\cdot\frac{999}{899}\cdots\frac{901}{801}.$$

together make a large proportion of the Ys in number, then, if we have no reason for making them contiguous, or otherwise limiting the equally probable arrangements, it may be said to be a moral certainty that some Xs are Zs.

In the first case, if we divide 43 times the product of m and n by 100 times η, it gives us a sufficient notion (not large enough) of the common logarithm of k, the odds in favour of some Xs being Zs being k to 1. Say there are 1000 Ys, and that 100 Xs are Ys and 100 Zs are Ys. Then $43 \times 100 \times 100$ divided by 100×1000 is 4·3, which is the logarithm of 20,000. It is then more than 20,000 to 1 that, in this case, one or more Xs are Zs. A more exact rule is as follows. To $43mn$ divided by 100η add its hundredth part, and to the result add such a fraction of itself as $m+n$

[The numerical value of the right-hand side of this equation can be found by actual reference to a table of logarithms; but, as seven-figure logarithms must be used to ensure accuracy to five figures, it is better to use an approximate formula, given by De Morgan in his Essay on Probabilities in the *Encyclopædia of Mathematics* and due to Laplace.

If n is large, $n! = n^n . e^{-n} . \sqrt{2\pi n.}\ (1 + 1/12n + \ldots)$, and so the formula above becomes

$$m + 1 = (1000)^{1000\cdot5}(800)^{800\cdot5}/(900)^{1801}$$

Either of the methods gives $\log(m + 1) = 4\cdot83281$, $m = 68047$.

De Morgan's approximation to the general formula,

$$\frac{\eta}{\eta - n}\,\frac{\eta - 1}{\eta - n - 1}\ \ldots\ \frac{\eta - m - 1}{\eta - n - m - 1},$$

may be easily obtained in the following way:—

Using the ordinary expansion formula for the logarithm of $\log(1 + n)$, where n is small, namely,

$$\log_{10}(1 + n) = (n - n^2/2 + n^3/3 - \text{etc.})\ (0\cdot43429\ldots),$$

we have

$$\log_{10}\frac{\eta - r}{\eta - n - r} = \log_{10}\left(1 + \frac{n}{\eta - n - r}\right) = \log_{10}\left[1 + \frac{n}{\eta}.\left(1 - \frac{n+r}{\eta}\right)^{-1}\right]$$

i.e., $\log_{10}\dfrac{\eta - r}{\eta - n - r} = 0\cdot4343\left[\dfrac{n}{\eta}\left(1 + \dfrac{n + r}{\eta} + \ ..\right) - \dfrac{n^2}{2\eta^2}(1 + ..)\right]$

approximately, neglecting cubes of $1/\eta$ and higher powers; that is,

$$\log_{10}\frac{\eta - r}{\eta - n - r} = 0\cdot4343\frac{n}{\eta}\left(1 + \frac{n + 2r}{2\eta} + \ ..\right).$$

Summing, from $r = 0$ to $r = m - 1$, we have

$\log_{10}(m + 1) = 0\cdot4343\dfrac{mn}{\eta}\left(1 + \dfrac{m + n}{2\eta}\right)$ approximately, the error being in defect.—J.M.C.]]

is of 2η. Thus $43mn \div 100\eta$ being 4·3, which, with its hundredth part is 4·343, and $m+n$ (200) being the tenth part of 2η (or 2000), we add to 4·343 its tenth part, giving 4·777, which is about the logarithm of 60,000, still under the mark. It is more than 60,000 to 1 that some Xs are Zs. When the fractions are very small, this rule is accurate enough, if η be considerable. Its result is, that if η be very considerable, and if a perceptible fraction of the Ys be Xs, and a perceptible fraction Zs, *and if we really have no reason to make the limitation of contiguity or the like,* then we are justified in treating it as a moral certainty that some Xs are Zs. But I suspect the relation of contiguity, to which I now proceed, better represents the actual state of the case in ordinary argument.

When the Xs which are Ys are contiguous, and also the Zs which are Ys, the probability that no Xs are Zs is the fraction having the product of $\eta-m-n+1$ and $\eta-m-n+2$ for nu-**[226]**merator, and the product of $\eta-m+1$ and $\eta-n+1$ for denominator. Thus in the example above proposed, 1000 Ys containing among them 100 Xs and 100 Zs (each set contiguous) we have 801×802 for numerator and 901×901 for denominator. This fraction is about 8-tenths; so that it is now 8 to 2, or 4 to 1, *against* any Xs being Zs.

In order to find the probability against the number of Xs which are Zs exceeding k, add k to both the multipliers in the numerator, which then become $\eta-m-n+k+1$ and $\eta-m-n+k+2$. For example, there are 100 Ys, containing 30 Xs and 60 Zs (each set contiguously): what is the chance against the number of Xs which are Zs exceeding 10? The numerator is 21×22: the denominator is 71×41. This fraction is 462 by 2911; whence it is 462 to 2449 *against,* or 2449 to 462 (more than 5 to 1) *for,* the number of Xs which are Zs exceeding 10.

The chances, it is to be remembered, are all *minima:* except when we mean that m Xs, *and not more,* are Ys, &c. These questions may serve to give some notion of the manner in which arguments not logically conclusive, may be morally so.

What is called *circumstantial* evidence is a species of induction by probability. The thing required to be found has the marks P,Q,R,S, &c. : this Y has the marks P,Q,R,S, &c. : there is then a certain amount of circumstantial evidence that this Y is the thing we want to find. If it can be shown that there is but one thing which has all these marks, then the circumstantial evidence is demonstrative. But if there were, say 100 Ys, of which 5 have the mark P, 5 the mark Q, &c., then having ascertained one Y which has all the marks, the question is, what chance is there against another Y having them all : the same chance, at least, is there that the Y found is the one sought. Instead however, of attempting the problem in this way, which is never resorted to for want of data (I mean that the resemblance which the rough processes of our minds bear to those of the theory of probabilities does not here exist) I take it as follows. If the possession of the mark P give a certain probability to the Y found being that sought, it is as a witness whose testimony has a certain credibility. Similarly for Q,R,S, &c. Compound these testimonies, when known, by the rule in page 195, and the result is the value of the circumstantial evidence.

[227] CHAPTER XII.

ON OLD LOGICAL TERMS.

In this chapter I propose to say something on a few terms of the old Logic, which though they keep their places in works on the subject, and have some of them passed into common language, are very little used. They relate generally to the simple notion, and the name by which it is expressed: and have little of special reference, either to the proposition or syllogism. They are mostly derived from Aristotle, whose incidental expressions became or give rise to technical terms, and whose single sentences were amplified into chapters. And here, as in other places, I have nothing to do with the degree of correctness with which Aristotle's meaning was apprehended, nor even with how much was drawn from Aristotle and how much added to him, but only with the actual phrases and their usual meaning.

The words *logic* and *dialectics* * are now usually taken as meaning the same thing: the old distinction is that dialectics is the part of logic in which common and probable, but not necessary, principles, are used. But the distinction is neither clearly laid down, nor faithfully adhered to, even by Aristotle himself.

The *term* (in this work always called *name*) was divided into *simple* and *complex* :[35] the simple term was the mere name, the complex term was what all moderns call the *affirmative proposition.* Thus *man* and *run* were simple terms : *man runs,* a complex term. Later writers rejected this confusion : and divided the acts of the mind considered in logic into *apprehension, judgment,* and *discourse,* taking cognizance of notions, propositions, and arguments. The common meaning of the word *discourse,* **[228]** (which now generally applies to something spoken) is derived from its place in this division. The word *argument,* which is now equivalent to *reasoning against opposition expressed or implied,* was originally nothing but the middle term of a syllogism.

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* Our language is capricious with regard to the use of singular and plural of words in *ic*: thus we have logic and dialectic*s*, arithmetic and mathematic*s*, physic and physic*s* for medicine and natural philosophy. Some modern writers are beginning to adhere uniformly to the singular, in which I cannot follow them, for I am afraid an English ear would not bear with *mathematic* as a substantive. Would it not better consist with the genius of our language if the plurals were to be always used, and the singulars made adjectives without the termination *al*?

The simple term was *universal* or *singular :* universal, when of more instances than one, as man, horse, star ; singular, when of one instance only, as the sun, the first man, the pole-star, this book. Singular names were called *individuals,* from the etymology of the word, as belonging to objects not divisible into instances to each of which the name could be applied. I have not dwelt upon the distinction between singular and universal, because it is ineffective in inference. And moreover, a singular proposition is only objectively singular, but ideally plural. " Julius Cæsar was a Roman " : in point of fact, there was but one Cæsar. But take any imaginary repetition of the circumstances of Cæsar's life ; such, for instance, as occurs to those who have thought of the possibility of the same course of events returning into existence after a certain cycle : and then the term Cæsar becomes plural. Or, even without so forced a supposition, we may say that, if we describe Cæsar, we must describe a Roman : that our definition of Cæsar is so close as to fit only one man that ever lived, makes no essential difference in the character of the proposition.

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[35] [See Mansel's *Aldrich,* p. 8, note g. The original expressions were *vox* (not *terminus*) *simplex, vox complex.* As Mansel observes, *vox* is the Latin equivalent of Aristotle's *φωνή,* articulate utterance, and Aristotle himself distinguishes the name and the λόγος, or discourse, as simple and complex *φωναί* respectively.—Ed.]

But a further distinction which was made divided singular terms into subjects of universal, and subjects of particular, propositions. A determinate (or definite) individual, as Cæsar, this man, was the former: a vague (or indefinite) individual, as a certain man, the first comer, was the latter. The distinction is that of "some man" and "this one man."

Certain notions of essence or relation, accompanying the apprehension of a name, were called *categories*, or *predicaments*,[36] meaning "modes of assertion with respect to" the object named. Aristotle gave ten categories, and might have given ten hundred. In their usual Latin form they were *substantia, quantitas, qualitas, relatio, actio, passio, ubi, quando, situs, habitus.*

The word translated by substance,[37] *οὐσία*, means mode of being: and its literal Latin is *essentia*, essence. It is called *substance* (that which stands under) as supporting *accidents*, presently explained. It is far too metaphysical a term to come into common life with-**[229]**out some degradation: and accordingly it there means that of which a thing is composed, whether material or not. Accordingly we have the material substance of a coat, the intellectual substance of an argument. But, as we use the word, its meaning belongs to the other predicaments. In fact, the substance of the old logicians stands, as to existence, in the same situation as *matter* (page 30) with respect to our sensible perceptions, or *object* with respect to our ideas. The substance, it was said, is *per se subsistens*, while the accident could not be said *esse*, but *inesse*. The distinction between the substance (mode of being) and the material substance (in the modern sense) may be helped by the distinction between *substantia prima* and *substantia secunda,* the first referring to the individual, the second to the general term. Thus the substance of John, as John, was *substantia prima ;* as man, *substantia secunda.* All these very metaphysical notions were the student's first introduction to logic, and were considered as of the utmost importance.

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[36] [*Predicamenta* is the Latin equivalent of *κατηγορίαι*. The full name was *σχήματα τῆς κατηγορίας*, "figures of predication," and the list was in effect a classification of the different senses of the copula "is" in affirmative propositions.—Ed.]

[37] [Linguistically the Latin equivalent of *οὐσία* is *essentia*; *substantia*, though often used to render Aristotle's *οὐσία*, is properly the equivalent of the Neo-Platonic (and non-Aristotelian) technical term *ὑπόστασις*.—Ed.]

The predicament of *quantity,* derived from the notion of whole and part, was conceived as either *continuous* or *discrete.* In continuous quantity, the unit was divisible, in discrete, indivisible. Thus ten feet is continuous, ten men discrete. The distinction is precisely that of magnitudinal and numerical.

Quality was subdivided into 1. *Habit* and *disposition,* the latter term being used for the imperfect state of the former 2. *Power* and want of it 3. *Patibilis qualitas* and *passio,* applied to the ideas of that which is undergone, the first permanently, the second for a time. 4. *Form* and *figure.*

Relation then, as now, referred to the suggestions derived from comparison of two things or ideas. It was divided into verbal and real (*secundum dici* and *secundum esse*). Thus the relation of *profit* to *profitable* was verbal : that of *father* to *son,* or of *above* to *below,* real. The two things related, or *correlatives,* were called *subject* and *term :* so that of two correlatives, giving two opposite relations, the subject of either was the term of the other. The *fundamentum* of the relation was that in which it took its rise, when it had a beginning.

Action and *passion,* the production and reception of an effect, requiring the producing *agent,* and the receiving *patient,* were divided into *immanent,* or enduring in the agent, and *transient,* **[230]** or passing out to another. Actions were *univocal,* or *æquivocal,* according as their effects were of the same or different species. A few years before the publication of Newton's Principia, it was taught in a work imported into Cambridge that when mice bred mice, the action was univocal, but when *the sun* bred mice (the writer must have been thinking of Aristotle and some of the schoolmen) æquivocal. There was also the *terminus à quo* and the *terminus ad quem* to represent the state before and the state after the action. Thus, when all this nonsense was sent to Coventry, the *terminus à quo* was an immense quantity of univocally bred learning of the preceding kind; the *terminus ad quem* was the rooting up of the wheat of logic with the tares.

The *where* (as to absolute position), the *when*, and the *site* (relative position) gave no peculiar terms of subdivision. The habitus (ἔχειν) referring to *possession* generally in the first instance, was materialized by some of the old logicians till it related to *dress* only, or *habit* in the thence acquired meaning.

The word predicament (and category as well) has been introduced into common language to signify a set of circumstances under which any thing takes place. It is then no longer confined to the above predicaments, nor is there any occasion that it should be.

The *predicables* (κατηγοϱούμενα) are distinguished from *predicaments* (κατηγοϱίαι) in that the latter belong to any simple notion or name, and may be predicated of it: the former belong to the connexion (when affirmative) between two names. They are said to be five in number, *genus*, *species*, *differentia*, *proprium*, and *accidens*.

The words *genus* and *species* have preserved their old meaning. If there be a number of names of which each is subidentical of the one which follows, say V, W, X, Y, Z: then of any two, say W and X, X is a *genus* containing the *species* W. Here Z is the *summum genus*, and V the *infima species*: X is the *genus proximum* of W, Y the *genus remotum*. In what I have called a *universe*, which is a *summum genus*, having for its *infima species* the individual instance of any name in it, the superidentical is the genus, the subidentical the species. Subcontraries (and contraries) are *opposite species*; supercontraries and complex particulars have no ancient name.

[231] The *differentia* is that by which one class (be it

species or genus, the difference being accordingly termed *specific* or *generic*) is distinguished from another. Thus the difference (or one difference) separating the species *man* from the other species of the genus *animal*, is the epithet *rational*.

The *proprium* (or property) is that which belongs to the species *only*, whether it be to all or only to some: thus to study, and to speak, are equally *propria* of man. But the old commentators give definitions of the property as follows. There are four kinds. 1. That which belongs to the species alone, but not to all. 2. To all the species, but not to that alone. 3. To the species only, and to all of it, but not at all times. 4. To the species alone, to all, and always.

The *accidens* (or accident) is that which may sometimes belong to the individual of a species, but not necessarily, nor to that species alone. In modern language, the term is limited to what is unusual and unexpected.

The word *cause* was used by the ancients in a wider sense than by us: more nearly in the sense of the Latin *causa*, or the Italian *cosa*. Causes were distinguished into *material*, *formal*, *efficient*, and *final*. The *material* cause was the very matter of a thing, considered as a kind of giver of existence; the *formal* cause was its form, in the same light; the *efficient* cause (our common English word) the agent or precedent; and the *final* cause, the ultimate end or object, considered as a reason for the existence of the thing. Sometimes writers still talk of final causes, and are as unintelligible to most readers as if they had talked of final beginnings.

The word form was used in a wider sense than that of figure or shape, to mean, as it were, law of existence, mode, disposition, arrangement. Mere figure or shape was only one of the *accidental forms*, as distinguished from *substantial forms*, belonging to the substance. And *motion* was as widely used as *form*: it meant any alteration. Thus, *corruption* was one of the *motions* of matter. Change from place to place, to which the modern word is confined, was *local* motion.

The original use of the terms *subject* and *object* is to denote a thing considered as that which may have something inherent in it, or attached to it, or spoken of it, &c.; and as that which may **[232]** be *objected* to the mind or reason, or made to come in its way. Thus it was said that matter is the *subject* of those properties which are the *objects* of the mind in natural philosophy. The transition to the modern sense of *object*, namely, end proposed, is natural enough. In modern times, subject and object are used * with respect to *knowledge:* the subject being the mind in which it is, the object being the external source from which it comes. For *subjective* and *objective* I have in this work used *ideal* and *objective* (page 29). *Adjunct* was the technical term for that which is in the subject.

A *modal* proposition was one in which the affirmation or negation was expressed as more or less probable: including all that is technically under probability (Chapter IX) from necessity to impossibility. The theory of probabilities I take to be the unknown God which the schoolmen ignorantly worshipped when they so dealt with this species of enunciation, that it was said to be beyond human determination whether they most tortured the modals, or the modals them. Their gradations were *necessary, contingent, possible, impossible;* contingent meaning more likely than not, possible less likely than not. These they connected with the four modes of enunciation, A, I, O, E, and when by *some* is meant *more than half*, the connexion is good. The controversy about *modal* forms continues up to this day among logicians who are not mathematicians: I should suppose that the latter would never give it a thought, except as a branch of the theory of probabilities, and except as to the consideration how the terms by which the non-mathematical logician indicates his degrees of belief are to be placed upon the numerical scale. In like manner he reads the thermometer by graduation, and though he admits the freezing and boiling point, which have an origin in nature, he leaves temperate, summer heat, blood heat, &c. to the fancy of those who choose to employ them.

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* See a full account of these words in Sir William Hamilton's notes to Reid, p. 806, &c.

At the same time it is clear that these modal forms were considered not merely as useful in expression of the nature and amount of belief, but as suggestive of real branches of inquiry, subservient to that great *à priori* inquiry into the nature of things to which [233] mediæval logic was applied. We are not fit to judge of the instrumental part of this philosophy, unless we consider also the materials on which it was founded. In an age in which much more faith was demanded of the student than now; when he was much more frequently required to decide in one way or the other upon a single testimony; when, in addition to the non-mythic wonders recorded in ancient writers, which there was no mode of contradicting, all that was known of immense regions and countries rested upon very few accounts, and those filled with stories quite as strange:—the absence of other means of distinguishing truth from falsehood obliged those who thought to lay much stress upon *à priori* considerations. It matters little to us whether we infer the *necessity* of man being a walking animal from the non-arrival of exceptions, and thence the *universality* of the rule, or the universality from the supposed perfect induction of instances, and thence the necessity. But it was of much more consequence to the old logician: of more *real* consequence. He did not know but that any day of the week might bring from Cathay or Tartary an account of men who ran on four wheels of flesh and blood, or grew planted in the ground like Polydorus in the Æneid, as well evidenced as a great many nearly as marvellous stories. As he could not pretend to inductive and demonstrative universality, even upon the question of the form of his own race, he was obliged to combine with his argument the antecedent testimony of his own and other minds, in the manner which the real doctrine of modals (page 205) shows to be necessary in all non-demonstrated conclusions. It is true that he frequently confounded the predisposition of minds with the constitution of objects ; the testimony with the thing testified about.

We shall never have true knowledge of the schools of the middle ages, until those who have studied both their philosophy, their physics, and their state of tradition, will look at their weapons of controversy as both offensive and defensive, and give a fair account of the amount of protection afforded by the first, in the existing state of the second and third. It would also be advisable to consider whether, looking at the power of communication by land and sea, and all the circumstances of literary intercourse, it would have been practicable to place the knowledge of the earth and its details upon any better footing of evidence.

[234] One leading feature of the schoolmen, acute as they were, and as to representation of notions, inventive, and which is shared by many more modern writers who have not disciplined themselves mathematically, is seen in their employment of quantity : there are instances of the strange use, the wrong use, and the no-use. Most of them arise from indistinct apprehension of continuity, which obliges them to accept such stages of quantity as are expressed by existing terms, without any effort to fill up gaps. There is also a slovenliness of definition in what relates to quantity. Thus dozens of instances might be given in which the *some* of the particular proposition is so defined that we might suppose it is " some, not all," instead of " some, it may be all," and the former is the express definition of some writers : and it is only when we find in rules that XY does not allow us to infer X:Y, nor to contradict X)Y, that we ascertain the real intended meaning. " Logicians," says Sir William Hamilton, " have referred the quantifying predesignations *plurimi*, and the like, to the most opposite heads ; some making them universal, some particular, and some between both." They must have had curious ideas of quantity who made the proposition " most Xs are Ys " either universal,

or between the universal and particular: I should suppose that those who did the latter must have imagined *some* to refer to a *minority*.

There is a strange notion of quantity revived in modern times, which consists in making *plurality of attributes* a part of the quantity of a notion. It is called its *intensive quantity*, or its *intension*, or *comprehension*. It is opposed to *extensive quantity*, or *extension*, which is the more common notion of quantity, referring to the number of species or of individuals (it may be either, the individual is the real *infima species*) contained under the name. Thus *man* is not so extensive as *animal*, but more intensive; the attribute *rational* gives greater comprehension. But "man residing in Europe" is less extensive and more comprehensive than either. It is said that the greater the intensive quantity the less the extensive, but this is not true, unless no two of the signs of intension be properties of the same species. Thus, according to such statements as I have seen, "man, residing in Europe, drawing breath north of the equator, seeing the sun rise before those in America," would be a more intensively quantified notion than "man residing in Europe"; **[235]** but certainly not more *extensive*, for the third and fourth elements of the notion must belong to those men to whom the first and second belong. Thus, in the Port-Royal Logic,[38] one of the earliest modern works (according to Sir W. Hamilton), in which the distinction is drawn, it is said that the *comprehension* of the idea of a triangle includes space, figure, three sides, three angles, and the equality of the angles to two right angles. But the idea of *rectilinear three-sided figure* has just as much extension.

The relation between comprehension and extension exists, and is useful: but not, I think, as that of different kinds of *quantity*. In page 148, where I hold that the proposition

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[38] [See the *Port-Royal Logic*, translated from the French by Thomas Spencer Baynes (2nd edition), Edinburgh, 1851, p. 49: "The comprehension of the idea triangle includes extension, figure, three lines, three angles, and the equality of these three angles to two right angles, etc."—Ed.]

is contained in its necessary consequence, the view is one of extension: the ordinary view is one of comprehension. "Every case in which P is true, is a case in which Q is true," tells us that all the P-cases are contained, as to extent (number and location of instances), among the Q-cases. But, as to comprehension, every P-case contains all that distinguishes a Q-case from other things. When, in page 47, it is said that the idea of man is contained in that of animal, I speak of extension: all the instances to which the first idea applies are among those to which the second applies. But, as to comprehension, the idea of animal is contained in that of man: all that defines animal goes to the definition of man, and other things besides. In page 50, the "*is* of possession of all essential characteristics," refers to comprehension; the "*is* of identity" to extension: both possessing equally the characters under which the verb may occur in logic. There is no distinction which affects inference: for X)Y has exactly the same properties whether we interpret it as expressing that Y has all the extension of X, and may be more; or that X has all that Y has in comprehension, and may be more.

In pages 115, &c. we have the mode of representing names of more or less comprehension. Thus, P, Q, R, &c. being characteristics, the obvious proposition PQ)P, illustrates the theorem that where the comprehension of one name has all that of a second (as PQ has that of P) the extent of the second is at least as great as that of the first. And the self-evident postulate in page 115, by which we may diminish the extent of a term universally used, or increase that of one particularly used, may be expressed in language of comprehension. That is, we may augment the com-**[236]** prehension of a universal, or diminish that of a particular. Thus, X)Y gives XP)Y, and X.Y gives XP.Y: but X)YP gives X)Y.

It will be easily seen that comprehension has the first attribute of quantity (page 174): there is *more* and *less* about it. But it is not of the *measurable* kind (page 175).

As to extent, 200 instances bear a definite ratio to 100, which we can use, because our instances are *homogeneous.* But different qualities or descriptions can never be numerically summed as attributes, to any purpose arising out of their number. Does the idea of *rational animal,* two descriptive terms, suggest any useful idea of *duplication,* when compared with that of *animal* alone. When we say that a chair and a table are more furniture than a chair, which is true, we never can cumulate them to any purpose, except by abstracting some homogeneous idea, as of bulk, price, weight, &c. To give equal quantitative weight to attributes, as attributes, seems to me absurd : to use them numerically otherwise, is at present impossible.

The reader will have seen the origin of several very common terms, which are used in a sense coinciding with, or at least much resembling, that put upon them by the schoolmen. But there is one which has diametrically changed its meaning ; it is the word *instance.* The word *instantia* (and also *ἔνστασίς*) implied a case *against*, not *for* ; the latter was *exemplum :* so that *instance to the contrary* would have been tautology.

I have referred the word *enthymeme* to this chapter, though it is always regularly explained in connexion with the syllogism. According to Aristotle, *᾿Ενθύμημά ἐστι συλλογισμὸς ἀτελὴς ἐξ εἰκότων καὶ σημείων*, an enthymeme is an imperfect syllogism from probables and signs : the modern critics reject the word *ἀτελὴς*, *imperfect,* as interpolated. The word *sign* seems to mean indication, symptom, or effect, which makes the cause almost necessary or highly probable. But the schools took the word *enthymeme* to mean a syllogism with a suppressed and implied premise, such as " He must be mortal, being a man." I cannot help suspecting that Aristotle * **[237]** made no difference between a suppressed premise, clearly intended and distinctly received, and one formally given. It seems to me that we might as well distinguish a written from a spoken syllogism, as to the logical character of the two.

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* He says all that is communicated (λέγετ) of the predicate, will be asserted in words (ῥηθήσεται) of the subject. These two different tenses of two different verbs are often both translated by *dicitur*. Why did they occur ? For various reasons, I allow myself to suspect, though not scholar enough to maintain, that λόγος generally meant communication, passage

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from one mind to another by any means, as much at least with reference to the receiving, as to the imparting, mind : and that it is here opposed to ῥῆσις, speech, in that sense. Throw the verbs back to their primary meanings, and it will be " That which *is picked up* of the predicate, shall *flow out* about the subject." If my conjecture be correct, the modern enthymeme is here put on the same footing as the fully expressed syllogism.

[The reference is to the text of Aristotle, *Analytica Priora*, II, 27, 2—the definition of Enthymeme. In Aristotle's language an "imperfect" syllogism means a syllogism in the second or third figure. But see the note to the Table of Contents where De Morgan withdraws this false etymology of ῥῆσις.—Ed.]

CHAPTER XIII.

ON FALLACIES.

THERE *is* no such thing as a classification of the ways in which men may arrive at an error: it is much to be doubted whether there ever *can be.* As to mere inference, the main object of this work, it is reducible to rules: these rules being all obeyed, an inference, as an inference, is good; consequently a bad inference is a breach of one or more of these rules. Except, then, by the production of examples to exercise a beginner in the detection of breaches of rule, there is nothing to do in a chapter on fallacies, so far as those of inference are concerned. Nevertheless, there are many points connected with the matter of premises, to which it is very desirable to draw a reader's attention: and above all to questions in which it is not at first obvious whether the mistake be in the matter or in the form; or in which it may be the one or the other, according to the sense put upon the words.

If there be anything *ridentem dicere verum quod vetat,* writers on logic have in all ages most grievously neglected the prohibition in treating this subject, and have given the student a prescriptive right to some amusement. One reason of this was, that the [238] Greeks endeavoured to try the new art by inventing inferences the falsehood of which could not be detected by its rules. These, as may be supposed, were whimsical efforts of reasoning: nevertheless, they have been handed down from book to book, unsurpassed in their way. Another reason is, that jests, puns, &c. are for the most part only fallacies so obvious that they excite laughter; and the greater number of them can be shown to break one or another of the rules of logic. Accordingly, they furnish striking examples of these rules; the application of which, in serious terms, has itself a taste of the ludicrous. Boccacio has, by his inimitable mode of narration, made a good story the jest of which could be described as consisting in nothing more than the assumption that what can be predicated of storks * in general can be predicated of roasted storks: which is what logicians would call the *fallacia accidentis*, or arguing *a dicto simpliciter, ad dictum secundum quid*.

The terms *fallacy*, *sophism*, *paradox*, and *paralogism*, are applied to offences against logic; but not with equal propriety. *Fallacy* and *sophism* may technically have been first applied to arguments in which there is a failure of logic: but it is now very common to apply them also to arguments in which there is a falsehood of fact, or error of principle, though logically treated; and if this last use be not correct, writers on logic have sanctioned it in their examples. Many persons go further, and call the erroneous statement itself a fallacy: that men are in the habit of walking on their heads, they would say is a very obvious fallacy. A *paradox* is properly something which is contrary to general opinion: but it is frequently used to signify something self-contradictory: thus the newspaper which recently avowed **[239]** its opinion that the repeal of the corn laws would make food both cheap and dear is said to have maintained a paradox. The modern use of the word implies disrespect, but it was not so formerly. Thus in the sixteenth century the opinion of the earth's motion was styled the *paradox of Copernicus* by writers who meant neither praise nor blame, but only reference to the opinion of Copernicus as *an unusual one.* The more precise writers of our day use the word paradox for an opinion so very singular and improbable, that the holder of it is chargeable with an undue bias in favor of singularity or improbability for its own sake. *Paralogism,* by its etymology, is best fitted to signify an offence against the formal rules of inference. It has been frequently abused by mathematical writers, who have signified by it errors of statement, and undue assumptions: but it is not completely spoiled for the purpose, and I shall therefore use it to denote a formal error in inference, as a particular class of fallacy or sophism, words which it would now be difficult to distinguish in meaning. Some have defined *paralogism* to be that by which a man deceives himself, and *sophism* that by which he tries to deceive others: on what grounds I do not know.

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* A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon table, the master desired to know what was become of the other leg. The man answered that storks had never more than one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him next day into the fields where they saw storks, standing each on one leg, as storks do. The servant turned triumphantly to his master: on which the latter shouted, and the birds put down their other legs and flew away. "Ah, Sir," said the servant, "you did not shout to the stork at dinner yesterday: if you had done so, he would have shown his other leg too."

The question of a premise being right or wrong in fact or principle, unless indeed it contradict itself, does not belong to logic: nor could it so belong unless logic were made, in the widest sense, that attempt at the attainment of the *cognitio veri* which some have defined it to be. All that relates to the collection of true premises with respect to the vegetable world belongs to botany; with respect to the heavenly bodies, to astronomy; with respect to the relation of man to his Creator, to theology. Even were it within the province of logic, it would be impossible, in less space than an encyclopædia, to enter upon questions connected with the matter of syllogisms. With regard to paralogisms, or *logical* fallacies, (so called, as an error about the measure of space is called a *geometrical* error) the classification under breach of rules would be good in form, but would afford no basis for the treatment of the subject. Those who bring them forward seldom proceed in direct defiance of rule, but in various modes of evasion. These it would be almost impossible to arrange in satisfactory order.

Aristotle made a classification of fallacies, which was of course [240] adhered to by the writers of the middle ages. In this, as in every other place, when I speak of Aristotle and his system, I speak of it as understood by those writers. How far they distinctly comprehended their master is a question into which I could not enter here, even if I were competent to write on the subject. It is, however, sufficiently apparent that the logic of Aristotle is not of the purely formal character which marked the dialectics of the middle ages: there is a much more decided introduction of the attempt to write on the matter of syllogism than many persons think there is. The classification of fallacies seems to be one proof of this: and the interpretation of that classification by the middle writers seems to add their testimony to the assertion: in this part of the subject they abandon technicalities almost entirely.

It ought to be especially remembered that we are very differently situated from those writers, not as to what is fallacy, but as to what the specimens of it produced are likely to be. Out of a world of general principles declared by authority, or declared to be self-evident by authority, they had to produce logical deductions; and, of course, the pure syllogism and its rules were to them as familiar as the alphabet. The idea of an absolute and glaring offence against the structure of the syllogism being supported one moment after it was challenged, would no more suggest itself to the mind of a writer on logic than it would now occur to a writer on astronomy that the accidental error (which might happen to any one) of affixing four ciphers instead of five in multiplying by a hundred thousand would be maintained after exposure. Accordingly, their formal chapters on fallacies would naturally relate, if not entirely to fallacies of matter, at least to those in which the fallacy of matter very closely hinges upon that of form. And so it is in all the old systems which I have examined. The Aristotelian division (or rather selection, for it is far from including everything) lends itself easily to this adaptation.

We, on the contrary, live in an age in which formal logic has long been nearly banished from education: entirely, we may say, from the education of the habits. The students of all our universities (Cambridge excepted) may have heard lectures and learnt the forms of syllogism to this day: but the practice has been small: and out of the universities (and too often in them) the very name of logic is a bye-word. **[241]** The philosophers who made the discovery (or what has been allowed to pass for one) that Bacon invented a new species of logic which was to supersede that of Aristotle, and their followers, have succeeded by false history and falser theory, in driving out from our system all study of the connexion between thought and language. The growth of inaccurate expression which this has produced, gives us swarms of legislators, preachers, and teachers of all kinds, who can only deal with their own meaning as bad spellers deal with a hard word, put together letters which give a certain resemblance, more or less as the case may be. Hence, what have been aptly called "the slipshod judgments and crippled arguments which every-day talkers are content to use." Offences against the laws of syllogism (which are all laws of common sense) are as common as any species of fallacy: not that they are always offences in the speaker's or writer's mind, but that they frequently originate in his attempt to speak his mind. And the excuse is, that he meant differently from what he said: which is received because no one can throw the first stone at it, but which in the middle ages would have been regarded as a plea of guilty. The current notions about what logic is, are beautiful and wonderful. I have heard a disputant, an educated man, a graduate, escape from allowing himself to be convinced that he was arguing with a middle term particular in both premises by declaring that *facts* were better than *syllogisms*: the form of his argument would have proved that men are plants, because both require air. "I" he said, "produce you *facts*, like Bacon: you quibble about their combination, like Aristotle."

The Aristotelian system of fallacies contains two subdivisions. In the first, which are *in dictione*, or *in voce*, the mistake is said to consist in the use of words: in the second, which are *extra dictionem*, or *in re*, it is said to be in the matter.

Of the first set six kinds were distinguished, as follows:—

1. *Æquivocatio* or *Homonymia*, in which a word is used in two different senses; giving really no middle term (if the middle term be in question) or a term in the conclusion which is not the same name as that used in the premises. For example, "All criminal actions ought to be punished by law: prosecutions for theft are criminal actions; therefore, prosecutions for theft ought to be punished by law." Here the middle term is doubly ambiguous, **[242]** both *criminal* and *action* having different senses in the two premises. But here, as in many other cases, the choice lies with the sophist to bring the fallacy under the head to which we refer it or not. It may please him to assert that he means the same thing by *criminal action* in both premises; in which case, the inference is logical, but one or the other premise must be denied as to the matter. Again, "Finis rei est illius perfectio; mors est finis vitæ; ergo mors est vitæ perfectio." Here the ambiguity may be thrown either on *finis* or on *perfectio*. The following example can be traced through books for three centuries. "Every dog runs on four legs; Sirius (the dog-star) is a dog; therefore Sirius runs on four legs." It has been the defect of many old works on logic that *all* their examples have been of that obvious absurdity, which is well enough in one or two instances. Such as "Nothing is better than wisdom and virtue; dry bread is better than nothing; therefore, dry bread is better than wisdom and virtue." Some of the old examples are "A mouse eats cheese; a mouse is one syllable; therefore one syllable eats cheese." And again, "Iste pannus est de Anglia; Angli est terra; ergo, iste pannus est de terra."

Where the syllogism is formally put, equivocation of the middle term is generally seen with great ease. The most difficult exception is, I think, the old fallacy, in which giving the name of the genus is confounded with giving the name of the species, and thereby, of course, giving the name of the genus. As in " To call you an animal is to speak truth ; to call you an ass is to call you an animal ; therefore, to call you an ass is to speak truth." This equivocation will puzzle a beginner as to its form, and the more so from the evident falsehood of the matter. The middle term is " He who says that you are *one* among all animals." He speaks truth ; and the one who calls you an ass or a goose, certainly says that you are *one* among all animals. The equivocation is in the two different uses of the word one ; in the first premise, it is an entirely indefinite *one* ; in the second it is a less indefinite *one*. This *one* is not attached to the quantity of the middle term, which is universal in the first premise, and particular in the second : but is part of the middle term itself.

The manner in which the serious fallacy of equivocation most frequently appears, is in the connection of the old associations of [243] a word which has shifted its meaning with the altered meaning of the same. The word loyal, for instance, originally meaning no more (and no less) than *lawful*, which, as applied to a man, meant one who respected the laws, and had not forfeited any right by misbehaviour, now means attached to the Crown and to the title of the holder of it. In contests for succession, the winner would, of course, assume that *lawful* men were on his side. In more recent times, the term was always self-applied, at elections, by those who supported the party which had the confidence of the Crown for the time being : but on such occasions, abstinence from the fallacy which the French call the *voie du fait* is the utmost which can be expected of human nature.

The word *publication* has gradually changed its meaning, except in the courts of law. It stood for *communication to others*, without reference to the mode of communication, or the number of recipients. Gradually, as printing became the easiest and most usual mode of publication, and consequently the one most frequently resorted to, the word acquired its modern meaning: if we say a man publishes his travels, we mean that he writes and prints a book descriptive of them. I suspect that many persons have come within the danger of the law, by not knowing that to write a letter which contains defamation, and to send it to another person to read, is *publishing a libel;* that is, by imagining that they were safe from the consequences of publishing, as long as they did not print. In the same manner, the well-established rule that the first publisher of a discovery is to be held the discoverer, unless the contrary can be proved, is misunderstood by many, who put the word printer in the place of publisher. I could almost fancy that some persons think rules ought to travel in meaning, with the words in which they are expressed.

A similar change has taken place in the meaning of the word to *utter,* the sense of which is to *give out,* but which now means usually to give out of the mouth in words. As yet, I am not aware that any person charged with the *utterance* of counterfeit coin has pleaded that no one ever uttered coin except the princess in the fairy tale: but there is no saying to what we may come, with good example, and under high authority.

It may almost be a question whether, in the time of Aristotle, successful equivocation, that is, undetected at the moment, would [244] not have been held binding on the disputant who had failed to detect it. The genius of uncultivated nations leads them to place undue force in the verbal meaning of engagements and admissions, independently of the understanding with which they are made. Jacob kept the blessing which he obtained by a trick, though it was intended for Esau: Lycurgus seems to have fairly bound the Spartans to follow his laws till he returned, though he only intimated a short absence, and made it eternal: and the Hindoo god who begged for three steps of land in the shape of a dwarf, and took earth, sea and sky in that of a giant, seems to have been held as claiming no more than was granted. The great stress laid by Aristotle on so many different forms of verbal deception, may have arisen from a remaining tendency among disputants to be very serious about what we should now call play upon words.

Governments permit what would otherwise be equivocation to take a strong air of truth, by legislating in detail against the principles of their own measures. The window-tax is a special instance. A newspaper calls it a tax upon the light which God's beneficence has given to all. The answer would be plain enough, namely, that it is an income tax levied upon a use of that light which (how truly matters not here) is asserted to be a fair criterion of income. But this answer is destroyed by the permission to block up windows, and thereby evade the tax: which is thus made to fall upon the light used, and not upon the means of using it which the size of the house affords. According to the principle of this impost, the blocked window is as fair a criterion of the income of the occupant as the open one, and should have been so considered.

Among the forms which the fallacy of equivocation frequently assumes, is that of the sophist altering or qualifying the known meaning of a word in his own mind, without giving the other party any notice: so that there may be, if not two meanings in one mind, yet different meanings in the two minds concerned. A person asserts that "Nobody denies, &c. &c." Should this go down, the point is gained; what nobody denies must be undeniable. But should it be contested (and it will generally be found that the things which nobody denies are matters of some difference of opinion, while those which nobody *can* deny are quite **[245]** sure to be points of constant controversy) the evasion is ready. It is no sensible person, or nobody that understands the subject, nobody that is anybody, in short: while perhaps it cannot be settled who does, or who does not, understand the subject, until, among other things, the very point in dispute is determined.

There is a wide range of equivocations arising out of meanings which are sometimes implied and sometimes not. A large class of them is made by the usual, but not universal, practice, of giving to the thing the name of that which it is intended to be, whether the attempt be successful or not. This is now abbreviation or courtesy; but it was the rule. According to old definitions, bad reasoning is reasoning, *syllogismus sophisticus* is a syllogism, and in an old book now before me, the fruits and effects of demonstration are science, opinion, and *ignorance*, the latter containing belief of falsehood derived from *bad* demonstration, which we should now call *no* demonstration.

One fallacy of our time, and a very favourite one, is the settlement of the merit of a person, or an opinion, not by arguing the place of that person or opinion in its species, but by arbitrary alteration of the boundary of the species, with the intent of excluding the individual in question altogether.

It is somewhat analogous to the proceeding of the landlord who unroofs the house to get rid of a tenant. Thus we have had the controversy whether Pope was a *poet*, not whether he was a good poet or a bad one, but whether he was a poet at all. The disputants, or some of them, claimed a right to define a poet, and decided that none but verse-makers of a certain goodness (to be settled by themselves) were poets. They might just as well have decided, on their own authority, that none but men of a certain amount of reasoning power were *men*. Had they done this last, as long as they fixed the amount at a figure which included themselves under the name, nobody would have thought they materially altered the extent of the term: it is not easy to see why they have rights so arbitrary, over words the objective definitions of which are nearly as well fixed as that of man.

Another form of the fallacy of equivocation is the assuming, without express statement, that the meaning of a phrase can be determined by joining the meanings of its several words: which is not always true in any language. When two words come to-[246]gether, it often happens that their dictionary meanings would never enable us to arrive at their known and usual (and therefore proper) compound meaning: though they might help us in explaining how that last meaning arose. A person undertakes to cross a bridge in an incredibly short time: and redeems his pledge by crossing the bridge as one would cross a street, that is, by traversing the breadth. Now, though it be true that, in general, to cross is to go over the breadth, or shorter dimension, yet in the case before us, the phrase is elliptical, and signifies crossing *the river* upon the bridge. Nor can it be said that this common meaning is incorrect: that which is common and well known is, in language, always correct. No reasonable person would say that a French newspaper is wrong in reporting an army to be *à cheval sur la rivière,* because a river is not a horse. This literal (or rather unlettered) mode of interpretation is adopted among gamblers in settling bets: and is of itself enough to raise a strong presumption that their occupation is not that of well-educated men.

It is common enough in controversy, for one side or the other to have fixed meanings of words in his own mind, on which he proceeds without any inquiry as to whether those meanings will be *conveyed* by the words to the other side, or to the reader. It is very difficult to avoid this form of the fallacy, without giving the meanings of the most essential terms, on the first occasions of their occurrence. It is not uncommon to meet with a writer who appears to believe, at least who certainly acts upon, the notion that the right over words resides in him, and that others are wrong so far as they differ from him. I do not only mean that there are many who have an undue belief in their own judgments, both as to words and things: but I speak of those who, though showing a proper modesty in respect to their own conclusions, seem to be unable to do the same with respect to their definitions of words. If all mankind had spoken one language, we cannot doubt that there would have been a powerful, perhaps a universal, school of philosophers who would have believed in the inherent connexion between names and things; who would have taken the sound *man* to be the mode of agitating the air which is essentially communicative of the ideas of reason, cookery, bipedality, &c. The writers of whom I speak, **[247]** are more or less of this school; they treat words as absolute images of things by right of the letters which spell them. "The French," said the sailor, "call a cabbage a *shoe;* the fools! why can't they call it a cabbage, when they must know it is one?"

Equivocation may be used in the form of a proposition; as for instance, in throwing what ought to be an affirmative into the form of a qualified negative, with the view of making the negative form produce an impression. Thus a controversial writer will assert that his opponent has not attempted to touch a certain point, except by the absurd assertion, &c. &c. &c. To which the other party might justly reply, "Your own words show that I have made the attempt, though your phrase has a tendency, perhaps intended, to make your reader think that there is none, or at least to blind him to the difference between *none* and *none that you approve of.*"

2. The *fallacia amphiboliæ,* or *amphibologiæ,* differs in nothing from the last, except in the equivocation being in the construction of a phrase, and not in a single term: as in confounding that which is Plato's (property) with that which is Plato's (writing). Or, as in "Qui sunt domini sui sunt sui juris; servi sunt domini sui; ergo servi funt sui juris." The ambiguities of construction in our language, arising from want of inflexions and genders are tolerably (and intolerably) numerous. The difficulty of determining the emphatic word often gives a doubt as to the meaning. But very often indeed there is a want of the distinction which the algebraist makes when he writes three-and-four tens as distinguished from three and four-tens: $(3+4).10$

and 3+4.10. It cannot, for instance, be said whether "I intend to do it and to go there to-morrow" means that it will be done to-morrow or not. It may be either—(I intend to do it and to go there) to-morrow, or—I intend to do it and (to go there to-morrow). The presumption may be for the first construction: but it is only a presumption, not a rule of the language. In an instance cited by Dr. Whateley —"If this day happen to be Sunday, this form of prayer shall be used and the fast kept the next day following," the construction is ambiguous, and the intended meaning probably against the presumption. There is a book of the last century, written by a "teacher of mathematics, and writing master to Eton College." Were mathematics taught at Eton, **[248]** or not? Punctuation may be an assistance; but it so often happens that the author leaves that point to the printer, that it is hardly safe to rely upon it. Printers punctuate correctly when the meaning is clear: but when it is ambiguous, they may be as apt to take the wrong meaning as any other readers.

3, 4. The *fallacia compositionis*, and *fallacia divisionis*, consist in joining or separating those things which ought not to be joined or separated. If we may say that A is X and B is Y, so that A and B is X and Y, we have no right to infer that we may form the compound and collective names "A and B," and "X and Y," and say that "A and B" is "X and Y." Thus two and three are even and odd: but five is not even and odd. Again, two and five are four and three; but neither is two four, nor five three. It must be remembered that the word *all*, in a proposition, is not necessarily significative of a universal proposition: it may be a part of the description of the subject. Thus in "all the peers are a house of Parliament," we do not use the words *all the peers* in the same sense as when we say "all the peers derive their titles from the Crown." In the second case the subject of the proposition is *peer*; and the term *all* is distributive, synonymous with each and every. In the first case the subject is *all the peers*, and the term *all* is collective, no more distinguishing one peer from another than one of John's fingers is distinguished from another in the phrase, " John is a man." The same remarks may be made on the word *some* ; as in " some peers are dukes," and " some peers are the committee of privileges." The *all* and *some* of the quantity of the proposition are distributive terms ; the all and some of the subject are collective. Again, all men are a species (of animals) which no number of men are, wanting the rest. *All men* here make the one individual object of thought of a singular proposition. This amounts to an ambiguity of construction, an *amphibologia*, as do most sources of fallacy falling under this head, which can therefore hardly be considered as anything more than a case of the last. We want another idiom or the algebraical distinction, as in " All (peers) hold of the Crown ; (all peers) are a house of Parliament."

5. The *fallacia prosodiæ* or *accentus* was an ambiguity arising from pronunciation, and its introduction seems to lead to very minute subdivision of the subject, and to ensure the entrance of **[249]** none but ludicrous examples. Burgersdicius does not think it unworthy of himself to descend to the following, " Omnis equus est bestia ; omnis justus est æquus, ergo omnis justus est bestia." An older writer has " Tu es qui es ; quies est requies ; ergo, tu es requies." These are mere puns ; and the makers of them were fairly beaten by the contriver of " Two men eat * oysters for a wager, one eat ninety-nine, the other eat two more, for he eat a hundred and won." But more serious fallacies may be referred to this head. A very forced emphasis upon one word may, according to usual notions, suggest false meanings. Thus, " thou shalt not bear false witness against thy neighbour," is frequently read from the pulpit either so as to convey the opposite of a prohibition, or to suggest that subornation is not forbidden, or that anything false except evidence is permitted, or that it may be given *for* him, or that it is only against *neighbours* that false witness may not be borne.

A statement of what was said, with the suppression of such tone as was meant to accompany it, is the *fallacia accentus.* Gesture and manner often make the difference between irony or sarcasm, and ordinary assertion. A person who quotes another, omitting anything which serves to show the *animus* of the meaning; or one who without notice puts any word of the author he cites in italics, so as to alter its emphasis; or one who attempts to heighten his own assertions, so as to make them imply more than he would openly say, by italics, or notes of exclamation, or otherwise, is guilty of the *fallacia accentus.*

To this fallacy I should refer one of very common occurrence, the alteration of an opponent's proposition so as to present it in a manner which is logically equivalent, but which alters the emphasis, either as noticed in page 134, or in any other manner. It is generally not reasoning, but retort, which is the object of the alteration: for inference cannot be altered by changing a proposition into a logical equivalent, but a smart repartee may be very effective against "Some Xs are Ys," but flat enough against "some Ys are Xs." And even when the proponent mistakes his own meaning, and miscalculates his own emphasis, still, if the mistake be obvious, there is fallacy in taking advantage of it; for he who communicates in such incorrect terms as show what the correct ones are, does, in fact, communicate in correct terms, to all who **[250]** see the showing. Of course, respect for logic never stood in the way of a successful retort from the time of Aristotle till now, nor will on this side of the millennium. A speculator once wrote to a scientific society, to challenge them to an (on his part) anti-Newtonian controversy, relying on it that he could contend in mechanics, though avowedly ignorant of geometry. He was answered by a recommendation to study mathematics and dynamics. His rejoinder was an angry pamphlet, in which, indignant at the unfairness, as he took it to be, of the recommendation, he exclaimed, "I did not confess my ignorance of dynamics." Had he been worth the answering, it would have been impossible to resist the reply, "No, but you showed it." Had he written, as he meant "It was not dynamics of which I confessed ignorance," and had an opponent written, as many would have done, "You say, sir, that you did not confess your ignorance of dynamics: indeed you did not, you contented yourself with an ample display of it," he would have used the *fallacia accentus.* Nor would he, in my opinion, have been clear of it though he had only taken advantage of a wrong, but evidently wrong, placement of emphasis on the part of the assailant. The use of such a weapon, as to its legitimacy, depends entirely upon the manner in which the question shall be settled how far irony is allowable. Where the answer is in the affirmative, a very obvious fallacy, as a sarcasm, may be permitted. But I may here observe, that irony itself is generally accompanied by the *fallacia accentus*; perhaps cannot be assumed without it. A writer disclaims attempting a certain talk as above his powers, or doubts about deciding a proposition as beyond his knowledge. A self-sufficient opponent is very effective in assuring him that his diffidence is highly commendable, and fully justified by the circumstances.

6. The *fallacia figuræ dictionis*, as explained, means literally a mistake in grammar and nothing else; as that because *fluvius* is *aqua* it is *humidA*, or that because *aqua* is feminine, so is *poeta*.[39]

All these fallacies *in dictione* come under the head of ambiguous language, and amount to nothing but giving the syllogism four terms, two of them under the same name. The fallacies *extra dictionem* are set down as follows.

1. The *fallacia accidentis*; and 2. That *à dicto secundum*

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[39] [This is not quite accurate as an account of Aristotle's "fallacy of figure of diction." Aristotle includes under this head all cases of confusion of one "figure of predication" with another, i.e., the fallacies arising from the "ambiguity of the copula." He gives as an illustration the fallacy of supposing "being well" to be an activity like "cutting" because both are expressed alike in Greek by the active voice of a verb. *De Sophisticis Elenchis*, c. 4, § 9.—ED.]

quid ad dictum simpliciter. The first of these ought to be called that [251] of *à dicto simpliciter ad dictum secundum quid,* for the two are correlative in the manner described in the two phrases. The first consists in inferring of the subject with an accident that which was premised of the subject only: the second in inferring of the subject only that which was premised of the subject with an accident. The first example of the second must needs be "What you bought yesterday, you eat to-day; you bought raw meat yesterday; therefore, you eat raw meat to-day." This piece of meat has remained uncooked, as fresh as ever, a prodigious time. It was raw when Reisch mentioned it in the *Margarita Philosophica* in 1496: and Dr. Whateley found it in just the same state in 1826. Of the first, we may give the instance "Wine is pernicious; therefore, it ought to be forbidden." The expressed premise refers to wine used immoderately: the conclusion is meant to refer to wine however used. This species of fallacy occurs whenever more or less stress is laid upon an accident, or upon any view of the subject, in the conclusion, than was done in the premises. As in the following:—"All that leads to such philosophy as that of the schoolmen, with their logic, must be unworthy to be studied, except historically." The intent of such a sentence is not formally to propose the false syllogism, "The schoolmen had that which led them to a false philosophy; the schoolmen had logic; therefore, logic led them to a false philosophy," but only to take the chance of the stress thus laid upon logic producing a disposition to suppose that the logic was in fault. The premises are really:—

The philosophy of the schoolmen (who paid particular attention to logic) } is { a false philosophy.

Every false philosophy } is { that the guides to which should be neglected, except as history.

whence it is rightly inferred that the guides to such a philosophy as that of the schoolmen (who studied logic) are only of historical use. And the same thing might equally be inferred of the schoolmen who ate mutton, a practice to which most of them were as much addicted, no doubt, as to making syllogisms. The art of **[252]** the sophist consists in making the accident which is either unfairly introduced, or withdrawn, or substituted, have an apparently relevant relation to the subject itself. Undoubtedly, the schoolmen's logic has a connexion with their philosophy which the mutton they ate has not: but as long as it is not *the* connexion which permits the inference, it is absolutely irrelevant.

All the fallacies which attempt the substitution of a thing in one form for the *same thing* (as it is called) in another, belong to this head: such as that of the man who claimed to have had one knife twenty years, giving it sometimes a new handle, and sometimes a new blade. The answer given by the calculating boy (page 54, note) was, relatively to the question, a worthy answer, and took advantage of the common notion that a bean, after being skinned, is still a bean, as before. More serious difficulties have arisen from the attempt to separate the *essential* from the *accidental*, particularly with regard to material objects. The Cartesians denied weight, hardness, &c. to be essential to matter, until at last they made it nothing but space, and contended that a cubic foot of iron contained no more matter than a cubic foot of air.

The law, in criminal cases, demands a degree of accuracy in the statement of the *secundum quid* which many people think is absurd: and it appears to me that the lawyers often help the popular misapprehension, and give it excuse, by confounding errors of things with errors of words, after the example of the world at large. Any error of any kind, provided it be small in amount, passes for a mistake in words only, by virtue of its smallness. By a mistake in words, I mean the addition or omission of words which, whatever they might do under another state of things, do not, as matters stand, affect the meaning.

Take two instances, as follows ; Some years ago, a man was tried for stealing a ham, and was acquitted upon the ground that what was proved against him was that he had stolen a portion of a ham. Very recently, a man was convicted of perjury, " in the year 1846," and an objection (which the judge thought of importance enough to reserve) was taken, on the ground that it ought to have been " in the year *of our Lord* 1846." There may, of course, be acknowledged rules, which, as long as they are rules, must be obeyed, and which may make the second mistake as ne-**[253]**cessarily vitiate an indictment as the first. But, in discussing the policy of the rules, it would seem to me that the two cases are entirely different. In both, no doubt, the rest of the indictment might, by implication, make good the meaning required : but there seems a great difference between allowing the remainder to correct an error, and allowing it to make good an insufficiency (supposing the date, in the second case, to be really insufficient). In the second case, the accused may see the omission as well as another, and may consider of his defence against every alternative : in the first, he may be actually led to appear in court with a defence not relevant to what will be brought against him. The second may be a hardship, the first is an injustice. And this, even on the supposition that the rest of the indictment is to be allowed in explanation : for we have no more right to suppose that the true parts will correct the erroneous ones, than that the erroneous parts will affect the construction of the true ones. But there is good reason to think that the sufficient description of one sentence may supply what is wanted in the insufficient description of another, when insufficiency is all.

But, perhaps, it will be held to be the better rule, that the remainder of the indictment should not be allowed in explanation. It will then be admitted by all that a material error, or a material insufficiency, should be allowed to nullify the charge. The difference between the law and common opinion entirely relates to what constitutes a material amount of one or the other. And here it is impossible to bring the two together: for the law must judge species, while the common opinion will never rise above the case before it. In the two instances, which by many will be held equally absurd, a great difference will be seen by any who will imagine the two descriptions, in each case, to be put before two different persons. One is told that a man has stolen a ham; another that he has stolen a part of a ham. The first will think he has robbed a provision warehouse, and is a deliberate thief: the second may suppose that he has pilfered from a cook-shop, possibly from hunger. As things stand, the two descriptions may suggest different amounts of criminality, and different motives. But put the second pair of descriptions in the same way. One person is told that a man perjured himself in the year 1846; and another, that he perjured himself in the year of our Lord [254] 1846. As things stand, there is no imaginable difference: for there is only one era from which we reckon. The two descriptions mean the same thing: nor can it even be said that one is complete and the other incomplete; but only that one is less incomplete than the other. The next question might have been, what lord was meant, our Lord Jesus Christ, or our Lord the King? both being phrases of law. The answer will be, that the number 1846 leaves no doubt which was meant. A very good answer, certainly; but equally conclusive as to the simple phrase "in the year 1846." The first case is one in which the two descriptions have a real difference of meaning: it is not so in the second.

3. The *petitio principii* is one of the logical terms which has almost found its way into ordinary life. It is translated by the phrase *begging the question*, that is, assuming the thing which is to be proved. This is also called *reasoning in a circle*, coming round, in the way of conclusion, to what has been already formally assumed, in a manner expressed or implied. I shall reserve what I have to say on the justice

of this translation, and take it for the present as good.

Every collective set of premises contains all its valid conclusions; and we may fairly say that, speaking objectively of the premises, the assumption of them is the assumption of the conclusion; though, ideally speaking, the presence of the premises in the mind is not necessarily the presence of the conclusion. But by this fallacy is meant the absolute assumption of the single conclusion, or a mere equivalent to it, as a single premise. If the conclusion be "Every X is Z" and if it be formally known that A and X are identical names, and also B and Z, then to assume "Every A is B" as a premise in proving "Every X is Z" would be a manifest *petitio principii*, or begging of the question. But even this must be said hypothetically; it is supposed fully agreed between the disputants that the two identities are granted. Let it be otherwise, and there is no *petitio principii*: it is *then* fair to propound A)B, which, if disputed, is to be proved, and afterwards to reason as in A)B+B)Z=A)Z, X)A+A)Z=X)Z. Strictly speaking, there is no formal *petitio principii* except when the very proposition to be proved, and not a mere synonyme of it, is assumed. This of course, rarely occurs: so that the fallacy to **[255]** be guarded against is the assumption of that which is too nearly the same as the conclusion required. And then the fallacy is nothing distinct in itself: but merely amounts to putting forward and claiming to have granted that which should not be granted. When this is done, it matters little as to the character of the fallacy, whether the undue claim be made for a proposition which is nearer to, or further from, the conclusion to be proved. When proof is offered, the advancement of the conclusion in other words is of course not *petitio* principii: when proof is not offered, the assumption of that which (with other things proved) would prove the conclusion, is a fallacy of the same character in all cases. There is an opponent fallacy to the *petitio principii* which, I suspect, is of the more frequent occurrence: it is the habit of many to treat an advanced proposition as a begging of the question the moment they see that, if established, it would establish the question. Before the advancer has more than stated his thesis, and before he has time to add that he proposes to prove it, he is treated as a sophist on his opponent's perception of the relevancy (if proved) of his first step. Are there not persons who think that to prove any previous proposition, which necessarily leads to the conclusion adverse to them, is taking an unfair advantage?

There is another case in which begging the question may be unjustly imputed. It should be remembered that *demonstrative* inference is not the only kind of inference: there is *elucidatory* inference, *recapitulatory* inference, &c. A proposition may have its asserted *explanation* presented as a syllogism, the inference of which, as demonstration, might well be called a result of *petitio principii*. Say "it never could have been doubted that men would apply science to the production of food." If there should be any hesitation about this, the explanation of *man* under the phrase which is exclusively characteristic of him, *rational animal*, would remove it: the animal must have food, the rational being will have science. But it would be begging the question to assert that the syllogism of elucidation "A rational animal is, &c.; man is, &c.; therefore man is, &c." is a demonstration. And out of this arises the fallacy of presuming that an author meant *demonstration*, when he can only be fairly construed to have attempted elucidation of what he supposed would, upon that elucidation, be *granted*. The forms of language are much the same in the two cases.

[256] It has been observed that Aristotle hardly ever uses the phrase *ἀϱχὴν ἀιτεῖσθαι*, *principium petere*: it is *τὸ ἐξ ἀϱχῆς* and *τὸ ἐν ἀϱχῇ*, that which is (ought to come) out of, or is in, the principle. By the word *principium* he distinctly means *that which can be known of itself*. He lays down five ways of *assuming* that which ought to come out of a self-known principle, of which begging the question is the first. The others are assuming the universal to prove the particular ; assuming a particular to help to prove the universal ; assuming all the particulars of which the universal may be composed ; and assuming something which obviously demonstrates the conclusion.

Among the earlier modern writers, as far as I have seen them, there is some diversity in their description of the *petitio principii.* That the *principium* was meant to be the thing known of itself, the *ἀρχή* of Aristotle, as far as the introduction of the word is concerned, seems clear enough. Was it not then by a mere corruption that it was frequently confounded with the conclusion, the " quod in *principio* quæsitum fuit ? " Did not the same inaccuracy,* which confounds the *τὸ ἐν αρχῇ* of Aristotle with the *ἀρχή* itself, govern the change of the word ? Most writers take the fallacy of the *petitio principii* as meaning that in which the conclusion is deduced either from itself, or from something which requires proof more, or at least as much, *ignotius aut æque ignotum.* But some, in their definitions, and still more in their examples, support the following meaning, which I strongly suspect to be the true derivation of the phrase, however the *principium* and *quod in principio* might afterwards have been confounded with one another. The philosophy of the time consisted in a large variety of general propositions (principles) deduced from authority, and supposed to be ultimately derived from intrinsic evidence, self-known, or else by logical derivation from such principles. These were at the command of the disputant, his opponent could not but admit each and all of them : the laws of disputation demanded † the assent which the geometer requires for his postu-[257]lates. Except when, now and then, literary society was shaken to its very foundations by a dispute which affected any of them, as a nominalist controversy or the like moral earthquake. The most frequent syllogism was one which having the form *Barbara*, had a *principium* for its major, and an *exemplum* for its minor: as in "All men are mortal (*principium*); Socrates is a man (*exemplum*); therefore Socrates is mortal." The *petitio principii*, then, occurred, when any one, to prove his case, made it an example of a principle which was not among those received, without offering to bring the former under the logical empire of the latter. And some writers define the fallacy as occurring *si contingat in syllogismo principium petere*; where by *principium* they mean the principle which generally occurs in the major premise, and by their instances they clearly show that they mean to include nothing but the simple syllogism of principle and example. They would leave us to infer that if any one should happen to construct a syllogism in which *both* premises are principles, one or both not received, the inference, though denied by simple denial of one or both premises, would not be considered as technically the *petitio principii*, which with them was, as it were, *petitio principii exemplum continentis*.

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* Sir W. Hamilton of Edinburgh (notes on Reid, p. 761), says that *principium* is always used for that on which something else depends.

† Does a traditional remnant of this convention still linger in the not unfrequent notion that a disputant is entitled to the concession of his *principia ?* We used to hear " You must grant me my first principles, else I cannot argue." Cardinal Richelieu's answer to his applicant's *il faut vivre*, namely, *Je n'en vois pas la nécessité*, had something of inhumanity in it : but, as applied to the *Mais, Monsieur, il faut se disputer* of the preceding assumption, it would generally be quite the reverse.

It has often been asserted that all syllogism is a begging of the question, or a *petitio principii* in the modern sense, an assumption of the conclusion. That all premises do, when the argument is objectively considered, contain their conclusion, is beyond a doubt: and a writer on logic does but little who does not make his reader fully alive to this. But the phrase, as applied to a good syllogism, is a misapprehension of meaning: for its definition refers it to what is assumed *in one premise*. The most fallacious *pair* of premises, though expressly constructed to form a certain conclusion, without the least reference to their truth, would not be assuming the question, or *an* equivalent. But a further charge has been made against the syllogism, namely that very often the conclusion, so far from being deduced from the principle, is actually required to deduce it: that for instance, in " All men are **[258]** mortal ; Plato is a man ; therefore Plato is mortal " we do not know that Plato is mortal because all men are mortal, but that we need to know that Plato is mortal, in order to know that it is really true that all men are mortal. There is much ingenuity in this argument : but I think a little consideration, not of the syllogism, but of how we stand with respect to the syllogism, will answer it.

When we say that A is B, we do not merely mean that the thing called A is the thing called B : if we spoke of objects as objects, it would not matter under what name, and " A is B " would be no other than " B is B " and the very proposition itself would be of its own nature a mere identity, an assertion that what is, is. It seems to me that between objects, thus viewed, there can neither be propositions nor syllogisms. A may remind us of a thing as suggesting one idea to our minds ; B of the same thing as suggesting another : and the proposition " A is B " then asserts that the two states of our mind are from the same external source. Our logic, in wholly separating names from objects, and dealing only with the former, makes a sort of symbolic representation of the distinction between ideas and objects.

Now the objection above stated to the syllogism appears to me to be founded upon thinking of the object, as if it had no names. Suppose all things marked, each with every name which can be applied to it. Undoubtedly then, each one marked *man* will have the mark *mortal* upon him, and some the mark *Plato*, it may be : and by the time all the marks are put on, and to a person who is supposed to be immediately cognizant of the simultaneous existence of two or more marks on the same thing, it would be an absurdity to attempt any syllogism at all. What coexistence of marks could there be which he must not be supposed to have noted in making the induction necessary for a universal proposition. When he collected the elements of " All men are mortal " he saw $\substack{\text{Plato} \\ \text{man}}$ among the rest and set it down.

But suppose that his knowledge is not acquired, as to different marks, all at once: but that each coincidence of marks is to be a separate acquisition to his mind. Then he does not know, by the time he has found out that "All men are mortal" whether Plato be mortal or not. Plato may be a statue, a dog, or a book written [259] by a man of that name. *Plato* does not carry *man* with it: his major tells him nothing about Plato, until he has the minor, "Plato is a man" and then, no doubt, he has absolutely acquired the conclusion "Plato is mortal." The whole objection tacitly assumes the superfluity of the minor; that is, tacitly assumes we know Plato to be a man, as soon as we know him to be Plato. Grant the minor to be superfluous, and no doubt we grant the necessity of connecting the major and the conclusion to be superfluous also. Grant any degree of necessity, or of want of necessity, to the minor, and the same is granted to the connection of the major and conclusion.

In the preceding case, the syllogism is looked upon as one of communication, by the authors of the objection; while at the same time it is tacitly assumed that the minor does not communicate: Plato, by virtue of our acquaintance with the name, is taken to be a man.

Moreover, it is to be noted that the proposition used in argument, whether to ourselves or to others, is very frequently not so much the mere attribution of one idea to another, as a declaration that *pro hac vice* the idea contained in the more extensive term is all that is wanted, and that the differences which constitute the species are not to the purpose. Or (page 234) it is the diminution of the comprehension which is necessary, and the increase of extension is only contingent. It is stripping the complex idea of the unnecessary parts, to prevent only what is requisite. Thus any one who will assert that, in the Mosaic account, no animal life whatever was destroyed by slaughter before the deluge, must be convinced by being reminded that an antediluvian (Cain) killed Abel who was a man and therefore an animal.

With the *petitio principii* may be classed (for it might also be referred to other fallacies) cases of the imperfect *dilemma*. Suppose we say "Either M or N must be true: if M be true, Z is impossible; if N be true, Z is impossible; therefore Z is impossible." Now if the disjunctive premise ought to have been "either M or N or Z is true," here would have been almost an express *petitio principii*. For example, say "A body must either be in the state A or the state B; it cannot change in the state A; it cannot change in the state B; therefore, it cannot change at all." Now, if the alternative A or B be necessary, the correct **[260]** statement may be "A body must either be in the state A, or in the state B, or in the state of transition from one to the other." Of this kind is the celebrated sophism of Diodorus Cronus, that motion is impossible, for all that a body does, it does either in the place in which it is, or in the place in which it is not, and it cannot move in the place in which it is, and certainly not in the place in which it is not. Now, motion is merely the name of the transition from the place in which it is (but will not be) to that in which it is not (but will be). It is reported that the inventor of this sophism sent for a surgeon to set his dislocated shoulder, and was answered that his shoulder could not have been put out either in the place in which it was, or in the place in which it was not; and therefore, that it was not hurt at all.

4. The *ignoratio elenchi*, or *ignorance of the refutation*, is what we should now call answering to the wrong point: or proving something which is not contradictory of the thing asserted. It may be considered either as an error of form or of matter; and it is, of all the fallacies, that which has the widest range. Such, for instance, as the case of a writer I have read, who admits that certain evidence, if given at all, would prove a certain point; and admits that such evidence has been given: but refuses to admit the point as proved, because the evidence was given in answer to objections, and in a second pamphlet. The

pleadings in our courts of law, previous to trial, are intended to produce, out of the varieties of statement which are made by parties, the real points at issue ; so that the defence may not be *ignoratio elenchi*, nor the case the counter-fallacy, which has no correlative name, but might be called *ignoratio conclusionis*. If a man were to sue another for debt, for goods sold and delivered, and if defendant were to reply that he had paid for the goods furnished, and plaintiff were to rejoin that he could find no record of that payment in his books ; the fallacy would be palpably committed. The rejoinder, supposed true, shows that either defendant has not paid, or plaintiff keeps negligent accounts ; and is a dilemma, one horn of which only contradicts the defence. It is plaintiff's business to prove the sale, from what *is* in his books, not the absence of payment from what *is not ;* and it is then defendant's business to prove the payment by his vouchers.

It is commonly said that no one can be required to prove a **[261]** negative, and often that no one *can* prove a negative. There is much confusion about this : for any one who proves a positive, proves an infinite number of negatives. Every thing that can be proved to be in St. Paul's Cathedral at any one moment is fairly proved *not* to be in more places than I can undertake to enumerate. What is meant is, that it is difficult, and may be impossible, to prove a negative without proving a positive. Accordingly, when the two sides of the question consist of a positive and negative, the burden of proof is generally considered to lie upon the person whose interest it is to establish the positive. This being understood, it is *ignoratio elenchi* to attempt to transfer the charge of proving the negative to the other party. But this rule is by no means without exception : there are many departures from it in the law, for example, though not under the most logical phrases. For instance, a homicide, as such, is considered by the law a murderer, unless, failing justification, he can prove that he had no malice. Here, in the language of the law, the homicide, supposed unjustifiable, is in itself a presumption of malice, which the accused is to rebut. It is not true, in point of fact, that such presumption exists on the mere case of homicide, independent of the manner of it: if the law will consult its own records, it will find that, for one homicide with malice of which it has had to take cognizance, there are dozens at least, done in heat of blood, and called manslaughters. But the case stands thus;—the alternatives are few, so that proving the negative of one, which the accused is called on to do, can be done by proving the affirmative one out of a small number. There are but malice, heat of blood, misadventure, insanity, &c. to which the action can be referred. Of these few things, it is easier for the accused to establish some one out of several, above all when motive is in question (of which only himself can be in possession of the most perfect knowledge) than it is for the prosecutor to establish a particular one. And the principle on which he is called to establish a negative (or rather another positive) is that the burden of proof fairly lies on the one to whom it will be by much the easiest. The proof of a negative, then, being as easy as, in fact identical with, the proof of one of the positive alternatives, such proof may, from the circumstances, lie upon a disputant, particularly when the number of the alternatives is few. But the *negative proof*, a **[262]** very different thing, is of its own nature hardly attainable, and therefore hardly to be required. A book has been mislaid; is it in one room or the other? If found in the second room, there is proof of the negative as to the first: and almost any one who can read can be trusted to say, on his own knowledge, that in a certain room there is a certain book. But to give negative proof as to the first room, it must be made certain, first, that every book in the room has been found and examined, secondly, that it has been correctly examined. No one, in fact, can prove more than that he cannot find the book: whether the book be there or not, is another question, to be settled by our opinion of the vigilance and competency of the searcher.

Controversialists constantly lay too much stress on their own negative proofs, on their *I cannot find*, even as to cases in which it is palpably not their interest to find.

Somewhat akin to the preceding is the constant fallacy of controversialists, conveyed in their strong assertion of the results of their own arguments. Few can bear to admit that there is a question for others to decide; and after summing up both sides, to separate the points which the reader is to pronounce upon. They must decide for him, and thus act both counsel and judge: probably because their arguments are not so convincing to their own minds as they wish them to be to the reader's. They prove, at the utmost, their own conviction that they have the right side: but the thing to be proved is that such conviction is well founded. They know the maxim *Si vis me flere, dolendum est primum ipsi tibi*, and think it will hold good of the reason, as well as of the feelings: as it will, to some. The consequence is, that the deliberate reader suspects them, and feels inclined rather to differ than agree: he will not dance to a writer who pipes too much. Just as "I'll tell you a capital thing," sets the hearer upon avoiding laughter, and gives him notice to try; so "I intend to give most unimpeachable proof," puts the judicious reader upon looking for inadmissible assumptions, and he is seldom allowed by such writers to look in vain. But, if the disputant who begins by declaring his intention to be irresistible, be suspicious, the one who ends by announcing that he is so, is absolutely self-convicted. If it be very clear, why should he say it? Does he tell his reader that he must remember to distinguish the black letters from the [263] white paper, or does he print at the top of the book "keep this side uppermost?" These things (essential as they are) he really does leave to the reader: but he dares not trust the latter to find out (though he says it is as clear as black and white) that his arguments are so strong and so good, that nothing but wilful dishonesty, or hopeless prejudice, can resist their force.

Another common form of the *ignoratio elenchi*, lies in attributing to the conclusion asserted some ultimate end or tendency. Thus, an argument in favour of checking the power of the Crown is called Jacobinism; of an increase of that power, absolutism: though the argument proposed may be sound, independently of its proposer's wishes. This is a case in which the result of the method is justifiable, though the method is wrong. Many readers will remember the advice given by an old judge to a young one, "Give your judgments without reasons; most likely your decisions will be right; and it is just as likely that your reasons will be wrong." This advice should be followed by many of those who judge or decide arguments. The proposer is of a known opinion, which gives him a strong bias towards the conclusion of the argument. He is a witness (page 205), and the effect upon the mind of the receiver is to be that of the united argument and testimony. The testimony is, in the receiver's mind, of a low order; the proposer is a radical, and the receiver is of opinion that a radical would pick a pocket: or else, perhaps, the proposer is a tory, and the receiver is of the belief that a tory must have picked a pocket. These opinions may be right or wrong; but they exist: and there is certainly no formal fallacy in admitting them, as affecting the testimony, to subtract from the probability of the truth of the conclusion. But there is a formal fallacy, a decided *ignoratio elenchi*, in throwing all the indisposition to receive upon the invalidity of the argument.

There is a much more culpable form of the same species. If such a conclusion were admitted, it would lead to such and such another conclusion, which is not to be admitted. In questions of absolute demonstration, this process is sound: if B be certainly false, and if it be the necessary consequence of A, then A must also be false. But it is unsound when it takes the form, "I believe B to be false; I believe it to follow from A; therefore I assume a right to disbelieve A whatever evidence may be **[264]** offered for it ?" This fallacy is sufficiently exposed in page 209. There is a tradition of a Cambridge professor who was once asked in a mathematical discussion "I suppose you will admit that the whole is greater than its part," and who answered, "Not I, until I see what use you are going to make of it." This was no doubt the extreme case ; the more ordinary one arises in a great measure from the great fallacy of all, the determination to have a particular conclusion, and to find arguments for it. Observe a certain person who is led on by a wily opponent in conversation : nothing is presented to him except what his reason fully concurs in, and no inference except what is indisputable. At a sudden turn of the argument, he sees a favourite conclusion, which he cares more for than for all the reasonings that ever were put together, upset and broken to pieces. He considers himself an ill-used man, entrapped, swindled out of his lawful goods ; and he therefore returns upon his steps, and finds out that some of the things which he admitted when he did not see their consequences, are no longer admissible. Neither he nor the opponent has the least idea of the nature of probable arguments, and of their opposition : both proceed as if the train of reasoning were either demonstration or nothing. The conclusion, formed perhaps upon testimony, which is more likely to be a guide to truth for the mind in question than any appreciation of argument which that mind could make, must, according to the maxims of the age, be referred to argument, and argument only. The perpetual and wilful fallacy of that mind is the determination that all argument shall support, and no argument shall shake, the conclusion. If there were only a distinct perception of another source of conviction, so strong that ordinary argument can neither materially weaken, *nor materially confirm it*, there would be sense in the conclusion ; sense, because there is truth. Right or wrong, such is the source of most convictions in, perhaps, most minds : such source ought therefore to be acknowledged. It would be an excellent thing, if, in any disputed matter, those who are better satisfied by authority of the truth of one side of the conclusion than of the validity of argument in general, would avow it, keep their own side, and let others do the same. But here is the difficulty: the persons who should avow such a state of mind are as much disposed to make converts as others: they do not like to [265] debar themselves from dissemination of their opinions. Accordingly they propound their best arguments, be they what they may, as what ought to produce all the conviction which themselves feel. On this point see page 194.

The whole class of *argumenta ad hominem*, having some reference to the particular person to whom the argument is addressed, will generally be found to partake of the fallacy in question. Such are *recrimination* and *charge of inconsistency*, as, "You cannot use this assertion, because in such another case you oppose it." But if the original argument itself should be a personal attack, then such a retort as the preceding may be a valid defence.

In many such *argumenta ad hominem*, it is not absolutely the same argument which is turned against the proposer, but one which is asserted to be like to it, or parallel to it. But *parallel cases* are dangerous things, liable to be parallel in immaterial points, and divergent in material ones. A celebrated writer on logic asserts, that no one who eats meat ought to object to the occupation of a sportsman on the ground of cruelty. The parallel will not exist until, for the person who eats meat, we substitute one who turns butcher for amusement. There is, or was, a vulgar notion that butchers cannot sit on a jury. Suppose that such a law were proposed, on the ground of the habits arising from continual infliction of death. Would it really be a counter-argument that men who eat meat have the same *animus* and are liable to acquire the same habits. It is contended (justly or not) that a desire to *take life* for *sport* is a cruel desire; to answer that those who eat flesh from which life has been taken by others have therefore also cruel desires, ought to be called arguing *a dicto secundum quid ad dictum* *secundum alterum quid*. The matter is clear enough. Cruelty of *intention* (the thing in question) must be settled by our judgment of the circumstance in which the sport consists. A person who seeks bodily exercise and the excitement of the chase, and who can acknowledge to himself that his object is gained on the birds which he misses, as well as upon those which he hits, even if thoughtless, cannot be said to act with cruelty of intention. But the sportsman, as he calls himself, who collects his game in one place, merely that he may kill, without exercise, or feeling of skill, is either culpably thoughtless, or else a savage, who delights in the infliction of death. Let any [266] man ask himself, whether in the event of his being called upon to vote for a perfectly absolute sovereign, he would feel much concerned to inquire whether the candidate was or was not a sportsman of the first kind: and then let him ask himself the same question with respect to the second.

The most amusing, and perhaps the most common, example of the *ignoratio elenchi*, is the taking exception to some part of an illustration which has nothing to do with the parallel. The word illustration (though it mean *throwing light* upon a thing) is usually confined to that sort of light which is derived from showing a process of difficulty employed upon an easier case. The first fallacy may be committed by the illustrator. He has before him the subject matter of the premises, their connexion in the process of inference, and the result produced. Either may be illustrated; thus, if it be doubtful whether such premises may be employed, the illustrator may throw away his mode of connexion, and choose another: if the process of inference be doubtful, he may choose other premises: and so on. But he may illustrate the wrong point: and this is a fallacy very common to teachers and lecturers. The greatest difficulty in the way of learners is not knowing exactly in what* their difficulty consists; and they are apt to think that when *something* is made clear, it must be *the* something. I am of opinion that the examples given of syllogisms in works of logic are examples of wrong illustration. The point in question is the form, the object is to produce conviction of the form, of its necessary validity. If the student receive help from an example stated both in matter and form, the odds are that the help is derived from the plainness of the matter, and from his conviction of the matter of the conclusion. If this be the case, he has not got over his difficulty. Many learners are puzzled to see that "Every Y is X" is not a necessary consequence of "Every X is Y." If the want of con-[267]nexion be established by an instance, as by appealing to their knowledge that every bird is not a goose, though every goose be a bird, their knowledge of the proposition is not logical. The right perception may, no doubt, be acquired by reflection on instances: but the minds which are best satisfied by material instances, are also those which give themselves no further trouble.

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* Every learner, in every subject, should accustom himself to endeavour to state the point of difficulty *in writing*, whether he want to show the result to another or not. I wish I had kept a record of the number of times which I have insisted on this being done, previously to undertaking the

The illustration being supposed correct, there is more than one fallacious mode of opposing it. Some persons will dispute the very method of illustration of form, in which the same mode of inference is applied to easier matter; but these are mere beginners, hardly even entitled to a name which supposes the possibility of progress. Others will deny the analogy of the matter, and these there is no means of meeting: for illustration is *ad hominem*, and the perception of it cannot be made purely and formally inferential: a denier of the force of an illustration is inexpugnable as long as he only denies. But when he attempts more, when he indicates the point in which the illustration fails, he very often falls into the error of attacking an immaterial point. If any one were to contend (as some do) that it is unlawful to take the life of any animal, he might be asked

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explanation, and of the proportion of them in which the writer has acknowledged that he saw his way as soon as he attempted to ask the road in precise written language. That proportion is much more than one half. Truly said Bacon, that writing makes an exact man.

what he would say if Guy Faux had trained a pigeon to carry the match to the vault, would it have been lawful to shoot the bird on its way or not? There are not a few who would think it an answer to say that he could not have trained the pigeon, or that pigeons were not then trained to carry.

5. The *fallacia consequentis* (now very often called a *non sequitur*) is the simple affirmation of a conclusion which does not follow from the premises. If the schoolmen had lived in our day, they would have joined with this the affirmation of logical form applied to that which wants it, a very common thing among us. A little time ago, either the editor or a large-type correspondent (I forget which) of a newspaper imputed to the clergy the maintenance of the "logic" of the following as "consecutive and without flaw." This was hard on the clergy (particularly the Oxonians) for there was no middle term, neither of the concluding terms was in the premises, and one negative premise gave a positive conclusion. It ran thus,

Episcopacy is of Scripture origin.

[268] The church of England is the only episcopal church in England,

Ergo, the church established is the church that should be supported.[40]

Many cases offend so slightly that the offence is not perceived. For instance "knowledge gives power, power is desirable, therefore knowledge is desirable" is not a syllogism; there is no middle term. It is a sorites, as follows, "knowledge is a giver of power, the giver * of power is the giver of a desirable thing, the giver of a desirable thing is desirable, therefore knowledge is desirable."

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[40] [De Morgan's example is unfortunate. The argument was not meant to be a syllogism but a train of reasoning with certain suppressed premises. The reasoning is valid, *if the implied premises are granted*. The argument might be expressed more fully thus:

(1) The form of Church Government which existed in apostolic times is the right form of Church Government. (As we see from the Scriptures), the form of Church Government which existed in apostolic times was Episcopacy. Episcopacy is the right form of Church Government.

(2) The Church with the right form of Government is the Church that ought to be supported.

The Anglican Church is the Church with the right form of Government (by the conclusion of syllogism (1)).

The Anglican Church is the Church that ought to be supported. The reasoning is "consecutive and without flaw"; the only question is whether the implied premises are, in point of fact, true.—Ed.]

It should be noted, however, that the copula "gives" resembles "is greater than" (page 5) and is an admissible copula in inferences with no conversion, provided that "A gives B and B gives C," implies "A gives C." The same may be said of the verbs to bring, to make, to lift, &c. And many of these verbs are, by the unseen operation of their having the effect of *is* in inference, often supplanted by the latter verb in phraseology. Thus we say "murder *is* death to the perpetrator" where the copula is *brings*; "two and two *are* four" the copula being "have the value of" &c. But this practice may lead to fallacies, as above shown: which must be avoided by attention to the class of verbs which communicate their action or state, such as make, give, bring, lift, draw, rule, hold, &c. &c. All these verbs are applied to denote the cause of the several actions: so, to give that which gives, or to bring that which brings, is to give or to bring. The boy who was said to rule the Greeks because he ruled his mother, who ruled Alcibiades, who ruled the Athenians, who ruled the Greeks, would have been correctly said so to do, if the matters of rule had been the same throughout.

6. The *non causa pro causa*. This is the mistake of imagining necessary connexion where there is none, in the way of *cause*, considered in the widest sense of the word. The idioms of language abound in it, that is, make their mere expressions of phenomena attribute them to apparent causes, without intent to assert real connexion. Thus we say that a tree *throws* a shadow, **[269]** to describe that it hinders the light. When the level of a billiard table is not good, the favoured pocket is said to *draw* the balls. A particular case of this fallacy, which is often illustrated by the words *post hoc, ergo propter hoc*, is the conclusion that

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* Because power is desirable. See page 115, as to this step.

what follows in time follows as a consequence. When things are seen together, there is frequently an assumption of necessary connexion. There is, of course, a presumption of connexion: if A and B have never been seen apart, there is probability (the amount of which depends upon the number of instances observed) that the removal of one would be the removal of the other. It is when there is only one instance to proceed upon that the assumption falls under this fallacy; were there but two, inductive probability might be said to begin. The fallacy could then consist only in estimating the probability too high.

As may be supposed, the *non causa pro causa* arises more often from mere ignorance than any other fallacy. To take the two instances that I happened to meet with nearest to the time of writing this page;—Walpole, remarking on the uniform practice among the old writing-masters of putting their portraits at the beginning of their works, remarks that these men seem to think their profession gives posterity a particular interest in their features. Probably they did not think about it: the usage of the day prevented any man from being chargeable with undue vanity who exhibited his physiognomy, and *most of the writing masters were themselves engravers*, and either did their own portraits, or more probably made use of their acquaintance with the more celebrated engravers for whom they did the under drudgery, to get themselves done on easy terms. Again, Noble (in his continuation of Granger) remarks that Saunderson had such a profound knowledge of music, that he could distinguish the fifth part of a note. The author did not know, first, that any person who cannot distinguish less than the fifth part of a note to begin with, should be bound over to keep the peace if he exhibit the least intention of learning any musical instrument in which intonation depends upon the ear; and secondly, that if Saunderson were not so gifted by nature, knowledge of music would no more have supplied the defect, than knowledge of optics would give him sight.

The *fallacia plurium interrogationum* consists in trying to get **[270]** one answer to several questions in one. It is sometimes used by barristers in the examination of witnesses, who endeavour to get *yes* or *no* to a complex question which ought to be partly answered in each way, meaning to use the answer obtained, as for the whole, when they have got it for a part. An advocate is sometimes guilty of the argument *à dicto secundum quid ad dictum simpliciter*: it is his business to do for his client all that his client might *honestly* do for himself. Is not the word in Italics frequently omitted? *Might* any man honestly try to do for himself all that counsel frequently try to do for him? We are often reminded of the two men who stole the leg of mutton; one could swear he had not got it, the other that he had not taken it. The counsel is doing his duty by his client; the client has left the matter to his counsel. Between the unexecuted intention of the client, and the unintended execution of the counsel, there may be a wrong done, and, if we are to believe the usual maxims, no wrong doer. The answer of the owner of the leg of mutton is sometimes to the point, "Well, gentlemen, all I can say is, there is a rogue between you." That a barrister is able to put off his forensic principles with his wig, nay more, that he becomes an upright and impartial judge in another wig, is curious, but certainly true.

The above were the forms of fallacy laid down as most essential to be studied by those who were in the habit of appealing to principles supposed to be universally admitted, and of throwing all deduction into syllogistic form. Modern discussions, more favourable, in several points, to the discovery of truth, are conducted without any conventional authority which can compel precision of statement: and the neglect of formal logic occasions the frequent occurrence of these offences against mere rules which the old enumeration of fallacies seems to have considered as sufficiently guarded against by the rules themselves, and sufficiently described under one head, the *fallacia consequentis*. For example, it would have been a childish mistake, under the old system, to have asserted the universal proposition, meaning the particular one, because the thing is true in most cases. The rule was imperative: *not all* must be *some*, and even *all*, when not known to be *all*, was *some*. But in our day nothing is more common than to hear and read assertions made in all the form, [271] and intended to have all the power, of universals, of which nothing can be said except that most of the cases are true. If a contradiction be asserted and proved by an instance, the answer is "Oh! that is an *extreme case*." But the assertion had been made of *all* cases. It turns out that it was meant only for ordinary cases; why it was not so stated must be referred to one of three causes;—a mind which wants the habit of precision which formal logic has a tendency to foster, a desire to give more strength to a conclusion than honestly belongs to it, or a fallacy intended to have its chance of reception.

The application of the *extreme case* is very often the only test by which an ambiguous assumption can be dealt with: no wonder that the assumer should dread and protest against a process which is as powerful as the sign of the cross was once believed to be against evil spirits. Where anything is asserted which is true with exceptions, there is often great difficulty in forcing the assertor to attempt to lay down a canon by which to distinguish the rule from the exception. Every thing depends upon it: for the question will always be whether the example belongs to the rule or the exception. When one case is brought forward which is certainly exception, the assertor will, in nine cases out of ten, refuse to see why it is brought forward. He will treat it as a fallacious argument against the rule, instead of admitting that it is a good reason why he should define the method of distinguishing the exceptions: he will virtually, and perhaps absolutely, demand that all which is certainly exception shall be kept back, simply that he may be able to assume that there is no occasion to acknowledge the difficulty of the uncertain cases.

The use of the extreme case, its decisive effect in matters of demonstration, may furnish presumption as to what it is likely to be in matters of asserted near approach. As in the following instance. It seems almost matter of course, when stated, to those who have not studied the subject of life contingencies, that the proper value of a life annuity is that of the annuity made certain during the average existence of such lives as that of the annuitant. That if, for example, persons aged 22 live, one with another, 40 years, an office which receives from every such person the present value of forty payments certain, will, without gain or loss, in the long run, be able to pay the annuities. If this be (as was **[272]** stoutly contended by some writers of the last century) a universal truth, it will hold in this extreme case. Let there be two persons, one of whom is certain to die within a year from the grant (and therefore never claims anything) and the other of whom is certain to live for ever. It is clear that the value of an annuity to both is 0 + the value of a perpetual annuity. But the *average life* of both is eternal: one perpetual duration makes the average of any set in which it is, perpetual. Hence by the false rule the value is *two* perpetual annuities, or just double of the truth.

We might suppose that most persons have no idea of a universal proposition: but use the language, never intending *all* to signify more than *most*. And in the same manner principles are stated broadly and generally, which the assertor is afterwards at liberty to deny under the phrase that he does not *carry them so far* as the instance named. It would not do to avow that the principle is not always true: so it is stated to be *always true*, but not capable of being *carried* more than *a certain length*. Are not many persons under some confusion about the meaning of the word *general?* In science it always has the meaning of *universal:* and the same in old English. Thus the catechism of the church of England asserts that there are two sacraments which are *generally* necessary to salvation: meaning necessary for all of the *genus* in question, be it man, Christian, member of the church, or any other. But in modern and vernacular English, *general* means only *usual*, and generally means usually.[41]

A great deal of what is called evasion belongs to this head, or to that of the *ignoratio elenchi*, as the sophist answers. The advocates, for instance, of the absolute unlawfulness of war never tell, unless pressed, what they think of the case of resistance to invasion. Is the country to be given up to the first foreigner who chooses to come for it ? Sometimes the extreme case comes into play : sometimes the assertion that no one will come ; which is irrelevant as to the question what would be right if he did come.

Among amusing modern evasions are " There is no *occasion* to consider that " and " I don't consider it in that *point of view*." Any one who watches the manner in which men defend their opinions will frequently see " A is B and B is C, therefore A is C " answered, not by denial of either premise, but by " that is not [273] the proper point of view " or " I don't see it in that light." This should be called the confusion between logic and perspective.

The denial of one universal is often made to amount to, or to pass into, the assertion of the opposite, or subcontrary, universal. This craving after general truths, the most manifest fault of the old logicians in their choice of premises, did not expire with them. Bacon[42] says " the mind delights in springing up to the most general axioms, *that it may find*

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[41] [De Morgan is certainly mistaken about the meaning of the word *generally* in the passage he quotes from the Catechism. The meaning intended is that the two sacraments in question are required in the *generality* of cases, but are not absolutely indispensable. They are *not* of necessity in all cases without exception. Thus the Catechism does not teach by implication, as it would do if De Morgan's exegesis were sound, that the penitent thief (who was certainly not present at the Last Supper) was lost in spite of Christ's promise to him. And De Morgan might have observed that the Anglican rubrics (1) forbid the administration of the Communion to children below the " age of reason," but (2) assert that baptized children, dying in infancy, are undoubtedly saved. This makes it clear that De Morgan's interpretation of the words " generally necessary " is mistaken.—Ed.]

[42] [*Novum Organum*, I, 20. " Gestit enim mens exsilire ad magis generalia, ut acquiescat."—Ed.]

rest." Many persons are desirous of "settled opinions," which is well; unless by settled opinions they mean universal, as is often the case. That some are and some are not is no settlement: it makes every case require examination, to see under which it falls. And with the above we may couple the tendency to believe that refutation of an argument is proof of the falsehood of its conclusion, and that a false consequence must be a false proposition. Hence it arises that so many persons dare not give up any argument in favour of a proposition which they fully believe: they think they abandon the proposition.

It sometimes happens that an assertion is made, which it is difficult to suppose can be anything but a case of a universal proposition: and yet the assertor takes care not to make his proposition universal, but persists in the particular case. A logician in our day has asserted that when Calvin says that *all* officers of the church should be elected by the people, he must be understood as speaking in reference to *deacons* only, because the assertion is made in the chapter on deacons. If it had been roundly stated that all universal propositions are to have their universes limited by the headings of the works or chapters in which they occur—for instance, that the assertion that all men are mortal, occurring in a history of England, is to be taken as made of Englishmen only—there would have been at least no ambiguity. But as it is, we are left to surmise whether this be meant, or whether the proposition be to apply to Calvin only, or to Reformers only, or to men whose names begin with C, &c. The odds are that the *application* of a universal proposition will be dictated by the heading of a chapter: but the extent to which a premise is *asserted as true* is not to be judged of by that to which it is *wanted for use:* and the less, the nearer we go to the day of the old logicians.

[274] Wrong views of the quantity of a proposition are as frequent as any fallacies. *Some*, meaning most, and *some*, meaning few, are frequently confounded. This is the neces-

sary consequence of the nature of human knowledge, in which we can but rarely form a definite idea of the proportion which the extent spoken of bears to the whole. It is part of the value of the mathematical theory of probabilities, that the mind is accustomed to the view of results drawn from perfectly definite supposed cases; as useless, it may be, in themselves, as many of the questions in a book of arithmetic, but nevertheless good for exercise. It is not surprising that fallacies about quantity should be capable of most striking exposure in questions concerning measurable quantity, that is, in questions of mathematics: nor that there should be classes of fallacy of which it is difficult to illustrate the detection by any other instances. What can be more clear, for example, to ordinary apprehensions than the broad statement that "of things of the same kind, that which is sometimes right must be better than that which is always wrong." But a little consideration will suggest that what is always wrong may be as good as that which is sometimes right, if we do not know how to distinguish the cases in which the latter *is* right: and also that what is not much wrong, generally, may be more useful than that which is mostly very wrong, when it is not absolutely right. A watch which does not go is right twice a day: but it is not so useful as one which does go, though very badly, and is never right.

To give an account of all the fallacies which depend upon wrong notions of quantity would require much space, and more assumption of mathematical knowledge in my reader than is consistent with my plan. But I may mention the mistaken use of absolute terms and notions in questions of degree. There can be, a disputant will say, but a right and a wrong; and if this be not right, it is wrong. Many persons will announce that their watches are *quite right*, absolutely at the true time, to a second: and will end by giving the time which was shown when they looked, as being accurately that of the instant at which they announce it. The proverb *Frustra fit per plura, quod fieri potest per* *pauciora*[43] contains an inaccuracy of degree: a bargain which costs twenty shillings and is worth fifteen, is not twenty shillings lost, but only five, though the vexation of the party overreached will seldom suffer him to see this.

[275] Proverbs in general are liable to this mistake. They are often used in exactly the same manner as the first principles of the old logicians. In fact, remembering that these first principles were bandied from mouth to mouth till they were perfectly *proverbial,* as we now call it, among the learned; and observing the application of our modern proverbs, as made by the mass of those who have not profited by mental discipline, we may see that the faults of the schoolmen are only those of the ordinary human mind. It is hard indeed if there be a purpose which a proverb cannot be found to serve: it is a universal proposition of no very definite meaning, sanctioned by usage, having the appearance of authority, and capable of stretching or contracting like Prince Ahmed's pavilion. One only is allowable—*In generalibus latet error*:[44] this destroys all the rest, and then, when closely looked at—commits suicide.

All mistakes of probability are essentially mistakes of quantity, the substitution of one amount of knowledge and belief for another. It is often difficult to convey a proper notion of the degree of force which is meant to be given; and still more so to retain it throughout the whole of a discussion. A person begins by stating an explanation as possible, or probable enough to require consideration, as the case may be. The forms of language by which we endeavour to express different degrees of probability are easily interchanged; so that, without intentional dishonesty (but not always) the proposition may be made to slide out of one degree into another. I am satisfied that many writers would shrink from setting down, in the margin, each time they make a certain assertion, the numerical degree of probability with which they think they are justified in presenting it. Very often it happens that a conclusion produced from a balance of arguments, and *first presented* with the appearance of confidence which might be represented by a claim of such odds as four to one in its favour, is afterwards *used* as if it were a moral certainty. The writer who thus proceeds, would not do so if he were required to write $\frac{4}{5}$ in the margin every time he uses that conclusion. This would prevent his falling into the error in which his partisan readers are generally sure to be more than ready to go with him, namely, turning all balances for, into demonstration, and all balances against, into evidences of impossibility.

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[43] [A form of the methodological principle known in modern—not in mediæval—times as "Ockham's Razor." This formulation of the principle is, however, older than Ockham (*d.* 1349) as it occurs in the writings of Ockham's teacher, the famous Oxford doctor Johannes Duns Scotus (*d.* 1308). See for a valuable historical notice W. M. Thorburn in *Mind,* N.S., No. 107, pp. 345–353.—Ed.]

[44] [A variant of the Latin proverb *Dolus versatur in generalibus*.—Ed.]

One of the great fallacies of evidence is the disposition to dwell **[276]** on the actual possibility of its being false: a possibility which must exist when it is not demonstrative. Counsel can bewilder juries in this way till they almost doubt their own senses. A man is shot, and another man, with a recently discharged pistol in his hand, is found hiding within fifty yards of the spot, and ten minutes of the time. It does not follow that the man so found committed the murder: and cases have happened, in which it has turned out that a person convicted upon evidence as strong as the above, has been afterwards found to be innocent. An astute defender makes these cases his prominent ones: he omits to mention that it is not one in a thousand against whom such evidence exists, except when guilty.

All the makers of systems who arrange the universe, square the circle, and so forth, not only comfort themselves by thinking of the neglect which Copernicus and other real discoverers met with for a time, but sometimes succeed in making followers. These last forget that for every true improvement which has been for some time unregarded, a thousand absurdities have met that fate permanently. It is not wise to toss up for a chance of being in advance of the age, by taking up at hazard one of the things which the age passes over. As little will it do to despise the usual track for attaining an object, because (as always happens) there are some who are gifted with energies to make a road for themselves. Dr. Johnson tells a story of a lady who seriously meditated leaving out the classics in her son's education, because she had heard Shakspeare knew little of them. Telford is a standing proof (it is supposed by some) that special training is not essential for an engineer.

The disposition to judge the prudence of an action by its result, contains a fallacy when it is applied to single instances only, or to few in number. That which, under the circumstances, is the prudent rule of conduct, may, nevertheless end in something as bad as could have resulted from want of circumspection. But upon dozens of instances, such a balance would appear in favour of prudence as would leave no doubt in favour of the rule of conduct, even in the instances in which it failed. The fallacy consists in judging from the result about the conduct of one who had only the previous circumstances to guide him. "You acted unwisely, as is proved by the result," is a paralogism, except when it implies "You did, **[277]** as it happens in this instance, take a course which did not lead to the desired result." Take a strong case, and the absurdity will be seen. A chemist makes up a prescription wrongly, and his customer leaves him for another: this other, so it may happen, makes it up still more wrongly, and poisons the patient. Who would venture to say that he acted unwisely, as is proved by the result, in leaving the tradesman whom he knew to be careless, for another of whom he knew no harm. The only way in which blame can be imputed, is when it can be said "You acted unwisely, in not finding out, as you might have done, that the result which has happened is the one which was likely to happen." One result proves very little as to the superior wisdom of the course which produced it; several may give a presumption of it, and the greater the number, the greater the presumption.

So little is this thought of, that the common phrase, "I acted for the best," meaning originally "I acted in the manner which under the circumstances, appeared likely to lead to the best results," very often loses its proper meaning, and is used as synonymous with "I acted with good intentions."

These, and many other points, I can only slightly touch on: I will proceed to notice a few other causes of error.

And first, of equivocations of style. I have before referred to such a phenomenon as the alteration of a good syllogism into a bad one, to make the sentence *read better*. But nothing ever reads well (for a continuance) except the natural current of a writer's thought. I should like it to be the law of letters, that every book should have inserted in it the printer's affidavit, setting forth the number of verbal erasures in the manuscript, fair copies being illegal. It would be worth at least one review.

There is a wilful and deliberate equivocation, which it is supposed the age demands. It is the use of synonymes, or supposed synonymes, to prevent the same word from occurring twice in the same passage. So far is the necessity of this practice recognized, that there are few printing-offices in London, the *readers* of which do not *query* the second introduction of any word which prominently appears twice. And then the author obeys the hint, strikes out one of the offenders, sticks in a dictionary equivalent, and would have been content if the printer's reader had done it for him. And so he writes a *good style*. To be sure, he does not [278] say what he meant, exactly; for synonymes are seldom or never logical equivalents: but what is that to elegance of expression?

The demand for non-recurrence of words arises from the public (I beg its pardon) not knowing how to read. If, when a word occurs twice, the proper emphases were looked for, and observed, there would be nothing offensive about the repetition. It is the reader who makes one and one into two, by giving both units equal value. Take this sentence from Johnson, (the first I happened to light on, in the preface to Shakspeare), and read it first as follows:—"He therefore indulged his natural *disposition*: and his *disposition*, as Rymer has remarked, led him to comedy:" and then as follows—"He therefore indulged his *natural disposition*; and *his* disposition, as Rymer has remarked, led him to comedy." This reading is what the context requires, and the ill effect of the repetition is next to nothing. Take the next sentence:—"In tragedy he often *writes*, with great appearance of toil and study, what is written *at last* with little felicity: but in his comic scenes he seems to produce, without *labour*, what *no* labour can improve." These were the first instances I found, from a chance opening of the *Elegant Extracts*, purposely chosen as a miscellany. The laws of thought generally dictate this rule, that the first occurrence of a word is the more emphatic of the two: the lesson of experience is, that a writer who prevents recurrence by the use of the dictionary of synonymes, is a good style-maker for none but a bad reader, and may very possibly be a good arguer for none but a bad logician. Of course, I should not deny that recurrence of both word and emphasis is a defect, if it be frequent.

The confusion between the means and the end, and putting one in the place of the other, is well enough known in morals: but there is a corresponding tendency to forget the distinction between the principle which is to be acted on, and the rule of action by which adherence to that principle is secured. A reference to the derived rule is in all respects as good as one to the first principle, between parties who understand both, and the connexion between them. But those who understand the rule only, are apt to forget that a rule may or may not be the true expression of a principle, according to the circumstances in which it is proposed to apply it. If, indeed, it were of universal appli-**[279]**cation, those who do and those who do not understand the principle might be on the same footing as to security: but there are few such rules.

The preceding caution may be applied in all departments of thought, in law and in logic, in morals and in arithmetic. It is impossible, for instance, to state the rule of three in such a manner as easily to include the cases in which it shall apply, and exclude those to which it does not. To say that it must be used where the fourth quantity, the one sought, is to be a fourth proportional to the three which are given, though correct, still leaves it open to inquiry what are the cases in which this condition is to be satisfied: and many cases might be, and are proposed, in which the inquiry is not easy to a beginner. In law, there are not only rules, but rules for their application. To an unlearned spectator, particularly in the courts of equity, in which the advocate addresses a judge, and not a jury, the argument takes that technical form which makes many persons think that the whole law is, at best, only arbitrary rule. It may be that some of those who there address the court can make nothing better of it: and just as there are arithmeticians, and good ones too, who are but the slaves, and never the masters, of their processes, so there may be advocates, and even judges, who have not one element of the legislator in them. But there are enough of a higher species.

The great art of using rules is to apply them in aid, and not in contravention, of the principles which they are intended to embody. A rule may have exceptions, it is said; but this is hardly a correct statement. A rule with exceptions is no rule, unless the exceptions be definite and determinable: in which case the exceptions are exclusions *by another rule.* The parallel is perfect between rules and propositions (page 143). Thus, "All Europe, except Spain and Portugal" is a universal proposition; but "All the states of Europe except two" is a particular one.[45] A rule which applies to all states except Spain and Portugal is a rule: but a rule which applies to all except two (unknown) is no rule. When it is stated, in ordinary language, that every rule is subject to exception, it is meant, for the most part, that the circumstances under which adherence to the rule gains the object, are those which most frequently occur, and that the circumstances under which adherence to the rule would defeat the object are [280] rare. If this were remembered, much confusion would often be saved. We want a word which shall so far express *rule*, that it shall imply that which will generally succeed, without the notion of *obligation* which accompanies that of *rule*, and which perpetually misleads. We want, in fact, the *rule nisi* of the courts, which is to be a rule unless cause be shown against it: and which will, in most cases, be ultimately made absolute, but is not absolute from the beginning.

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[45] [So in Edition 1, but the meaning is clearly, "All Europe except Spain and Portugal" is [the subject of] a universal proposition; but "All the states of Europe except two" is [the subject of] a particular one.—Ed.]

The common mistake is, that the rule *nisi* is an absolute rule, and that therefore it may be substituted for its leading object or first principle, and that even the very words which express that object gained, may be taken as equally expressive of satisfaction of the rule, and *vice versâ*. For instance, it is commonly stated that the rule by which a discoverer is determined, is publication; that he who first publishes the discovery, is *to be held* the discoverer; one lapse more, and it is said that he *is* the discoverer; yet one more, and it will be said that the publication is the discovery. The very remarkable circumstances attending the recent discovery of the planet *Neptune*, involving points of peculiar interest and delicacy, have caused this rule to be much discussed, and have brought out every variety of statement of it. The thing to be determined is the *actual truth of the question*, the real history of the human mind with regard to it. No one has a right under any rule, no matter what its authority, nor by whom imposed, to substitute the thing which is not, for the thing which is, or the less probable for the more probable. If philosophers were to attempt, by a law of their own framing, to substitute the conventional result for the real one, the common sense of mankind would dispute their authority, and reverse their decision. The first rule (*nisi*) is undoubtedly that the first printer is the first publisher, the second, that the first publisher is the discoverer. These will, unless cause be shown against them, be made absolute in every case. A notion which is very prevalent, namely, that the first publisher has *therefore* the rights of the discoverer, is as incorrect as that the first *printer* is therefore the first *publisher*. To take the current language, one would suppose that printing one hundred copies would be held better than circulating one thousand in manuscript, and that even though the first publisher could be proved to have plagiarised, he has still the rights of discovery.

[281] Just as (page 244) early notions make laws of literal interpretation supersede those of intended meaning, so, in the earlier stages of law, rules are often made to over-ride the principles on which they profess to be founded, and to defeat truth and common sense. There is more excuse here than there would be in a question of science, for peace and convenience are main objects of law, and it may be that rigid adherence to a rule, as a rule, at a certain avowed sacrifice of truth and justice, may be the only practicable means of preventing a larger sacrifice of both. In old times, the rule of affiliation, *Pater est quem nuptiæ demonstrant,* was held so absolutely, that the husband of the mother would be the legal father, though the two had been confined in two different jails a hundred miles apart for twelve months preceding the birth of the child. The modern law has made this rule to be no more than it ought to be, namely, one which must hold unless the contrary be proved.

It is not uncommon, in disputation, to fall into the fallacy of making out conclusions for others by supplying premises. One says that A is B; another will take for granted that he must believe B is C, and will therefore consider him as maintaining that A is C. But it may be that the other party, maintaining that A is B, may, by denying that A is C, really intend to deny that B is C. In religious controversy, nothing is more common than to represent sects and individuals as *avowing* all that is esteemed by those who make the representation to be what, upon their premises, they ought to avow. All parties seem more or less afraid of allowing their opponents to speak for themselves. Again, as to subjects in which men go in parties, it is not very uncommon to take one premise from some individuals of a party, another from others, and to fix the logical conclusion of the two upon the whole party: when perhaps the conclusion is denied by all, some of whom deny the first premise by affirming the second, while the rest deny the second by affirming the first. Any sect of Christians might be made atheists by logical consequence, if it were permitted to join together the premises of different sections among them into one argument. This is a fallacy which, however common, could easily be avoided, and would be, if those who use it cared for anything but victory. But there is another form of the same, which every one is subject to, and which it is **[282]** not so easy to perceive. It is that of drawing upon our former selves for the premises which are to guide us for the time being. Conclusions remain in our minds long after the grounds on which they were formed are abandoned: and it may happen that one premise of an argument will still have force, when the very reasons on which the second premise is now admitted are contradictory of those which once induced us to admit the first. Thus many who have learnt to advocate the legal toleration of opinions which they still believe, by force of education, to be absolute crimes against society, are logically the advocates of toleration of crime; whereas, the arguments which they have learned to think valid for the first premise, ought, if worth anything, to teach them to deny the second. I have myself heard from one mouth in one conversation (of course not in one part of it) that all sins against the Creator are sins against society, that all sins against society ought to be punished by society, that certain opinions then named are sins against the Creator, and that it is the height of injustice to punish any one for his opinions.

In printed controversy, the statement of the opposite opinion or assertion may be made by description without *citation* (by chapter or page), by description with citation, or by *quotation* with or without description. The first is not allowable. The presumption is strong that a person who opposes an opinion, imputes an error, or makes a charge, upon the writings of another, is bound at least to cite, in a manner which cannot be mistaken, the part of those writings to which he refers. There are writers who refer descriptively and even commentatively, putting the reference of citation, and thus (as Bayle says Moréri constantly does) lead the reader to suppose that the words of their paraphrase and comment are those of the passage itself. I do not see that quotation is obligatory, though highly desirable: but the reader must remember, when there is only citation, that it is not the author cited who speaks, but the person who brings him forward. It is a man's own account of his own witness: with the advantage of an apparent offer of enabling the reader to go and verify the statement for himself. If the citer be honest, the passage in question exists: if judicious, it is to the effect stated. Consequently, whenever the citer's honesty or judgment is expressly in question, no mere citation is admissible.

[283] When citations are few they ought perhaps to be quotations: when they are many, it may be impracticable to make them so. But extensive citation ought to be encouraged. Lazy readers do not like it: they are not pleased to have a power of verification offered of which they do not mean to avail themselves; and they would rather, in case of being misled, have to throw the blame upon the author than upon their own non-acceptance of the offered means of verification. Accordingly, they express their disgust at "pages loaded with references." But the more diligent readers consider every citation as a boon. At the same time it is to be remembered that there are writers who, relying on the common disinclination to verify, add a large number of citations and give the appearance of a

strong body of authorities, which are often nothing to the purpose, and sometimes not taken from actual examination, but copied from other writers.

Perhaps the greatest and most dangerous vice of the day, in the matter of reference, is the practice of citing citations, and quoting quotations, as if they came from the original sources, instead of being only copies. It is in truth the reader's own fault if he be taken in by this, or by the false appearance of authority just alluded to; for it is in his own power to certify himself of the truth: though there may be difficulty when the citations are many, or when some of them are from very rare books. Honesty and policy both demand the express statement of every citation and quotation which is made through another source. If a person quote what he finds of Cicero in Bacon, it should be "Cicero (cited by Bacon) says, &c." It has happened often enough that a quoter has been convicted of altering his author, and has had no answer to make except that he took the passage from some previous quoter.

Quotations are frequently made with intentional omission and alteration. But no rule ought to be more inflexible than that all which is within the marks of quotation ought to be a literal transcript of the book quoted. Sometimes the omission is made because part of the sentence is unnecessary, *as the quoter thinks*. But this is just the point which he has no business to decide without letting his reader know that he *has* decided it, which is easily done by the recognized mark of omission (.....) If a person would quote the Æneid for the antiquity of Carthage, **[284]** he has no business to write down, as from Virgil, "Urbs antiqua fuit Carthago:" it should be "Urbs antiqua fuit...... Carthago," if he decide upon omitting "Tyrii tenuere coloni." In this case, not only may the omission make the proposition appear more categorical than it is in the original, turning it from "There was an old city, Carthage," rather towards "Carthage was an old city;" but a reader may choose to think that the omitted words qualify the epithet, or even offer proof destructive of it. What if he should deny the antiquity of Tyre? The *omission* may (or may not) be right, but the omission without notice, or *suppression*, is certainly wrong.

Moreover, it is dangerous to truth to shorten without notice, inasmuch as those who quote the quotation will be apt to do the same thing; that is, thinking they have the whole passage, to shorten it further. What this may end in, no one can predict: but mistakes have been brought about in this way quite as absurd as any that ever were made. It may reasonably be supposed that many very ludicrous errors arise thus. A good many years ago, I succeeded, by means of a shortened quotation, put away until it was wanted, in arriving at, and publishing, the conclusion that Archimedes was once supposed to have been an ancestor of Henry IV. of France. The real purport of the sentence was that he was supposed to have been an ancestor of the Sicilian martyr St. Lucia, on whose day Henry IV. was born. It has happened that A has been said to have asserted in a second book, that B related the death of C, when the truth is that A said in the first book that B died many years before C (See the *Companion to the Almanack* for 1846, page 27). I do not speak of omissions made because the part omitted would prove more than the quoter likes: this of course is fraud.

Unjustifiable as unnoted omissions may be, still more so are additions and alterations. Writers have sometimes inserted glosses of their own, into the text which they quote, either as addition or alteration. Explanatory *additions* may easily be made within brackets [ ], which are understood marks of such a thing: but alterations are intolerable. But why, the reader may ask, are such things insisted on? Is not the simple rule, *Be honest*, enough to include these and hundreds of things like them, without detail? To this I reply that within a twelvemonth before **[285]** the time I write this, a clergyman, a man of high education and character both, published a sermon in which he gave a verse from the Bible within marks of quotation, in which he wilfully struck out one word, and inserted another, without notice: and his sermon went through several editions, either without detection, or without that detection leading to successful remonstrance. I do not suppose there was dishonesty here; but rather the following reasoning;—"I am sure it was meant; therefore I may state that it was said." Such reasoning is one of the curses of our literature.

There is one alteration within the marks of quotation which may at first seem reasonable: it is alteration of grammar to bring the quoted phrases into connected English with the quoter's context. As when a man says "I know" and another person, quoting him, says "He knows." But it is surely just as easy to put down He says "I know." There is often an alteration of emphasis in this adaptation of grammar, and generally an introduction of irony: and it is the *premier pas* to something worse. As far as I have seen, those who do it as a matter of course, are apt sometimes to put their own paraphrases under marks of quotation. A writer should suit his own grammar to that of his quotation, and not the converse.

Omission of context, preceding or following the quotation, may alter its character entirely: and this is one of the most frequent of the fallacies of reference, both intentional and unintentional. The only way to insure full confidence is to give the egg in its shell: that is, to begin at a point which clearly precedes the immediate subject of quotation, and to continue until the matter is as clearly past: to give a sentence preceding and a sentence following the matter quoted for its own sake, distinguishing the latter. This is not always conclusive: because the subject may be resumed in a sentence or two, or in another part of the book. But it will inform the reader, in most cases, whether he is or is not likely to differ from the quoter as to the meaning of the part quoted. And this refers particularly to quotations of opinion: those of fact may often be more briefly treated with safety.

In quoting ancient authors, in cases where the text is not notorious, the various readings should be given, especially when it **[286]** is an author whose text has an indifferent reputation for accuracy. Or if this cannot be done, the edition should be cited. Shameful things have occurred in controversy, by omission of a part of the ordinary text, which the quoter *chose to consider* as an interpolation, without choosing to consider that the reader ought to have liberty to judge for himself on that point.

Among the cases of indirect citation, should be included that in which a book is mentioned as existing, not on the authority of the writer's own eyes, but on that of a catalogue. The number of nonexisting books which are entered in catalogues and copied, as to their titles, into other works, is greater than any one who has not examined for himself would suppose possible. In those who know this, confidence is destroyed; and this sometimes affects questions of opinion. I am told that Dugald Stewart, who had a strong notion of the practical impossibility of presenting Euclid in a syllogistic form, never would believe that it had been done by Herlinus and Dasypodius. Such a work is entered in catalogues: but I must say that the state of catalogues is such that Stewart or any one else had full right to doubt of any work, upon no other than catalogue evidence. The work does exist, and I have a copy of it. But, seeing how matters stand, no one has a right to declare that an old book ever was written, without informing his reader on what sort of evidence he relies.

CHAPTER XIV.

ON THE VERBAL DESCRIPTION OF THE SYLLOGISM.

In page 75, I have made a first attempt to express the relations of propositions in language which will make syllogisms capable of verbal description, and the inference of their conclusions matter of self-evidence. It is desirable that this should be more fully done, and I accordingly renew the attempt, with the best words of description which I can find or make. Any one who can suggest words which better convey the meaning to himself, will find it easy to substitute them for those which I have used.

[287] The conditions to be satisfied are, that the words should have as much imported meaning as possible, that every word and its contrary should have the connexion of contrariety well marked, and that the verbal descriptions should be capable of being easily formed from the symbolic notation. As may be supposed, these conditions are to some extent contradictory of each other: the sacrifice of either to the others is then to be made to the most advantageous effect.

There are two ways in which it may be necessary to describe the syllogism. First, the one hitherto used throughout this work, in which one concluding term is referred to the other by the intervention of the middle term: what X is of Y, and what Y is of Z, determine what X is of Z. Secondly, that in which the two terms are referred to one another by comparison of both with the middle term: what X and Z severally are of Y determine what X is of Z.

In the first mode, the middle term is mentioned, and its description is middle in the sentence; while the reference term is understood in the predicate of each description.

Thus when we say "a subcontrary of a supercontrary is a subidentical," it is that a subcontrary of a *supercontrary* (*of Z*) is a subidentical (of Z); and the *supercontrary of Z* is the middle term.

In the second mode, the middle term is understood in the subject, and the concluding terms in the predicate, of the description of the syllogism. Thus when we say "genus and species are genus and species," it means that two terms which are severally genus and species of the middle term (one entirely containing, the other entirely contained in, the middle term) are genus and species to one another (the first genus, the second species).

Now it will be very easily seen, that the way to change the first description into the second is as follows. Say the description runs thus, "P of Q is R." If Q be its own correlative, as happens when Y and Z are convertibly connected, then "P of Q" merely becomes "P and Q:" but if Q have another, Q^0, for its correlative, then "P of Q" becomes "P and Q^0." Again, if R be its own correlative, its plural takes its place: but if R have R^0 for its correlative, it becomes "R and R^0." Thus "subcontrary of supercontrary is subidentical" of the first mode, becomes "subcontrary and supercontrary are subidentical and superidenti-**[288]**cal" meaning that $C_{\prime}$ and $C^{\prime}$ of the middle term are $D_{\prime}$ and $D^{\prime}$ of each other. But "subcontrary of superidentical is subcontrary" becomes "subcontrary and subidentical are subcontraries."

I need hardly say that "P of Q is R" with respect to X in terms of Z, must be read "Q^0 of P^0 is R^0" with respect to Z in terms of X. This rule we have already used.

It is thus shown that it is only necessary to dwell on the first mode; and now arises the question what words are to be employed in describing the eight standard propositions. After a good deal of consideration, I prefer to denote the universal relations by positive terms, and their contrary particulars by the corresponding negative ones: not without full perception of the sacrifice which ensues of the first condition above mentioned to the third.

The words *genus* and *species* immediately suggest themselves to denote the relation of Y to X and X to Y in X)Y. These are to be understood as employed up to their limit; or the genus and species may be coextensive. For two names which have nothing in common, as in X.Y, I propose to say that they are *externals* of each other. And for two names which have nothing out of one or the other, as in x.y, that they are *complements* of each other. Remember that complemental does not mean *only just complemental* (which is contrary), but may be contrary or supercontrary.

In X:Y, I call X a *non-species* of Y, and Y a *non-genus* of X. These words have not as much as I could wish of imported meaning, nor are there any positive terms which I can propose to supply their places. They appear as synonymous with *not entirely contained in* and *not containing the whole*. In XY, let X and Y be *non-externals*; and in xy, let X and Y be *non-complements*. Accordingly, in describing what X is with respect to Y, we have as follows, showing the substitutions which occur in reading the syllogistic symbols into this language.

$\mathrm{A}_{\prime}$, species	$\mathrm{O}_{\prime}$, non-species.
A', genus	O', non-genus.
$\mathrm{E}_{\prime}$, external	$\mathrm{I}_{\prime}$, non-external.
E', complement	I', non-complement.

If we consider *genus* and *complement* as *larger terms*, and *species* [289] and *external* as *smaller ones*, and if we put down each universal followed by its two weakened particulars, writing first that which is of the same accent, we have

Universal.	First weakened form.	Second weakened form.
A' Genus	I' non-complement	$\mathrm{I}_{\prime}$ non-external.
$\mathrm{A}_{\prime}$ Species	$\mathrm{I}_{\prime}$ non-external	I' non-complement.
E' Complement	O' non-genus	$\mathrm{O}_{\prime}$ non-species.
$\mathrm{E}_{\prime}$ External	$\mathrm{O}_{\prime}$ non-species	O' non-genus.

Thus it appears that the primary weakened form of a larger name contains a larger name, and of a smaller a smaller: and the contrary for the secondary forms. The words primary and secondary do not refer to importance, but only to order of derivation: thus $A_{,}$ was in our table X)Y, weakened into XY, before it became y)x, weakened into yx or xy.

The rules for forming particular syllogisms by weakening universal premises may now be repeated. In a universal syllogism, substitute for the *first* premise and for the conclusion their *primary* weakened forms, or for the *second* premise and for the conclusion their *secondary* weakened forms. In a strengthened syllogism, substitute for the *first* premise its *secondary* form, or for the *second* premise its *primary* form.

I now write down the whole body of syllogisms, that the reader may exercise himself in the independent comprehension of their meaning, and in assent to their inferences; deducing the particular syllogisms from the universals only.

Universal and particular Syllogisms.

Symbol.	Description of X with respect to Z.
$A_{,}A_{,}A_{,}$	Species of species is species.
$I_{,}A_{,}I_{,}$	Non-external of species is non-external.
$A_{,}I'I'$	Species of non-complement is non-complement.
$A'A'A'$	Genus of genus is genus.
$I'A'I'$	Non-complement of genus is non-complement.
$A'I_{,}I_{,}$	Genus of non-external is non-external.
$A_{,}E_{,}E_{,}$	Species of external is external.
$I_{,}E_{,}O_{,}$	Non-external of external is non-species.
$A_{,}O'O'$	Species of non-genus is non-genus.
[290]	
$A'E'E'$	Genus of complement is complement.
$I'E'O'$	Non-complement of complement is non-genus.
$A'O_{,}O_{,}$	Genus of non-species is non-species.
$E_{,}A'E_{,}$	External of genus is external.
$O_{,}A'O_{,}$	Non-species of genus is non-species.
$E_{,}I_{,}O'$	External of non-external is non-genus.

Symbol.	Description of X with respect to Z.
E'A,E'	Complement of species is complement.
O'A,O'	Non-genus of species is non-genus.
E'I'O,	Complement of non-complement is non-species.
E,E'A,	External of complement is species.
O,E'I,	Non-species of complement is non-external.
E,O,I'	External of non-species is non-complement.
E'E,A'	Complement of external is genus.
O'E,I'	Non-genus of external is non-complement.
E'O'I,	Complement of non-genus is non-external.

Strengthened Syllogisms.

A,A'I'	Species of genus is non-complement.
A'A,I,	Genus of species is non-external.
A,E'O'	Species of complement is non-genus.
A'E,O,	Genus of external is non-species.
E,A,O'	External of species is non-genus.
E'A'O,	Complement of genus is non-species.
E,E,I'	External of external is non-complement.
E'E'I,	Complement of complement is non-external.

No person could propose to himself a better exercise in the acquisition of command over language, than practising the demonstrations of these relations, or more properly their reduction into specific showing, as to the matter of the inference, in what its extent consists. For instance, "the complement of a non-complement is a non-species": How, and by how much? The non-complement leaves something which is neither in the term understood, nor in that non-complement. This, the complement of that non-complement must fill up: and by this then, at least, the complement of the non-complement is not in the term understood, of which it is therefore so far non-species.

[291] In the preceding view, I have particularly considered the connexion between contrary forms, and the adaptation of language to that connexion. But in the first derivation of the simple syllogisms (page 88) the universals were related, not to their contraries, but to their particular concomitants. I now proceed to the consideration of this view, and to the justification, on self-evident principles, of the assertion that there is a real and striking affinity between the universal syllogism and its concomitants, as $A_{\prime}A_{\prime}A_{\prime}$ and $O^{\prime}A_{\prime}O^{\prime}$, $E^{\prime}E_{\prime}A^{\prime}$ and $E^{\prime}I^{\prime}O_{\prime}$, &c.

The complex propositions $D_{\prime}$, $D^{\prime}$, and $C_{\prime}$ contain each a universal which, in common language, is generally confounded with it, and a particular, the existence of which is therefore for the most part supposed in thought to accompany the universal. The remaining universal, $E^{\prime}$, is differently circumstanced: if we say that X and Y complete the universe, we should generally mean that they only just complete it, and should not think of the supercontrary relation, or of their overcompleting it. To be contained but not to fill; to contain with room to spare, or to *overfill;* to exclude and be excluded without completion; and to exclude and be excluded with completion (or to complete and be completed without inclusion);—are our most usual ideas of the relations of the extent of names.

The reduction of the complex proposition to the simple universal, when done by removal of the concomitant *particular,* is in all cases a lowering of the quantity, by the removal of an excess, as follows:—

$D_{\prime}$ means that X is contained in Y, and more is contained.
$D^{\prime}$ means that X contains Y, and contains more.
$C_{\prime}$ means that X excludes Y, and excludes more.
$C^{\prime}$ means that X completes Y, and more* than completes.

Drop the second clauses, and $D_{\prime}$, &c. are reduced to $A_{\prime}$, &c. Drop the first clauses, and it would seem as if we had still the **[292]** complex propositions; for *more* will contain its tacit reference to that which it *is more than.* Let this tacit reference be dropped, and then we have, instead of the whole complex proposition, only its particular. And this abandonment is actually made in common language, by what would be called perhaps a lax, but is a very logical, use of the word *more*. "There are more than fish on the dry land," would be perfectly intelligible, and not as implying that there were any fish: "he was actuated by more than the motive, &c." very often means "other than the motive," &c.

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* The alteration of grammar here seen is in deference to the word *complete,* the best I can get. In this proposition, the verb refers to *the universe,* and it is X(joins in completing the universe)Y and joins in completing more (than the universe).

Now, in the complex syllogism, as we have seen (page 81), the excessive part of the conclusion (whence comes its second clause, its additive *more*) is the sum of the excessive parts of the premises. If one of the complex premises be deprived of its assertion of excess, or lowered into a simple universal, the conclusion still remains, though not *à fortiori*, necessarily. This being done, *the valid excess of the conclusion depends upon the excess of the remaining premise;* and the concomitant particular syllogism, considered as part of the mixed complex syllogism, is the expression of this, without the rest. Finally, the excess may be used in the lax, or non-correlative, sense, and then the concomitant syllogism stands by itself.

For example, $O_{\prime}A'O_{\prime}$ may be read thus:—Consider $O_{\prime}$ as concomitant of A' in D'. "X contains more than [something that is not in] Y; Z contains X; therefore, Z contains more than [something that is not in] Y." If "more than Y" mean "Y and more," this would be $D'A'D'$. Again, $O'E_{\prime}I'$ is "more than X [something not X] is contained in Y; Y excludes Z; therefore, X excludes more than Z [something not in Z]." If "more than X" were "X and more," &c.: this would be $D_{\prime}E_{\prime}C_{\prime}$. And so on for other cases.

I now proceed to what I may call the *quantitative* description of the syllogism: by which I mean the expression of its cases in terms of the quantities *only* of its names and propositions, leaving the alternative of affirmation and negation to be settled by the law of these quantities. My reason for the presentation of the system in so many different points of view will be obvious enough: that which claims to be complete, must show itself to contain just the same, and no more, as to results, whatever may be the principle which is chosen as the basis of construction.

[293] Every proposition, in speaking of two names, speaks of their contraries, and (page 63) of the four terms, two direct and two contrary, two are universal and two are particular. Since universal and particular are themselves properly contraries, (for " Every X " is " Xs, *known* to be all " and " Some Xs " are " Xs, *not known* to be all ") let us signify the universal and particular forms of the *proposition* by V and v. Again, speaking of a *name*, let its mode of entry, universal and particular, be denoted by T and t. Writing down V (or v) applied to T (or t), T (or t) we can make eight varieties, which give us the eight standard forms applied to one order, say XY ; as follows :—

$$\begin{array}{l|l|l|l} A_1 = V(Tt) & A' = V(tT) & E_1 = V(TT) & E' = V(tt) \\ O' = v(Tt) & O_1 = v(tT) & I' = v(TT) & I_1 = v(tt) \end{array}$$

Thus I' or xy, may be described as the particular in which both terms are universal : for X and Y are both universal in xy, or x:Y, or y:X. And v(TT) describes it thus.

If, understanding the order to be XY, YZ, XZ, we write down any three propositions, we make an attempt at a syllogism, valid or not, as the case may be : as in

$$V(Tt).v(tt).V(tT) \text{ or } VvV(Tt,tt,tT)$$

which must be A_1I_1A'. It will assist the memory to observe that sub-symbols have VT or vt at the beginning, super-symbols vT or Vt. Also, that affirmatives have an even number of capitals (*none* * or *two*) and negatives an odd number (*one* or *three*). A universal and its particular concomitant have the same entries of T and t, and contranominals have inverted modes of entry of these letters. The convertibles have T in both places, or t : the inconvertibles have T and t.

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* The reader must here follow the mathematician in considering 0 as an even number.

First, it is unnecessary to write down the term-letters of the conclusion, for they must be taken from the premises, in every case in which the conclusion is the strongest that can be drawn from the premises; and our system has no others (nor, indeed, **[294]** has the Aristotelian any other except *Bramantip*). Thus, TT,tt being the term letters of the premises, strike out the second T and the first t, which refer to the middle term, and Tt must belong to the conclusion. To prove this, observe that we know that t in the premise cannot give T in the conclusion: therefore T cannot give t; for if, the term being Z, T gave t, then, putting z properly in its place, t would give T, which it cannot. Again, we know that the valid forms, as to propositions, are VVV, VVv, vVv, Vvv; so that v occurring once only, must come third, and V must come in the first pair. Further, in the four term letters of the premises, VVV, vVv, Vvv, require Tt, or tT, to come in the middle, while VVv alone requires TT, or tt. Observe these laws, and every formation which can take place under them leads to a valid syllogism. Putting dots to represent a blank place, we form the eight universal syllogisms by filling up the blanks in VVV (..t,T..) and VVV(..T,t..); the eight strengthened syllogisms from VVv(..T,T..) and VVv(..t,t..); the eight particulars which begin with a universal from Vvv(..t,T..) and Vvv(..T,t..); and the eight particulars which begin with a particular from vVv (..t,T..) and vVv(..T,t)..). And, under the rules just given, we have no other cases.

Taking the preceding as a basis, we might make the rules of accentuation follow from it. For, since the first blank in our symbol, and the first concluding term, must agree, and since accents depend only on the first two letters in the symbol of a proposition, we may proceed as follows. Let K and L, each of them, mean T or t, as the case may be, but with the proviso that what it means in either place it shall mean in the other. Then, in VVV(KT,tL,KL) and in vVv(KT,tL,KL), in which symbols of conclusion are introduced, we see that the first and third accents must agree, which is part of the direct rule. As to the first and second accents, they agree in the first instance above, if K be t, which puts an even number of capitals in the first symbol VKT, or an affirmative proposition at the commencement : they differ if K be T, which puts a negative proposition first. In the second instance, they agree if K be T, which puts an affirmative first, &c. I leave it to the reader to deduce the other cases of this rule, the inverse rule, and also that premises give an [295] affirmative, or a negative, conclusion, according as they have like or unlike signs. And thus it will appear, that the symbolic rules given in chapter V, are really expressions of the general rules of quantity.

It will be observed that the concomitant syllogisms of a universal have the same term letters as that universal, and only change VVV into Vvv, or vVv. Also, that the inverted syllogisms of page 96 only invert the order of all the term letters, and the letters of the premises, when different.

Thus, E,A'E, being VVV(TT,tT), its concomitants I'A'I' and E,O,I', are vVv(TT,tT) and Vvv(TT,tT). But the inverted form A,E,E, is VVV(Tt,TT). Contranominals have different quantities in all the term-letters. The weakened forms of a universal change the first premise letter and the first term letter, or the second of both. Thus, E,E'A, being VVV(TT,tt), its weakened forms, O,E'I, and E,O,I', are vVv(tT,tt) and Vvv(TT,tT).

The forms of the numerical syllogism (page 161) may be recovered by few and easy rules, in which the premises as they stand determine the conclusion, as follows :—Let ξ be designated as the number of X, and ξ' as that of x ; and so on. Let a term of the conclusion be called *direct* when it is in the premise, and *inverse* when its contrary is in the premise. Then,

1. In every case, the conclusion has the sum of the quantities mentioned in the premises, as part of the expression of its quantity.

2. For every inverse term in the conclusion, the number of its direct term appears in the quantity of the conclusion, subtracted. Thus, x in a premise, with X in the conclusion, must have $-\xi'$ in the concluding quantity. But the direct terms of the conclusion never introduce anything into the concluding number.

3. When the entrances of the middle term are similar (YY, or yy), the terms of the two forms of conclusion are both direct and both inverse, with subtraction of the number of the middle term in the former, addition of the number of its contrary in the latter. Thus, yy gives $-\eta'$ in the direct, $+\eta$ in the inverse form.

4. When the entrances of the middle term are dissimilar (Yy, **[296]** or yY), each form of conclusion has one direct and one inverse term; and no number from the middle term enters the concluding quantity.

Thus, the conclusions from $mx\mathrm{Y}+n\mathrm{YZ}$ are immediately written down as

$$(m+n-\eta)\mathrm{xZ} \text{ and } (m+n+\eta'-\xi'-\zeta)\mathrm{Xz}:$$

while those from $m\mathrm{xY}+n'\mathrm{yz}$ are at once

$$(m+n'-\xi')\mathrm{Xz} \text{ and } (m+n'-\zeta')\mathrm{xZ}$$

There are relations existing between the forms of the syllogism which I have not considered. For instance, the description of X with respect to Z being that it is a species, ($\mathrm{A}_\prime$), the description of its contrary, x, is that it is a super-contrary, (E'). If then we give the name of *contradescriptives* to $\mathrm{A}_\prime$ and E' we find that A' and $\mathrm{E}_\prime$, $\mathrm{I}_\prime$ and O', I' and $\mathrm{O}_\prime$, are also contradescriptives. The arrangement of syllogisms by contradescriptives, and the laws of connexion thence resulting, will be an easy exercise for the student.

APPENDIX.

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I.

Account of a Controversy between the Author of this Work and Sir William Hamilton of Edinburgh; and final reply to the latter.

THIS appendix contains an account of a controversy in which some of the matters treated in the preceding work involved me with Sir William Hamilton, Professor of Logic and Metaphysics in the University of Edinburgh. It has produced four publications (to which I shall refer as I, II, III, IV) namely:

I. " Statement in answer to an assertion made by Sir William Hamilton, Bart. .... by Augustus De Morgan," .... (London, octavo, R. and E. Taylor, pp. 16, published April 30, 1847.)

II. " A letter to Augustus De Morgan, Esq. .... on his claim to an independent rediscovery of a new principle in the theory of syllogism. From Sir William Hamilton, Bart. Subjoined, the whole previous correspondence, and a postscript in answer to Professor De Morgan's ' Statement,' " (London and Edinburgh, octavo, Longman and Co., Maclachlan and Co. pp. 44, exclusive of " Prospectus " hereinafter mentioned: received by me May 22, 1847.)

III. Letter from me to Sir W. Hamilton, dated May 24, published in the *Athenæum* Journal of May 29.

IV. Letter from Sir W. Hamilton to me, dated June 2, published in the same Journal of June 5.

There are two questions involved, one concerning my character, the other purely literary. The former stands thus. March 13, Sir W. Hamilton informed me by letter that (the Italics are his own words) *to* him *it is manifest that* for a certain *principle* I was *wholly indebted to* his *information*, and *that if* I *should give* it *forth as a speculation of* my *own* (which I had done to himself, and meant to do, as he knew, and have since done, in print) I should, even *though recognizing always* his *priority, be guilty both of an injurious breach of confidence towards* him *and* *of false dealing towards the public.* This *hypothetical charge,* and *derogatory supposition of which he may formerly have surmised the possibility* (such are his subsequent qualifications of it) is unreservedly retracted at the *beginning* and *end* of II : but it is frequently insinuated in the *middle,* by proposing things as difficult to be explained otherwise, by hint that *others* may believe it, by *hopes* that they will not, by charges of falsehood, &c. &c. For the *formal* charge is substituted imputation of [298] lapse of memory, intellectual confusion, &c. The following is the programme of the first intended argument, (II. p. 4).

" I confess, that, for a time, I regarded your pretension, as an " attempt at plagiarism, cool as it was contemptible.

" From this view, feeling, information, reflection turned me ; " and I now, Sir, tender you my sincere apology, for admitting, " though founded on your own statements, an opinion so derogatory of one, otherwise so well entitled to respect.

" In itself, this view was, to me, painful and revolting.—The " character, too, which you bear among your friends, I found to " be wholly incompatible with a supposition so odious. You " are represented as an active and able man, profound in Mathematics, curious in Logic, wholly incapable of intentional deceit, " but not incapable of chronological mistakes. Your habitual " confusion of times is, indeed, remarkable, even from our correspondence. Your dates are there, not unfrequently of the " wrong month, and not always, even of the right year. With " much acuteness, your works show you deficient in architectonic " power, the concomitant of lucid thinking ; and, that you are " not guiltless of intellectual rashness is sufficiently manifest, " from your pretention to advance Logic, without having even " mastered its principles."

With regard to the subsequent insinuation of a retracted charge, my explanation (believing as I do, that Sir W. Hamilton always speaks subjective truth) is that his mind insensibly fell back to its old bias as he felt that the substitute for his charge wanted strength : my conclusion is, that it is unnecessary henceforward to notice any thing he may say or write on my character : and my determination is to act accordingly.

Sir W. Hamilton's pamphlet contains about a score and a half of quotations, on which hang sundry jokes and sneers, some of them at mathematicians in general, and myself as one of the body. On these I shall only say that my notions of the common sense of controversy, and my determination to persist, generally, in the tone of respect to my opponent's learning and character which I have hitherto preserved, would, were there nothing else, prevent my adopting the habit of which they are specimens. But as no man willingly stands an unreturned fire of facetiæ without desiring to prove that his forbearance does not arise from want of ammunition, I will permit myself (disclaiming the *animus* under which such things are usually written) just to show that quotation, application, allusion, sneer, joke, and fling at an opponent's studies, are all among the weapons which I could have employed, if I had thought them worthy of my antagonist, or of those whom I want to convince.

I might, for instance, have written something like the following ;—

Among the assets of the old logicians, discovered when the schools were swept out, there was found, as is well known, the question *Utrum chimæra bombinans in vacuo posset comedere secundas intentiones :* a very good title, as Curll would have said, wanting nothing but a treatise written to it. Now whether it be *comĕdere*, or whether the schoolmen invented *comédere*, Sir W. Hamilton, on whom their mantle has fallen, has written the treatise, and successfully maintained the affirmative. His [299] notion that his communication could give any hint, is clearly and aptly described by *chimæra*, his style by *bombinans*, his proof by *vacuum :* and the *second intentions*, above noticed, chewed up and given forth with his first ones, are a practical example of the possibility of the Q.E.I. He, or rather the bombinating chimæra which has personified itself in his form, as the *οὖλος ὄνειϱος* did in that of Nestor, is thus both retractor and detractor. But though the transition from slops to solids generally indicates convalescence, yet, as here made manifest, the passage from liquid to dental may be only the growing weakness, the periscence, of the case.

I assert the following documents to be all that are relevant with respect to the literary part of the controversy. They are given at the end of this appendix.

A is an extract from a communication of mine to the Cambridge Philosophical Society, made before I received any communication whatsoever from Sir W. Hamilton. I assert it to contain a distinct announcement and use of the principle of *quantification of the middle term*, be that middle term subject *or predicate*. On this point the reader is to judge.

B is a communication from Sir W. Hamilton to me. The reader is to judge first, whether it contain anything which is intelligible with respect to any system of syllogism ; secondly, whether, if it should so contain anything, that something would have been information to me who had written A, on some matter afterwards found in C.

C is the relevant part of an addition made by me to A, when the latter came before me in proof. The reader is to judge first, whether C contain anything more than an application of A; secondly, if so, whether that something more is derived from anything intelligibly hinted at in B.

The only bare fact on which Sir W. Hamilton and myself are at issue is this. I assert and maintain that the matter of C was written in my possession before I received B: Sir W. Hamilton holds me mistaken, and thinks he can prove from the correspondence that in this point my memory has failed. This I continue to treat as irrelevant: for we are both agreed that the *corpus delicti*, if *delictum* there be, lies in C containing something not substantially contained in A, but sufficiently hinted at in B. Any reader who thinks that C does contain something suggested by B which is not in A, may declare against the correctness of my memory; any one who thinks the contrary, will hold it of no consequence whether my memory on the disputed fact be good or bad. With the first reader I have no case: with the second I have all I think worth caring about.

Sir W. Hamilton maintains my letters to be essential parts of the case. They *may* become so, as soon as it is *pointed out* what C contains which is hinted at in B, and not contained in substance or principle, in A. When Sir W. Hamilton *points out*, by citation from C, what he alleges to have been taken, and by citation from B, what he thinks it has been taken from, and when I thereupon fail to produce equivalent [300] knowledge from A or else to expose the irrelevance of his citation from B—then those letters may become of importance. This he has not done, though specially challenged to do so: and when I come to discuss III and IV, it shall appear that he admits he has not done it.

I now give the best account I can of the origin of the dispute, premising, that up to this 3d of September, 1847, I do not absolutely know what the system is which I am charged with appropriating. There is *a* system which I think is most probably the thing in question: but a system containing a defect of so glaring a character, that I will not attribute it to Sir W. Hamilton, who describes his own as "adequately tested and matured" until he expressly claims it, or until I have the most indubitable proof.

In the common, or Aristotelian proposition, the quantities of the subject and predicate are determined, the first *by expression* or *implication*, the second *by the nature of the copula* (see page 57 of this work). And the only quantities considered are *all* and *some;* the latter meaning anything that *not none* may mean, some, it may be all but not known to be all, perhaps not more than one. The matter contained in A suggested itself to me in the summer of 1846, and was forwarded to Cambridge with the rest of the memoir on the 4th of October.

I will now introduce Sir W. Hamilton's description of the various kinds of quantity (II. p. 31, 32).

" Your ' Statement ' is chiefly plausible from a wretched con- " fusion of distinct things. This confusion, with which you " delude yourself, and many of your readers, is of two independent " schemes of logical quantification ; the one, asserting *an increase* " in the expressly *quantified terms*, the other, *a minuter division* " *of the forms of quantification itself.* To disintricate this entangle- " ment, we have simply to consider, in their contrasts, the *three* " following schemes of quantification :—

" The *first* scheme is that which logically—confines all expressed " quantity to the *Subject*, presuming the *Predicate* to be taken— " in *negative* propositions, always determinately in its *greatest* " and *least* extension (universally and singularly), in *affirmative* " propositions, always indeterminately in *some part* of its exten- " sion (particularly).

" The *second* scheme is that which logically—extends the " expression of quantity to *both* the propositional terms, and " allows the *Predicate* to be of *any quantity*, in propositions of " *either quality*. This not only supplies a capital defect, but " affords a principle on which Logic obtains a new and general " development.

" The *third* scheme is that which logically—admits *more* " *expressed quantities* than a determinately least or greatest " extension (quantity singular and universal), and an indeter- " minately partial extension (quantity particular). This, though " it corrects, *perhaps*, an omission, yields no principle for a general " logical development.

" The first doctrine is the common or Aristotelic ; the second " is mine ; and in the third—in so far as you have gone, and " apart from the consideration of right or wrong—I do not ques- " tion your originality.

" Now, the second and third schemes are both opposed to the " first, but in different respects ; consequently the second and " third may, each [301] of them, combine with itself, *either* the " whole other, *or* that part of the first to which it is not itself " opposed. More is impossible.

“ Let the following be noted :—* *Your* OLD *view* (*that in the body*
“ *of the Cambridge Memoir*) *is a combination of the* THIRD *scheme*
“ *of quantification with the* FIRST ; *your* NEW *view* (*that in its*
“ *Addition*) *is a combination of the* THIRD *scheme of quantification*
“ *with the* SECOND : *and the confusion, of which you are* NOW
“ *guilty, is the recent and uniform, and perverse identification, in*
“ *your* PRESENT ‘ *Statement,*’ *of the* SECOND *scheme with the*
“ THIRD.

“ Before, however, proceeding to comment on your confusion
“ of the second and third schemes, I may also relieve a confusion
“ in the term *definite* and its reverse, *indefinite,* as applied to
“ logical quantification.

“ In the *first*, common, or Aristotelic meaning, *definite*, or more
“ precisely *predefinite* (διοϱιστὸς, ῶϱοςδιοϱιστὸς,) is equivalent
“ to *expressed*, *overt*, or, more proximately, to *designate* and *pre-*
“ *designate ;* in this sense, *definite quantity* denotes *expressed,*
“ in opposition to merely *understood, quantity.*

“ In the *second* meaning, that which I have always used, (and
“ certain ancients, I find were before me,) *definite* is equivalent
“ to determinately *marked out ;* a sense in which *definite quan-*
“ *tity* is *extension undivided or indivisible, universal* or *singular*
“ (this including any collected plurality of individuals) as opposed
“ to *particular quantity.*

“ In the *third* meaning, which you have usurped, *definite* is
“ equivalent to *numerically specified ;* and in this sense, a *definite*
“ is an *arithmetically articulate* quantity, as opposed to one arith-
“ metically inarticulate.—This your meaning of the word I did
“ not, before the appearance of your ‘ Statement,’ apprehend ;
“ for of course I presumed you to use it in its first or common
“ meaning, from which you never hint that you consciously
“ intend to deviate.”

Three schemes of quantity are here mentioned.

First, the ordinary one.

Secondly, that in which the *ordinary quantities*, *all* and *some*, are applied in every way to both subject and predicate.

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* Let the following also be noted :—My *old* view (that in the body of the Cambridge paper) is entirely on the *first* scheme, except in one *digressive* section and one subsequent paragraph (from both of which A is quoted) in which the *second* and *third* are combined : my next view (that in the addition) is also a combination of the *second* and *third* schemes : and my “ Statement ” contained also a uniform, but not recent, identification of the same *second* and *third* schemes, which I never separated in thought until I saw this paragraph. Any one who can form an opinion of the way in which the subject would present itself to the mind of a mathematician, will see that the second scheme would present itself concomitantly with, and as an essential part of, the *algebraical* form of the third. A. De M.

Thirdly, that in which numerically definite quantity is applied to subject or predicate or both : the essential distinction of this case is *numerical* definiteness : it really contains the second system, when numerical quantity is algebraically expressed. Of these, it appears, Sir W. Hamilton claims the second, or rather, the application of such a scheme to the syllogism. What then is it ? I suppose it to be the following. My order of reference is XY.

[302] *All X is all Y* means that X and Y are identical : it is my D. *All* X *is some* Y is A_1. *Some X is all Y* is A'. *Some* X is *some* Y is I_1. As to negative propositions, *All X is not all Y* is E_1. *Some X is not all Y* is O_1. *All X is not some Y* is O'. *Some X is not some Y* is true of all pairs of terms one of which is plural. In its indefinite form, it is what I have in Chapter VIII. called *spurious*.

The propositions of this system are then the complex D, or A_1+A', the six Aristotelian forms A_1, A', E_1, O_1, O', I_1, and the spurious form, which may be called U. In looking over (Sept. 5) Sir W. Hamilton's pamphlet, I happened to light on the assertion (incidentally made) that his system gives *thirty-six* valid moods in each figure. On examining the preceding system, I find this to be the case. I should not have published the results, had not Sir W. Hamilton made it necessary for me to comment on them. I shall denote the proposition U, or " Some Xs are not some Ys " by X::Y ; and I shall, supposing each case to be formed in the first figure, then transpose it into my own notation.

1. There are *fifteen* forms in which D enters. Whenever D is either of the premises, the other premise and conclusion agree. Thus we have A_1DA_1, DUU, &c. &c.

2. *Fifteen* Aristotelian forms $A_1A_1A_1$, $A'A'A'$; $A_1E_1E_1$, $E_1A'E_1$; $A_1O'O'$, $O_1A'O_1$; $A'O_1O_1$, $O'A_1O'$; $A'I_1I_1$, $I_1A_1I_1$; E_1I_1O', $I_1E_1O_1$; $A'A_1I_1$; $A'E_1O_1$, E_1A_1O'.

3. *Six* more U syllogisms $A'O'U$, O_1A_1U ; $A'UU$, UA_1U ; $I_1O'U$, O_1I_1U.

The two things to be considered are ;—the introduction of the identical proposition ; and that of the spurious one, as I call it.

It is, I suppose, a fundamental rule of all formal logic, that every proposition must have its denial, its contradiction. Now D has no simple contradiction in this system : that O' and O_1 both contradict it (and also E_1) is true : but the mere contradiction is the disjunction " O' or O_1." A person who can show that one or the other of these is true, has demonstratively contradicted D, even though it could be proved impossible to determine which of the two it is.

The proposition U is usually spurious. But if we introduce it, we must introduce its contradictory also. Now if either X or Y be plural names, it must be true: consequently, the contradiction of U is "X and Y are singular names, and X is Y." When a syllogism having the premise U is introduced, either that premise may be contradicted, or it may not. If it may, there is no form to do it in: if it may not, then it is a spurious proposition, and cannot, by combination with others, prove anything but a like spurious conclusion.

Let X::Y denote "Some Xs are not some Ys," and X_1Y_1 denote "there is but one X and one Y, and X is Y." Then either X::Y or X_1Y_1 must be true, and one only. A logical system which admits one and not the other, which contains an assertion incapable of contradiction without going out of the system, can hardly be said to be "adequately tested and matured," and is not self-complete. The proposition X_1Y_1 includes in itself the conditions of D, and is a kind of *singular form* of D.

[303] I presume, from the number of Sir W. Hamilton's moods, *thirty-six*, as above obtained, that the contradiction neither of D nor of U finds a place. Admit them, and the contradiction of U alone (call it V) demands sixteen new moods in each figure. I will now proceed.

In my publication, speaking now of (A) what was sent to Cambridge before I communicated with Sir W. Hamilton, I had no quantification intermediate between the ordinary one, and the numerical one applied to either subject or predicate, as wanted in the *canon of the middle term* there given. Look at the last of the seven syllogisms in the *second extract*, where *both the predicates*, being of the middle term are quantified, and the condition of validity is quantitatively stated. But for "Y_1+Y_2 less than 1" should be read "y_1+y_2 greater than 1." The equivalence of this to "Y_1+Y_2 less than 1" is a mistake. In the *first extract*, the general canon is given which is afterwards used in C.

Up to the time when Sir W. Hamilton published his letter in reply to my statement, (II), I never had separated the idea of his second scheme of quantification from that of the third.

Thus then we stood on October 3, when I sent my paper to Cambridge. Sir W. Hamilton had been teaching the application of the ordinary quantities to both subject and predicate: I had arrived at the algebraical representation of the numerical quantification of terms, whether subject or predicate matters not, as long as they were middle terms.

1846, *October* 6. My communication (containing A) was in the hands of Dr. Whewell (as he informs me) for transmission to the Cambridge Society: I never saw it again till the next February. *October* 7, Sir W. Hamilton wrote to me, in answer to an application of mine on the *history* of the syllogism, further informing me that he taught an extension and simplification of its *theory*, which he offered to communicate. *November* 2, (the offer having been accepted) Sir W. Hamilton forwarded the communication B, which I give entire; consisting of a letter, and the *Requisites* which he had furnished to his students, for a prize Essay. *December* 28, he wrote again, forwarding a printed *Prospectus* of his intended work on logic. This is not material; for, on receiving it, I thought certain, what from the previous communication I had thought possible, that Sir W. Hamilton was in possession of the theory of numerically definite syllogisms (but this was a mistake of mine, as will presently appear). I accordingly, to preserve my own rights, immediately forwarded (as will presently be stated more in detail) an identifying description of the sheets of paper on which my numerical theory was written, and an account of both my systems (in letters dated *December* 31, 1846, and *January* 1, 1847). Of this, Sir W. Hamilton (who has published both letters) is my witness. 1847, *February* 27, I dated the addition to the proof sheet of my Cambridge paper, which was despatched to Cambridge the next day. This addition contains C, which itself contains (in substance) all that part of my letter of *January* 1 which refers to the disputed point. *March* 13, Sir W. Hamilton wrote the letter containing the charge of plagiarism; having been for two months prevented by illness from resuming the subject. **[304]** All subsequent correspondence referred to proceedings, and not to the subject matter of the charge.

Many days before the middle of October, I had applied the system of quantification in the manner shewn in C. Sir W. Hamilton thinks my memory has failed here: I know better. My memory does not depend upon a date, but upon the opening of the University College Session, which takes place in the middle of October. But it matters nothing, for the notion of the complete quantification of a predicate, *when wanted because it is the middle term*, will prove the possession of that process as well as quantification *in all cases whether wanted or not*. On receiving B, I looked with curiosity at 2°, on which, in fact, Sir W. Hamilton grounds his declaration of having made a communication. He demands of his pupils,

"The reasons why common language makes an *ellipsis* of the *expressed quantity*, frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought."

On looking at this, and seeing mention of the quantities which the terms have in thought, *in common language*, I took it for granted that the common quantities were spoken of: namely, that of the subject from the tenor of the proposition, that of the predicate from the nature of the copula. I never should have imagined that in the common language of common people, there were any other quantities, even if, in their minds, the *predicate* have these. Had this been all, I should have passed it over, as referring to common quantities, and making common people a little more of logicians, as to the predicate, than I have found them to be. That this *common language* meant the language of any scientific system, I had not the least idea: still less that it referred to the language of the writer's own unprinted system, current only between himself and his hearers. And, though I gained a suspicion that Sir W. Hamilton might have (which he had not) adopted numerical quantification, it was not from this passage, which by itself was nothing, but from what is now coming, which made this passage ambiguous.

On looking further into B, (which see) I found that Sir William's system, whatever it might be, noted defects in the *conversion of propositions*, and a *general canon of syllogism*. Now I had two systems, each of which had its own way of adding to the conversions, and each its own canon of syllogism. In my first system (which has now grown into Chapter V) the permanent introduction of the *contranominals* is a completion of conversion: and the reduction, by the remarks in pages 96, &c. of all syllogisms to universal affirmative premises, was the canon of syllogism. In the second, seen in A and C, which has grown into Chapter VIII, there is the universality of simple conversion, and the canon of the middle term. Sir W. Hamilton may deny (I believe he does) that these are canons: let it be so; but I took them for canons, and thought of them when I saw the word *canon* in his summary. And then the question was, had Sir W. Hamilton one of these systems, or a third one? I had been throughout our correspondence well pleased with the idea that I had hit upon something in common with Sir W. Hamilton; and in my answer to communication B I said,

[305] "I am not at all clear that I shall *not* have to claim only

"secondary originality on several points. When I see 'defects "of the common doctrine of conversion' and a 'supreme canon' "of categorical syllogism, I must wait for further information. " .... I think I may yet be able to flatter myself that I have "followed you in some points unknowingly."

The reader will observe that this instructive communication is supposed to tell me, that in my thoughts the predicate has all kinds of quantity: though in truth *both have their quantities* is not English for *either may have any one of two species of quantity*. Sir W. Hamilton has expressed (perhaps) the dictum which is to have taught me new quantification, in terms of that new quantification unknown. By *both have quantities* he seems to assert that he meant *both have all quantities*. That both have their quantities, is true in the common system: these words, which express a truth of the common system, Sir W. Hamilton declares to be a sure mode of communicating the difference between his system and the common one. This may do in his own lecture room, in which he has the *arbitrium et jus et norma loquendi* in his own hands. A distinctively unmeaning phrase may, in virtue of his explanations, pass current between him and his pupils: and a private bank, of course, must receive its own notes. But they are not lawful tender anywhere: nor good tender out of the neighbourhood.

I shall now proceed to the letters in the *Athenæum* (III and IV). These contain the issues raised by the pamphlets: my short letter contains the strength of my case: I am to presume that my opponent's letter contains the strength of his answer, and I think it does so. At least I can see nothing stronger in his pamphlet.

MR. DE MORGAN.	SIR W. HAMILTON.
I take this mode of acknowledging the receipt of your printed letters to me. I promised you an answer, if you would bring forward the grounds of your assertion that I had acted with breach of confidence and false dealing. But you now admit that your grounds are no grounds; you declare your conviction that (though chargeable with con-	In reply to your letter in the last number of the *Athenæum*:—you were not wrong to abandon your promise "of trying the strength of my position;" for never was there a weaker pretension than that, by you, so suicidally maintained. You would, indeed, have been quite right had you never hazarded a second word; for every additional sentence you have

fusion, want of memory, &c. &c.) I have acted with good faith; and you offer a proper retraction and apology. You state in various places and manners, that though you are satisfied of my integrity, all may not be so; and, thereupon, you call for an answer. But I think that others will be [306] quite satisfied with your own answer to your own charge.

There is nothing left which I care to discuss with you. Our views of logic, their coincidences, their differences, their first dates, my memory, &c. I am content to leave to those who will read my statement and your letters, with two remarks.

written is another mis-statement, calling, sometimes, for another correction.

There is no strength in an abandoned position. My pamphlet was published in defence of my own *character*: when Sir Wm. Hamilton retracted his charge of breach of confidence and false dealing, there was nothing to which I stood engaged, nothing I cared to write separate pamphlets on, especially when the approach of this present publication was considered. Any one who reads page 9 of my pamphlet, in which the promise was made, will see that it has reference to what I there call " the infamy which would attach to any one who had deserved the terms he used for the conduct he described." I certainly forgot to say " unless you retract : " but as he had already refused to retract (though he had proposed to *suspend* the charge, provided I would then undergo an examination) it did not enter into my head to provide for such a contingency. The assertions about weakness, misstatement, &c. are for the reader's judgment. I did not, in this letter, allude expressly to Sir W. Hamilton's various insinuations that the old charge might be true: both because, at the first hurried reading, I did not become aware of their extent; and also, because I wished to take time before I made up my mind as to the way of treating what I saw of them.

MR. DE MORGAN.	SIR W. HAMILTON.
1. As soon as the question of character was disposed of, it was your business to show that my *Addition*,* written after I communicated with you, contained some principle not contained in my *Memoir*, † written before I communicated with ‡ you. This you do not do. You assert, and you describe, and you sum up; but you do not quote,—except a few words, which are not in that part of my *Memoir* which I declared to contain the principles used in my *Addition*.	You do not deny, that your correspondence asserts a claim to the principle communicated to you by me; but you complain that I have not shown that your *Addition* involves a new doctrine, uncontained in *that part!* [from the overt contradictions of its *other* parts I had] of your Memoir which you declared to contain the principles used in your *Addition*. And this you can say, when I explicitly stated that "*throughout the whole paper* (the Memoir) not only is there much in contradiction—there **[307]** is absolutely *nothing* in (more then fortuitous) conformity with the theory of a quantified predicate" (L. p. 34). This, too, you can say whilst before your eyes, *unanswered*, there was lying "my *formal request*, that you would point out *any passage* of your *previous writings* in which this doctrine (that asserted in your '*Statement*,' of a quantification of the middle term, be it subject or predicate) is contained" (Ibid)—for *I could find none; and none has by you been indicated.*

I do deny, in one sense, that my correspondence asserts a claim to the principle communicated by Sir W. Hamilton: for I deny that he *communicated any principle.* I presume of course that the Prospectus and letter sent on the 28th of December are out of the question: since I gave the system on which the charge was made by return of post. Sir W. Hamilton has very properly confined himself, in his pamphlet, to his communication (B) of *November* 2, as containing the communication which he asserts me to have used. Let the reader look through it and ask himself what *new principle* is communicated, and where.

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* Here given in C. † Here given in A, so far as relevant.

‡ Sir W. Hamilton's part of this is B.

Sir W. Hamilton asserts that he has shown my *Addition* to contain a new doctrine, not contained in *one definite part* of my *memoir*, by the contradictions of its *other parts*. Let P, Q, R, be parts of a memoir; and S an addition. By showing that P and Q contradict one another, Sir W. Hamilton thinks he shows that S contains a doctrine not involved in R. The fact is, that all my memoir except "Section iii. *On the quantity of propositions*" and one other paragraph (from both which A is taken) belongs to the system of Chapter V. in this work: while Section iii., the other paragraph, and the *addition*, belong to Chapter VIII. Let the reader take notice that Sir W. Hamilton (who, by the way, seems to consider "I explicitly stated" as a sufficient answer to "you have not shown") *does* find *something* in my memoir *in conformity with the theory of a quantified predicate.* He says it is *fortuitous*: but it did not seem to him requisite to bring it forward, and point out its *fortuitousness*. This point is for the reader to judge of. "How dare you," he says, "rob me of my quantified predicate." "Good Sir," I answer, "I had it before I knew you." "What if you had," he replies, "it is enough if I inform you that it was only by accident."

Sir W. Hamilton cannot find either in the *memoir* or the *addition* (he says here only in the *previous writings*, but in his pamphlet (p. 34) he states it of both *memoir* and *addition*), any thing about the doctrine of quantification of the middle term, whether it be subject or predicate, which doctrine he says *is repugnant to all that is there taught*. It is [308] true that in the next sentence he refers to *previous writings*, as cited. I will therefore conclude that Sir William included the *addition* by mistake, and meant the *memoir* only. Whether my Section iii. (A) is or is not full of quantification of the middle term, without reference to whether that middle term be subject or predicate, I am quite content to leave to the reader. Sir W. Hamilton says he cannot find it. This I believe, and wonder at: but it does not follow that it is not there. Let the reader look.

Again, when Sir W. Hamilton asserted that C contains something which I got from him, and which is therefore not in A, I repeat that he ought to have pointed out *what it is*. His assertion that he *cannot find* it in A neither proves that it *is not* in A, nor that it *is* in C.

This is the pinch which obliged him to write forty-four pages of *accusation* in answer to sixteen of *defence:* and this is the point on which the question will finally turn. I am tediously often obliged to bring the whole matter to its A B C; but what else can I do with an opponent who writes an *ignoratio elenchi* of forty-four pages long.

Sir W. Hamilton is not good at finding. Immediately after what he has quoted from himself as above, comes the following passage:—

"In regard to your third assertion, that '*perfectly definite* "*quantification destroys the necessity of distinguishing subject and* "*predicate;*' this is altogether a mistake. It is not 'definite "quantification,' (in whatever sense the word *definite* be em"ployed), but the quantification of *both the terms* which 'destroys "the necessity of distinguishing subject and predicate;' and "this by showing, that propositions are merely *equations*, and "enabling us to convert them *all—simply*."

I now quote from myself. Of the two sentences now coming, Sir W. Hamilton quotes the first, *omits the second*, which shows that my phrase "*perfectly* definite" means *definite in both terms*, and then makes the preceding remark.

"In fact, perfectly definite quantification destroys the neces"sity of distinguishing subject and predicate. To say that some "20 Xs out of 50, are all to be found among 70 Ys, or that 20 "out of 50 Xs are 20 out of 70 Ys, is precisely the same thing "as saying that 20 out of 70 Ys are 20 out of 50 Xs."

In a writer of whom dishonest intention might be concluded, we should know how to explain the omission of the second sentence. But there is no dishonesty in Sir W. Hamilton: the omission must be referred to the same disposition which prevents him from seeing quantification of the middle term in A. What I take that disposition to be, matters nothing to my reader. Perhaps this sentence alone will enable some to detect that I had not any idea of the second system of quantification independently of the third.

MR. DE MORGAN.	SIR W. HAMILTON.
2. All the alleged inconsistencies which you find in my letters, &c. will not help you till you have [309] done this:—and even then, you will have to show that your communication was intelligible. In glancing over my letters	You say, that my exposure of your inconsistencies is unavailing, except "I show that my communication was intelligible." You forget that it is for *you* to explain how, having "*subscribed to*," as having "*rightly understood*," twenty-

and the mass of notes which you have written on them, I see that I have several times used inaccurate language, as people do in hurried letters. Still more often you have misunderstood me. If my occasional inaccuracy and your occasional misunderstanding should be held to furnish some excuse for you when you precipitately charged me with dishonourable conduct, I shall be better pleased than not.

two sentences of my prospectus (L. pp. 19, 16), you could subsequently declare that communication to be *unintelligible ! !* (L. p. 59). I have now no doubt, however, that you then "subscribed to" more sentences than, by you, were "rightly understood." Indeed, had you only betimes avowed that all you had "subscribed to, as rightly understood," was to you really unintelligible, and that the repetition of my doctrine was in your mouth mere empty sound, two pamphlets might have easily been spared.

First, the *prospectus* is not the "communication." The communication is that of November 2 (B). Let the reader look at it, and see whether it be intelligible communication of new principle.

In my pamphlet I have several times spoken of *the* communication, though there were two. This was natural enough, inasmuch as there was *one* communication (that of Nov. 2), on which the charge was made against which that pamphlet was a defence. Sir Wm. Hamilton has never ventured to maintain that I derived anything from the communication of Dec. 28, containing the prospectus, to which I replied on the evening I received it, as presently mentioned. But he makes, in various places of which the above is one, a mixture of the two communications.

Secondly, I have looked carefully at pages 19 and 16 of Sir Wm. Hamilton's letter, and at all the rest of our correspondence, without finding that I have ever admitted that I subscribed to any part of the prospectus as by me "rightly understood." Page 59 is no doubt a misprint for 39. I have neither found, nor have I the slightest remembrance of, any subscription of mine to any thing Sir Wm. Hamilton ever wrote as "rightly understood."

I repeat the account given in my pamphlet of the manner in which I subscribed to this prospectus ;—

"The next communication is dated *Dec.* 28, and consisted of
"—1. A letter. 2. A printed prospectus of Sir William Hamil-
"ton's intended work on logic. Nothing turns on this, for the
"simple reason that my answer contained the most express and
"formal proof that, come by it how I might, I was *then* in the

" most complete *written* possession of all I have since published. " ...The prospectus which accompanied this letter [310] is very " full on the *results* which Sir William Hamilton can produce " from his principles ; but gives nothing, I think, certainly " nothing intelligible to me, on those principles themselves.

" As soon as I saw these *results*, I instantly saw that many of " them agreed with my own. I had then no doubt that we possessed something in common ; and I said so very distinctly " in my reply. As the reader will presently see, this first impression has not been confirmed. Feeling it now time to secure " whatever of independent discovery might belong to me, I " answered Sir William Hamilton in two letters, dated December " 31 and January 1. In these letters—

" 1. I returned the printed prospectus with the results underlined which *my* system would produce.

" 2. I stated that I had a system written on certain sheets of " paper, which I described as to number, size, &c., adding the " head words of each page. I felt inclined to get the signature " of some good witness put upon these papers ; but at the same " time I felt reluctant that Sir William Hamilton should see, " if it ever became necessary to produce these papers, that I had " been taking *precautions* against him. I therefore determined to " make himself my witness.

" 3. I stated distinctly the first principles of *both* my systems, " and the syllogistic formulæ to which they lead."

Thirdly, I substantiate the above, so far as the subscription is concerned, by quoting two passages from Sir W. Hamilton's publication of my letter of December 31.

" I received your obliging communication *this morning* and " am now fully satisfied that I have, in one of my views of syllogism, arrived at your views in substance, or something so like " them, that I could subscribe in my own sense to a great part " of your paper ...... This chapter [meaning the one on the " sheets of paper above referred to] I might express in your " words wherever they are underlined in the prospectus which I " return, hoping you will send another."

Where are those words " rightly understood " which Sir W. Hamilton attributes to me three times in one paragraph ?

He must have been quoting from memory. Seeing *his results*, I found they were also *my results ;* so I told him that I could " subscribe " (and I cannot find I have used this word more than once, and it is in page 19 referred to by Sir William) " in my own sense to a great part of " his " paper." If words can speak meaning, I here tell him that I subscribe in *my own* sense, leaving it to the future to show whether I subscribe in *his*, that is, whether I *understand him rightly*.

[I was reading this for the press, when I found out the words which, applied in one sense hypothetically to *one* of his results, Sir W. Hamilton has transferred in a different sense to all. One of his results, speaking of the moods, is the establishment of "Their numerical *equality* under all the figures," the Italics being his. I could not make out the English of this. The others I *understood* in the grammatical sense. For example, "The abrogation of the special laws of syllogism" is intelligible: I did not know whether my sense of these words, that is to say, my [311] abrogation of those laws, was the same as Sir W. Hamilton's; still that he did abrogate certain laws was clear. But numerical equality of moods I could only understand as referring to the numerical quantities which I supposed (the reader will remember that I sent back the prospectus by the next post, and had little time to look at it) Sir W. Hamilton's system to contain. It means, I find, that there are the same number of moods in all figures: but to attribute *numerical equality* to different things is a mode of saying that there is *the same number of them in different sets* to which I was unaccustomed. Having, however, as I thought, divined what the English of this might mean, I underlined it, adding (as Sir W. Hamilton states in one of the foot-notes, which I never remarked till now) these words, "If I understand this rightly I may underline it I think." I meant, "If I can make out *the words*." This *understand rightly*, Sir W. Hamilton actually takes from this sentence, joins it to my "subscription" mentioned in *another document*, and represents me as declaring that I have "*subscribed to as rightly understood*" *twenty-two sentences*, &c., and himself as quoting from one passage.]

But, had I *betimes* avowed my non-understanding, two pamphlets might have been spared. Where are we now? I did avow my not understanding the first communication, and my subscribing to the second *in my own sense*. To which Sir W. Hamilton subsequently answered to the effect that I spoke false, that I did understand the *first*, *for* that I had sent him, in letters written immediately after the second was received, his "fundamental doctrine" and "many of its most important consequences." What have I been contending for all along, except that the doctrine of his *first* communication *was* to me mere empty sound, and that all I was able to produce when I received the *second*, was my own? But Sir W. Hamilton actually gives me a right to say, with reference to the *second*, the more developed and more intelligible communication, that I did not understand it, insists upon my saying it, and reproaches me for not saying it. Well then, to use a Scottish phrase, the less I lie when I say I did not understand the *first,* which is the point at issue. So that, as to the matter of our controversy, Sir W. Hamilton admits that there was (fortuitous he calls it) entrance of the theory of the quantified predicate in my writings prior to his communications ; and as to the conduct of it, he admits that I *did not* understand his communication ; and in the face of fact, reproaches me with maintaining that I *did* till after the pamphlets were written : when it was of the essence of my statement, first, that I did not understand, secondly that neither I nor any one else could have understood, save only the pupils to whom the requisites were addressed.

MR. DE MORGAN.	SIR W. HAMILTON.
Your copious and slashing criticisms on my intellect (by which you avenge yourself for the retraction of your aspersion on my integrity), I will profit by so far as I [312] discover them to be true : the rest shall amuse me ;—and the whole will be good for the printer. Take one retort from me on the same terms. You have much skill in forming new words ; and, as is fair, you put your own image and superscription on your own coinage. I think you have got into the habit of assuming the same authority over that already existing portion of our language which is commonly said to belong to the Queen—and that you need an interpreter. If I can arrive at your meaning by the time I write the preface to my work on logic, I will state your claim, accompanied by your own words ; if not, I can still state your own words. Till then, I have nothing more to say.	I disregard your misrepresentation that " I avenge myself for the retraction of my aspersion on your integrity by my copious and slashing criticisms on your intellect." When your (excusable) irritation has subsided, you will see that I *could only* secure you from a verdict of plagiarism by bringing you in as suffering under an illusion. What, however, is all in all ;—my criticisms will not, I think, be found untrue. If guilty of lese majesty by reference to the Queen's English, have I not my accuser as abettor ? For you not only passed my mintages (*quantify* and *quantification*) as current coin ; but, in borrowing, actually " thanked me for the words " (L. p. 22). However, my verbal innovations are, at least, not elementary blunders. I do not, for example, confound a *term* with a *proposition,* the *middle* with the *conclusion* of a syllogism.

Sir W. Hamilton unconsciously adapts his language to a very true supposition, namely, that he has, in his pamphlet, made himself the jury in this case. He is unfortunate about the mintage. I say to him " You make new words well, but I am afraid you alter the old ones." To which he replies " Why, you thanked me for my new words." So I did, and so I do again : but what has that to do with the *lese majesty* part of my insinuation.

Sir W. Hamilton says that I have somewhere (where he does *not* say) used *term* for *proposition*, *middle* for *conclusion*, *collectively* for *distributively*. This may be ; such slips of the pen are common enough. He sets them down as blunders of ignorance. I am not afraid the reader will follow him. He ought to have said where they occur, that is, when he first mentioned them, in his pamphlet. Till I put these letters together, I was satisfied, on Sir Wm. Hamilton's statement, that I had done all these enormities : but now, after the case of " *rightly understood* " which I have just had to discuss, I do not feel so well satisfied.

SIR W. HAMILTON.

Finally, I beg leave to remind you.—There is now evidence in your possession that for *seven years*, at least, the doctrine of a quantified predicate has been publickly taught by me ; whilst, on your part, there is a counter assertion or innuendo, which, as you cannot prove, it concerns *your character* formally to annul.

I never denied that Sir W. Hamilton had taught *a* doctrine of the quantified predicate. By the time I wrote my pamphlet, I was pretty [313] sure that it was not the same as mine. Sir W. Hamilton's answer confirmed me in this, as appears in page 300.

I now come to mention a part of the discussion which I should perhaps have omitted, if I had not pledged myself in my pamphlet to give an account of a certain offer which I there made to Sir William Hamilton, in the event of that offer not being accepted. It is a curious instance of that disposition to hold a correspondent or an opponent capable of solving enigmas, and bound to do it, which appears in his presuming that (see B, paragraph 2°) an obscure reference to what is done in *common language* would enable me to guess at the *uncommon language* of his system and his lectures. I insert it, also, as a specimen of the various misunderstandings and misapprehensions which Sir W. Hamilton imputes to me, referring to a matter which readers will separately comprehend. Had I space or inclination to deal with them all, I believe I could serve them all in the same way.

Oct. 7, 1846, I learnt from Sir Wm. Hamilton that his doctrine had obtained considerable publicity through the *notes and essays* of his students. In my reply, referring to this system, and to his offer of communicating it, I asked if he had a pupil whom he could trust with the communication; the answer was B, presently given. But, *Dec.* 28, in sending the prospectus, Sir W. Hamilton informed me that, before forwarding it (the first communication in which that he had other than Aristotelian quantification was intelligibly announced) he had waited for a reply from Mr. ——. "That gentleman" continues Sir W. Hamilton, in words some of which I place in Italics "was a pupil of mine "six years ago, and obtained one of the highest honours of the "class; he was therefore *fully competent to afford you information,* which I begged him to do, in regard to my logical doctrines "as they were taught so far back. I knew him to be a graduate "of your College, and he tells me that he was for three years a "pupil of your own. If you are still interested in the matter, "you can therefore obtain from him as an acquaintance, what "information you wish, more agreeably than from a stranger. "When he *attended me,* besides the twofold wholes in which the "syllogism proceeds, *the quantification of the predicate,* and the "effect of that on the doctrine of conversion, on the doctrine of "syllogistic moods, on the special syllogistic rules, &c., were "*topics discussed,* and partly *given out for exercises. They were,* "*in fact, then mere commonplace.*"

Jan. 13, 1847, Mr. —— called on me at University College, after an evening lecture of mine, put his *notes* into my hands, and has since stated (in which I have no doubt he is correct, though I do not remember it) that he informed me he *was doubtful whether they contained exactly what I wanted,* and that he would gladly furnish any additional information. Now I conceived, as I thought it was intended by Sir W. Hamilton I should do, that the notes of one of the best students, even if not exactly what I wanted, were sure to contain *something* of the *mere commonplace* (by which I took to be meant the ordinary matter of the lectures) which was *discussed,* and *given out as exercises* to those [314] who attended. But in these notes I found nothing on quantification (I had now this key word, which did not appear in the main communication B) differing from what is usual; and after expressing this in my pamphlet, I proceeded as follows:

"But if there really be anything in which Sir William Hamilton "has preceded me, I shall be, of all men except himself, most "interested in his having his full rights. And I make him this "offer, and will take his acceptance of it as reparation in full

" for his suspicions and assertions. With the consent of the " gentleman to whom these notes belong, which I am sure will " not be refused to our joint application, I will forward to him " a copy of their table of contents, having more than a hundred " and fifty headings. From these Sir William Hamilton shall " select those which are, in his opinion, sure to contain proof of " his priority on any point which I have investigated. Of these " I will have copies made and sent to him : and will print in the " work on Logic which I am preparing (and in some one part of " it) the parts which he shall select as fit to prove (or to show that " he could prove, let him call it as he likes) his case, or the germs " of his case (as he pleases, again). Provided always, that the " matter shall not run beyond some eight or a dozen octavo pages " of small print. And I on my part propose that I shall be " allowed to print, to one-half the amount selected by Sir William " Hamilton, of additional extract : but if this be refused I will " not insist on it. With this I will put a heading fully descriptive " of the reason and meaning of the insertion, and such distinct " reference and account at the beginning of the preface as shall " be sure to call the reader's attention to it. So that my book " shall establish the claim, if it can be established from the notes " of one of the best students. If this offer be not accepted, an " account of it will take the place of any other result. If Sir " William Hamilton, or any one else, can propose anything to " make this offer fairer, I shall probably not be found indisposed " to accept the addition. And though, I will frankly say, my " present conviction is that the acceptance of the offer would " alone cause my work to knock Sir William Hamilton's assertions to atoms, yet I will pledge myself, in any case, to abide " by it."

Had our places in this discussion been changed, I should have taken care that no reader of my answer should have been left in ignorance of so fair an offer on the part of my opponent : more especially if that opponent had been accused by me of fraud and falsehood, in a manner which I felt obliged formally to retract. But Sir Wm. Hamilton does not notice *the offer*, even by an allusion : and refers to the notes in the following way :

" In regard to Mr. —— and his Notes, I beg leave to say, that " in my relative letters, neither to that gentleman nor to you, " did I ever refer to his *Notes* of my lectures, but exclusively to " his *personal information* in regard to them. And for a sufficient reason. The Paragraphs on Logic dictated to, and taken " down by, my students, on which I afterwards prelect, were " written so far back as the year 1837, and prior to many of my

" new views, and to the *whole* doctrine of a quantified predicate. " These views, as developed, were, and are, introduced in a great " [315] measure as corrections of the common doctrine ; in the " older Notes especially, they may, therefore, not appear in the " dictated and numbered Paragraphs at all ; whilst, frequently, " (particularly at first,) they were given out as data, on which, " previous to farther comment, the students were called on or " excited to write expository Essays. I distinctly recollect, that " in the Session during which Mr. —— attended my course of " Logic (1840–1) it was required, on the hypothesis of a quanti- " fied predicate,—to state in detail, the valid moods of each " syllogistic figure ; and I, further, distinctly recollect, that Mr. " —— was one of those who essayed this problem. If wrong " on this point, I shall admit that my memory is as treacherous " as yours. It was, indeed, quite natural, that Mr. —— should " give, and that you should receive, his Notes ; but, of course, " you could have sought or obtained no personal information " from him, in reference to the point in question, without men- " tioning the fact. .... Were it, however, requisite to give proof " from *Notes* of so manifest a fact, I doubt not that scores of " students would be willing to place theirs at my disposal."

On the appearance of Sir W. Hamilton's pamphlet, Mr. —— wrote him a very straightforward letter, of which he sent me a copy, with permission to both of us to use it. The general tenor is that Sir W. Hamilton is correct in his statements of what he had taught (which statements I never impugned as to fact ; I did not know what they meant). On the point in question Mr. —— says (the Italics are mine) ;—

" During the Session in which I attended your lectures (1840 " and 1841) your new system, based on the thorough going quan- " tification of the predicate (the second of the three systems men- " tioned in page 31 of your published letter) and its consequences " in making all propositions simply convertible &c. *was not " developed by you in your ordinary series of Lectures. I believe " it was not touched upon in them, but it was partly explained to " the class verbally,* and then given out as a subject for Essays.* " When the Essays were given in they were read aloud in the " class, and commented upon by you, and in so doing you fully " explained the system as ' a full extension and thereby a com- " plete simplification of the syllogistic theory.'

" These facts which were strongly fixed in my memory, because " I believe on that occasion I happened to be the only Essayist

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* I think this should be *extempore*: meaning that Sir W. Hamilton usually reads his lectures.

" who had rightly apprehended and worked out the thesis, will " account for the circumstance that my notes, which were originally taken in shorthand, although containing a full Report " of all your ordinary Lectures, are completely silent on the " subject."

The reader may find out, if he can, where Sir W. Hamilton referred to *personal information* as distinguished from *notes*, or to his teaching of his new system, as a matter distinct from that of his ordinary lectures: and must judge what his success is in saying what he means [316] to say. And he may find out further, how I was to guess that the *mere commonplace* of the *topics discussed* in Sir William's teaching was to come, after an interval of six years, from his old pupil's *personal information*, and not from the full and (as I found them) excellent *notes* which he made at the time.

I should add that Mr. ——, subsequently to the printed controversy, answered every query which I put to him on Sir W. Hamilton's system, but did not feel justified (as in a like case I should not either) in answering positively as to the minute details of it, after laying it by for years.

I have mentioned one or two instances in which, as seems to me, Sir W. Hamilton has a strange idea of the sense of his own words: I will now take one of the cases in which he has dealt as strangely with mine. The way in which we use language, is one of the means which the reader has, for forming his judgment on the whole of this dispute: and he must decide which of us is incapable of giving to the phrases of the other their proper signification.

When I returned to Sir W. Hamilton his prospectus, with those parts underlined which I could interpret in my own sense, the more important parts relating to logical mood and figure were not thus underlined. In the accompanying letter, I used these words, " To mood and figure, I have attended but little; what I get on these points will be from your hint, or from your book." The whole letter was on what I had done in the way of investigation, not of elementary reading: and I may safely say that it is clear I meant that I had not made mood and figure, as constituent parts of a theory of syllogism, subjects of *investigation*, with a view to new properties. But Sir W. Hamilton, in two places, makes me avow ignorance of the ordinary system of mood and figure. In a foot-note to the above, he says, " And " yet, though confessedly to seek in the very alphabet of the " science, Mr. De Morgan would be a logical inventor! What " is here acknowledged in terms, is sufficiently manifested from "mistakes." * And in his pamphlet (II. p. 9), he represents me as "no proficient—no thorough student,—in the science;" and refers to this paragraph of mine as the ground of the assertion. It would have been strange, if, avowing ignorance of the ordinary doctrine of mood and figure, I had said that what I should get on these points must be from Sir W. Hamilton's hint or unpublished book, when any ordinary treatise would have given it: so strange, that this clause ought, I think, to have suggested the obvious meaning. Is Sir W. Hamilton's interpretation a *fair* one? I do not doubt that he meant it to be fair. What I ask is, has he the power to read fairly as well as the will?

The two preceding cases (that of the *notes* and that of the *avowed ignorance*) are specimens of Sir W. Hamilton's *give and take*, of the [317] manner in which he expects to be understood, and of that in which he claims a right to understand. They are also, of course, specimens of my own.

In (A), the symbols A, E, I, O, are the $A_{\prime}$, $E_{\prime}$, $I_{\prime}$, $O_{\prime}$, of this work: and a, e, i, o, are the A', E', I', O'.

(A) *From the paper as sent to Cambridge before I had any communication whatsoever from Sir William Hamilton (without any corrections).*

SECTION III. *On the quantity of propositions.*

"The logical use of the word *some*, as merely 'more than none,' needs no further explanation. Exact knowledge of the extent of a proposition would consist in knowing, for instance in 'some Xs are not Ys' both what proportion of the Xs are spoken of, and what proportion exists between the whole number of Xs and of Ys. The want of this information compels us to divide the exponents of our proportion into 0, more than 0 not necessarily 1, and 1. An algebraist learns to consider the distinction between 0 and quantity as identical, for many purposes, with that between one quantity and another: the logician must (all writers imply) keep the distinction between 0 and a, however small a may be, as sacred as that between 0 and $1-a$; there being but the same form for the two cases. We shall now see that this matter has not been fully examined.

"Inference must consist in bringing each two things which are to be compared into comparison with a third. Many comparisons may be made at once, but there must be this process in every one. When the comparison is that of identity, of *is* or *is not*, it can only be in its ultimate or individual case, one of the two following :—' This X is a Y, this Z is the very same Y, therefore this X is this Z ; ' or else ' This X is a Y, this Z is not the very same Y, therefore this X is not this Z.' And collectively, it must be either ' Each of these Xs is a Y ; each of these Ys is a Z ; therefore each of these Xs is a Z ; ' or else ' Each of these Xs is a Y, no one of these Ys is a Z, therefore no one of these Xs is a Z.'

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* Sir W. Hamilton should have cited a few: but when he declares I have made elementary blunders, he does not give so much as a reference. The plan is a safe one.

" All that is essential then to a syllogism is that its premises shall mention a number of Ys, of each of which they shall affirm either that it is both X and Z, or that it is one and is not the other. The premises may mention more : but it is enough that this much can be picked out ; and it is in this last process that inference consists.

" Aristotle noticed but one way of being sure that the same Ys are spoken of in both premises ; namely, by speaking of all of them in one at least. But this is only a case of the rule : for all that is necessary is *that more* Ys *in number than there exist separate* Ys *shall be spoken of in both premises together.* Having to make $m+n$ greater than unity, when neither m nor n is so, he admitted only that case in which one of the two m or n, is unity and the other is anything except 0. Here then are two syllogisms which ought to have appeared, but do not,

[318]	Most of the Ys are Xs Most of the Ys are Zs	Most of the Ys are Xs Most of the Ys are not Zs
	∴ Some Xs are Zs	∴ Some of the Xs are not Zs

And instead of most, or $\frac{1}{2}+a$, of the Ys, may be substituted any two fractions which have a sum greater than unity. If these fractions be m and n, then the middle term is *at least* the fraction $m+n-1$ of the Ys. It is not really even necessary that all the Ys should enter in one premiss or the other : for more than the fraction $m+n-1$ of the whole may be repeated twice.

" And in truth it is this mode of syllogising that we are frequently obliged to have recourse to ; perhaps more often than not in our universal syllogisms. ' *All* men are capable of some instruction ; all who are capable of any instruction can learn to distinguish their right and left hands by name ; therefore all men can learn to do so.' Let the word *all* in these two cases mean only *all but one* and the books on logic tell us with one voice that the syllogism has particular premises, and *no conclusion can be drawn.* But in fact idiots are capable of no instruction, many are deaf and dumb, some are without hands: and yet *a* conclusion is admissible. Here m and n are each very near to unity, and $m+n-1$ is therefore near to unity. Some will say that this is a probable conclusion: that in the case of any one person it means there is the chance m that he can receive instruction, and n that one so gifted can be made to name his right and left hand: therefore $m \times n$ (very near unity) is the chance that this man can learn so much.

" But I cannot see how in this instance the probability is anything but another sort of inference from the demonstrable conclusion of the syllogism, which must exist under the premises given. Besides which, even if we admit the syllogism as only probable with regard to any one man, it is absolute and demonstrative in regard to the proposition with which it concludes.

" But this is not the only case in which the middle term need not enter universally: this however is matter for the next Section. I now go on to another point."

Extract II.

" I now take the two cases in which particular premises may give a conclusion: namely

$$I_{II} \qquad XY+ZY=XZ \qquad\qquad XY+Y{:}Z=X{:}Z \qquad O_{io}$$

on the supposition that the Ys mentioned in both premises are in number more than all the Ys. If Y_1 and Y_2 stand for the fractions of the whole number of Ys mentioned or implied in the two premises, and y_1 and y_2 for the fractions of the *ys* implied or mentioned, we shall by a [319] repetition of the process on $YX+YZ=XZ$ (the other being obtained in the course of the process) arrive at the following results or their counterparts: remembering that Y_1+Y_2 is greater or less than 1, according as y_1+y_2 is less or greater.

Designation.	Syllogism.	Condition of its existence.
I_{II}	$YX+Y\ Z=XZ$	Y_1+Y_2 greater than 1
O_{io}	$YX+Y{:}Z=XZ$	....
i_{oo}	$Y{:}X+Y{:}Z=xz$	....
O_{oi}	$X{:}Y+yz\ =X{:}Z$	Y_1+Y_2 less than 1
i_{ii}	$yx+yz\ =xz$	....
O_{oi}	$X{:}Y+yz\ \ X{:}Z$	....
I_{oo}	$X{:}Y+Z{:}Y=XZ$	....

(B) *Communication received on the 4th or 5th of November from Sir William Hamilton, being the pretext for his charge that I have, with injurious breach of confidence towards himself, and* *false dealing towards the public, appropriated his "Fundamental Doctrine of Syllogism" privately communicated to me: and, after the retraction of that charge, noticed in pages* 297, 8, *for the assertion that I have done the same thing unconsciously.*

"16, Great King Street,
November 2nd, 1846.

"DEAR SIR,—I have been longer than I anticipated in answering your last letter. I now send you a copy of the requisites for the prize Essay, which I gave out to my students at the close of last session. It will show you the nature of my doctrine of syllogism, in one of its halves. The other, which is not there touched on, regards the two wholes, or quantities in which a syllogism is cast. I had intended sending you a copy of a more articulate statement which I meant, at any rate, to have drawn up; but I have not as yet been able to write this. I will send it when it is done. From what you state of your system having 'little in common with the old one,' and from the contents of your First Notions, we shall not, I find, at all interfere, for my doctrine is simply *that of Aristotle, fully developed.*

"It will give me great pleasure if I can be of any use, in your investigations concerning the history of Logical doctrines. I have paid great attention to this subject, on which I found, that I could obtain little or no information from the professed historians of Logic; and my collection of Logical books is probably the most complete in this country. But, as I mentioned to you in my former letter, it is only in subordinate matters that in *abstract* Logic there has been any progress.

"I remain, dear Sir, very truly yours,
"W. HAMILTON."

[320] *Essay on the new Analytic of Logical Forms.*

Without wishing to prescribe any definite order, it is required that there should be stated in the Essay,—

1°. What Logic *postulates* as a condition of its applicability.

2°. The reasons why common language makes an *ellipsis* of the *expressed quantity*—frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought. [*This paragraph is the one on which Sir W. Hamilton principally relies.*]

3°. *Conversion of propositions*—on the *common* doctrine.

4°. Defects of this.

5°. *Figure* and *Mood* of Categorical syllogism, and Reduction, on *common* doctrine (General statement).

6°. Defects of this (General statement).

7°. The one *supreme Canon* of Categorical Syllogisms.

8°. The evolution, from this canon, of all the *species* of Syllogism.

9°. The evolution, from this canon, of all the *general laws* of categorical Syllogisms.

10°. The error of the *special laws* for the several Figures of Categorical Syllogism.

11°. *How many Figures* are there.

12°. What are the *Canons* of the *several Figures*.

13°. *How many moods* are there in all the Figures: showing in concrete examples, through all the Moods, the *unessential* variation which Figure makes in a syllogism.

(Those which follow 13° were wrong numbered.)

15°. What relation do the Figures hold to *extension* and *comprehension*.

16°. Why have the *second* and *third* Figures *no determinate major* and *minor* premises and *two* indifferent conclusions: while the *first* Figure has a *determinate major and minor premise*, and a single *proximate conclusion*.

17°. What relation do the Figures hold to *Deduction* and *Induction*.

N.B.—This Essay open for competition to all students of the class of Logic and Metaphysics during the last or during the ensuing session.

April 15th, 1846.

(C) *Extract from the Addition to my Paper, taken, as can be shown, from the papers which I gave the means of identifying in January last, and which papers (though I hold it immaterial) I assert to have been written before I received any logical communication from Sir William Hamilton. (To be compared with the extracts given in A).*

"Since this paper was written, I found that the whole theory of the syllogism might be deduced from the consideration of propositions in a form in which *definite quantity* of assertion is given both to the subject and the predicate of a proposition. I had committed this view to paper, when I learned from Sir William Hamilton of Edinburgh, that [321] he had for some time past publicly taught a theory of the syllogism differing in detail and extent from that of Aristotle. From the prospectus of an intended work on logic, which Sir William Hamilton has recently issued, at the end of his edition of Reid, as well as from information conveyed to me by himself in general terms, I should suppose it will be found that I have been more or less anticipated in the view just alluded to. To what extent this has been the case, I cannot now ascertain ; but the book of which the prospectus just named is an announcement, will settle that question. From the extraordinary extent of its author's learning in the history of philosophy, and the acuteness of his written articles on the subject, all who are interested in logic will look for its appearance with more than common interest.

" The footing upon which we should be glad to put propositions, if our knowledge were minute enough, is the following. We should state how many individuals there are under the names which are the subject and predicate, and of how many of each we mean to speak. Thus, instead of ' Some Xs are Ys,' it would be, ' Every one of a specified Xs is one or other of b specified Ys.' And the negative form would be as in ' No one of a specified Xs is any one of b specified Ys.' If propositions be stated in this way, the conditions of inference are as follows. Let the *effective number* of a proposition be the number of mentioned cases of the *subject*, if it be an affirmative proposition, or of the *middle term*, if it be a negative proposition. Thus, in ' Each one of 50 Xs is one or other of 70 Ys,' is a proposition, the effective number of which is always 50. But ' No one of 50 Xs is any one of 70 Ys ' is a proposition, the effective number of which is 50 or 70, according as X or Y is the middle term of the syllogism in which it is to be used. Then two propositions, each of two terms, and having one term in common, admit an inference when 1. They are not both negative. 2. The sum of the effective numbers of the two premises is greater than the whole number of existing cases of the middle term. And the excess of that sum above the number of cases of the middle term is the number of the cases in the affirmative premiss which are the subjects of inference. Thus, if there be 100 Ys, and we can say that each of 50 Xs is one or other of 80 Ys, and that no one of 20 Zs is any one of 60 Ys ;—the effective numbers are 50 and 60. And $50+60$ exceeding 100 by 10, there are 10 Xs, of which we may affirm that no one of them is any one of 20 Zs mentioned.

" The following brief summary will enable the reader to observe the complete deduction of all the Aristotelian forms, and the various modes of inference from *specific particulars*, of which a short account has already been given.

" Let a be the whole number of Xs ; and t the number specified in the premiss. Let c be the whole number of Zs ; and w the number specified in the premiss. Let b be the whole number of Ys ; and u and v the numbers specified in the premises of x

and z. Let $X_t\,Y_u$ denote that each of t Xs is affirmed to be one out of u Ys: and $X_t{:}Y_u$ that each of t Xs is denied to be any one out of u Ys. Let $X_{m,n}$ signify m [322] Xs taken out of a larger specified number n; and so on. Then the five possible syllogisms, on the condition that no contraries are to enter either premises or conclusion, are as follows:—

1. $X_tY_u+Z_wY_v=X_{t+w-b,t} \quad Z_w=Z_{t+w-b,w}X_t$.
2. $X_tY_u+Y_vZ_w=X_{t+v-b,t} \quad Z_w=Z_{t+v-b,w}X_t$.
3. $Y_uX_t+Y_vZ_w=X_{u+v-b,t} \quad Z_w=Z_{u+v-b,w}X_t$.
4. $X_tY_u+Z_w{:}Y_v=X_{t+v-b,t}:Z_w$.
5. $Y_uX_t+Z_w{:}Y_v=X_{u+v-b,t}:Z_w$.

"The condition of inference expresses itself; in the $X_{m,t}$ of the conclusion, m must neither be 0 nor negative. The first case gives no Aristotelian syllogism; the middle term never entering universally (of necessity) into any of its forms, under any degree of specification which the usual modes of speaking allow. The other cases divide the old syllogisms among themselves in the following manner: they are written so as to show that there is sometimes a little difference of amount of specification between the results of different figures, which changes in the reduction from one figure to another. The Roman numerals mark the figures.

2.	$t=a,\ v=b$	$Y)Z_w+X)Y_u=X)Z_{u,w}$	*Barbara* I.
	$t=a,\ v=b$	$X)Y_u+Y)Z_w=Z_{u,w}X$	*Bramantip* IV.
	$t<a,\ v=b$	$Y)Z_w+X_tY_u=X_tZ_{u,w}$	*Darii* I.
	$t<a,\ v=b$	$X_tY_u+Y)Z_w=Z_{t,w}X_t$	*Dimaris* IV.
3.	$u=b,\ v=b$	$Y)X_t+Y)Z_w=Z_{b,w}X_{b,t}$	*Darapti* III.
	$u<b,\ v=b$	$Y_uX_t+Y)Z_w=Z_{u,w}X_{u,t}$	*Disamis* III.
	$u=b,\ v<b$	$Y)X_t+Y_v)Z_w=Z_{v,w}X_{x,t}$	*Datisi* III.
4.	$t=a,\ v=b,\ w=c$	$Y.Z+X)Y_u=X.Z$	*Celarent* I.
	$t=a,\ v=b,\ w=c$	$Z.Y+X)Y_u=X.Z$	*Cesare* II.
	$t=a,\ v=b,\ w=c$	$X)Y_u+Z.Y=Z.X$	*Camestres* II.
	$t=a,\ v=b,\ w=c$	$X)Y_u+Y.Z=Z.X$	*Camenes* IV.
	$v=b,\ w=c$	$Y.Z=X_tY_u=X_t{:}Z$	*Ferio* I.
	$v=b,\ w=c$	$Z.Y+X_tY_u=X_t{:}Z$	*Festino* II.
	$t=a,\ v=b,$	$X)Y_u+Z_w{:}Y=Z_w{:}X$	*Baroko* II.
5.	$u=b,\ v=b,\ w=c$	$Y.Z+Y)X_t=X_{b,t}:Z$	*Felapton* III.
	$u=b,\ v=b,\ w=c$	$Z.Y+Y)X_t=X_{b,t}:Z$	*Fesapo* IV.
	$v=b,\ w=c$	$Y.Z+Y_uX_t=X_{u,t}:Z$	*Feriso* III.
	$u=b,\ w=c$	$Y_v{:}Z+Y)X_t=X_{v,t}:Z$	*Bokardo* III.

I conclude by submitting to the reader what I began with, namely, that until Sir William Hamilton produces something from C, intelligibly hinted at in B, and neither substantially contained in the matter, nor [323] immediately deducible from the principles, of A, he has no right whatever to assert that I have borrowed from him consciously or unconsciously. I have not found any person who thinks that such a thing can be produced : and I leave every reader to form his own opinion whether it can be done or not.

APPENDIX II.

On some forms of inference differing from those of the Aristotelians.

I THINK it desirable to state all I know of any attempt to deal with the forms of inference otherwise than in the Aristotelian method. Since the time of Wallis, three well known mathematicians have written on the subject, Euler, Lambert, and Gergonne: there may have been others, but I have not met with them.

Euler's "Lettres à une Princesse d'Allemagne sur quelques sujets de Physique et de Philosophie" (3 vols. 8vo. Petersburg, 1768–1772, according to Fuss) contain the representation of the syllogism by *sensible terms,* namely, areas. There was a Paris edition by Condorcet and Lacroix, in 1787, as is stated by Dr. Henry Hunter, who published an English translation from it and from the original edition, London, 1795, 2 vols. 8vo. Euler makes use of circles to represent the terms. In a tract published (or completed) in 1831, in the Library of Useful Knowledge, under the name of "the Study and Difficulties of Mathematics" I fell upon this method before I knew what Euler had done, using, for distinction, squares, circles, and triangles, as in Chapter I. of this work. The author of the "Outlines" presently mentioned, has what I consider a very happy improvement on Euler. The proposition "some X is Y," is represented by the latter as the circle of X, partly inside and partly outside the Y. The author of the "Outlines" puts a broken segment of the circle of X inside the circle of Y, leaving it unsettled whether the rest of the circle is united to the broken piece, or transferred elsewhere.*

But Euler had been preceded in the publication of this idea by Lambert, in his "Neues Organon, &c." Leipzig, 1764, 2 vols. 8vo. In this work, the terms are represented by lines, and identical extents by parts of the lines vertically under one another, as in page 79. The whole notion is represented by continuous line, the part left indefinite in particular propositions by dotted line. Some of the contranominal forms are more distinctly mentioned than is usual, but there is no introduction that I can find of any form of inference which is not Aristotelian.

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* I should say that Euler does not use the numerical, but the magnitudinal notion (see page 48 of this work).

[324] In the seventh volume of the *Annales de Mathématiques* (Nismes, 1816 and 1817, 4to.) there is a paper by the editor, M. Gergonne, entitled *Essai de dialectique rationelle.* I did not see this paper, nor Lambert's work, until after my memoir in the Transactions of the Cambridge Society had been published. The second would have given me no hint: the first might have done so. There is the idea, and some formal use, of a complex proposition: but the division is erroneous. The subidentical, identical, and superidentical forms are there; these are not easily missed: the others which Gergonne uses are, the *complete exclusion* (the *contrary or subcontrary* of my system, which, disjunctively, are only the common universal negative) and *partial inclusion with partial exclusion* (the *complex particular, or supercontrary*, of mine). The use of contraries is expressly * forbidden, the old conversion by contraposition formally declared *false*, and the particular proposition asserted to be incapable of being made universal. But M. Gergonne's complex propositions, such as they are, are used in a manner resembling that in chapter V, of this work, though requiring a separate *tâtonnement* for many things the analogues of which appear as connected results of my system. Accordingly, I am bound to attribute to M. Gergonne the first publication of the idea of *a* complex syllogism, and of the comparison of the simple one with it. But numerical statement is not hinted at.

Sir William Hamilton's system dates, as to its publication in lectures, from 1841, as far as has yet appeared. What I have to say of it will be found in another appendix.

In 1842, there was published anonymously "Outline of the laws of thought"; London and Oxford (Pickering and Graham) *octavo in twos* (small). The author is the Rev. Wm. Thomson, tutor of Queen's College, Oxford. It is a very acute work, and learned. The system of propositions is extended by the introduction of both the common quantifications of the predicate into the *affirmatives only*, which introduces the propositions U and Y, as the author calls them, or "All Xs are all Ys," and "Some Xs are all Ys."

The memoir in the Cambridge Transactions in which I gave the first account of what has since grown into Chapters IV, V,

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* I am told that some works on logic used in the Irish colleges formally announce that the truth of the [ordinary] laws of syllogism depends upon the exclusion of contraries: but I have not met with any of them.

VIII, and X, of this work, is described as to date in the preceding appendix. With reference to the subject of chapter V, I may note the following defects of that memoir: 1. That only one arrangement of X and Z as premises being taken, only half the system is given, and many correlative arrangements are not obtained (see page 140). 2. That owing to my not seeing distinctly that each universal proposition has *two* weakened forms, the syllogisms $A_{\prime}A'I'$ and $E'E'I_{\prime}$ are considered as a class apart. 3. That much of the power of forming easy rules is not gained, by the order of reference being made XY, ZY, XZ, instead of XY, YZ, XZ. The former appears at first the more natural order, and is certainly [325] more easily described; namely, to refer each of the concluding terms to the middle term, with which both are compared. I observe, since, that M. Gergonne adopts this last order of reference: but the other is by an immense deal more convenient in its results, as I think I have shown.

With respect to the numerical quantification, what I did in the *Memoir* and *Addition* is given in full in the preceding appendix. Sir William Hamilton, who distinctly renounces all claim to the "arithmetically articulate" system, and doubts whether it afford any basis for a logical development, states that he had formerly obtained the "ultratotal quantification" (page 317) and thrown it away as a cumbrous and useless subtlety, without publishing it, as I understand, in any way. To his reply, he appends a note which I think it desirable to republish at length, as a document in the history of this speculation, and that I may make that history complete (II. p. 41).

"I have avoided, in the previous letter and postscript, all
" details in regard to the *third* scheme of quantification (p. 32);
" because that scheme except in so far as it is confounded with
" the *second*, has no bearing in the controversy; and I admit
" that whatever Mr. De Morgan has therein accomplished, he
" has accomplished independently of me. Further, I shall not
" deny him any claim of priority to whatever he may have stated
" in our correspondence, in reference to this third scheme.
" Finally, I shall acknowledge, for I think it not improbable,
" that his syllogism (p. 19) suggested a reconsideration, on my
" sickbed, of a certain former speculation, in regard to the ultra-
" total quantification of the middle term in both premises
" together;—a speculation determined by the vacillation of the
" logicians, touching the predesignations *more, most,* &c. but
" which I had laid, aside, as a useless and cumbrous subtlety.

" Aristotle, followed by the logicians, did not introduce into

" his doctrine of syllogism, any quantification between the abso-" lutely universal and the merely particular predesignations, " for valid reasons.—1°, Such quantifications were of no value " or application in the one whole (the universal, potential, " logical), or, as I would amplify it, in the two correlative and " counter wholes (the logical,—and the formal, actual, meta-" physical,) with which Logic is conversant. For all that is out " of classification, all that has no reference to genus and species, " is out of Logic, indeed out of Philosophy ; for Philosophy tends " always to the universal and necessary. Thus the highest " canons of deductive reasoning, the *dicta de Omni et de Nullo*, " were founded on, and for, the procedure from the universal " whole to the subject parts ; whilst, conversely, the principle " of inductive reasoning was established on, and for, the (real " or presumed) collection of all the subject parts as constituting " the universal whole.—2°, The integrate or mathematical whole, " on the contrary, (whether continuous or discrete) the philo-" sophers contemned. For whilst, as Aristotle observes, in " mathematics genus and species are of no account ; it is, almost " exclusively, in the mathematical whole, that quantities are " compared together, through a middle term, in neither premise, " equal to the whole. But this reasoning, in [326] which the " middle term is never universal, and the conclusion always " particular, is,—as vague, partial, and contingent,—of little " or no value in philosophy. It was accordingly ignored in " Logic ; and the predesignations *more, most*, &c., as I have said, " referred, to universal, or, (as was most common) to particular, " or to neither, quantity. This discrepancy among Logicians " long ago attracted my attention ; and I saw, at once, that the " possibility of inference considered absolutely, depended, exclu-" sively on the quantifications of the middle term, in both pre-" mises, being, together, more than its possible totality—its " distribution, in any one. At the same time I was impressed " —1°, with the almost utter inutility of such reasoning, in a " philosophical relation : and 2°, alarmed with the load of valid " moods which its recognition in Logic would introduce. The " mere quantification of the predicate, under the two pure quan-" tities of *definite* and *indefinite*, and the two qualities of *affirma-" tive* and *negative*, gives (abstractly) in each figure, *thirty-six* " valid moods ; which, (if my present calculation be correct,) " would be multiplied, by the introduction of the two hybrid " or ambiguous quantifications of *a majority* and *a half*, to the " fearful amount of *four hundred and eighty* valid moods for each " figure. Though not, at the time, fully aware of the strength

“ of these objections, they however prevented me from breaking down the old limitation; but as my supreme canon of Syllogism proceeds on the mere formal possibility of reasoning, it of course comprehends all the legitimate forms of quantification. It is;—*What worst relation of subject and predicate, subsists between either of two terms and a common third term, with which one, at least, is positively related;—that relation subsists between the two terms themselves:* in other words;—*In as far as two notions both agree, or one agreeing, the other disagrees, with a common third notion:—in so far, those notions agree or disagree with each other.* This canon applies, and proximately, to all categorical syllogisms,—in extension and comprehension,—affirmative and negative,—and of any figure. It determines all the varieties of such syllogisms; is developed into all their general, and supersedes all their special, laws. In short, without violating this canon, no categorical reasoning can, formally, be wrong. Now, this canon supposes that the two extremes are compared together, through the *same common middle;* and this cannot but be, if the middle, whether, subject or predicate, in both its quantifications together, exceed its totality, though not taken in that totality in either premise.

“ But, as I have stated, I was moved to the reconsideration of this whole matter; and it may have been Mr. De Morgan's syllogism in our correspondence (p. 19), which gave the suggestion. The result was the opinion, that these two quantifications should be taken into account by Logic, as authentic forms, but then relegated, as of little use in practice, and cumbering the science with a superfluous mass of moods. As to Mr. De Morgan's statement in our correspondence (p. 21) of the principle on which (by his later system) such syllogisms proceed, this, to use his own expression, ‘ I did not comprehend at [327] all; ’ nor do I now,* having, to speak with the Rabbis,

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* The passages which Sir William Hamilton does not understand, are the following, and also that relating to the effective terms, in C of the preceding appendix.

“ Now suppose propositions in which the quantitative part of the preceding is made more definite. Say that

$X_t \quad Y_u$ |	and $X_t : Y_u$ |
mean	
Every one of t Xs is one or other of u Ys	No one of t Xs is any one of u Ys

Let the *effective number* of cases in a proposition be the number which it makes effective in inference. Then the effective number in a positive proposition is the number of cases of the *subject.*

The effective number in a negative proposition is the number of cases of the *middle term.*

And the criterion of inference being possible, is that the sum of the effec-

" 'reserved it for the advent of Elias.' I saw however, that, " be it what it might, it had no analogy with mine ; indeed, " even from the fuller exposition of his doctrines, contained in " the body of the Cambridge Memoir and its Addition, which I " afterwards received, I can find no indication of his having " generalised either, 1° *the comprehensive principle of all inference, that the two quantifications of the middle term, should, together, exceed it as a single whole ;* or, 2°, *under a non-distributed middle, the* TWO *exclusive forms of its quantification.* On " receipt, however, of Mr. De Morgan's Cambridge Memoir, I " saw, or thought I saw, in the body of the paper, on his *old* " view, some manifestation of a less erroneous doctrine upon " this point, than that afterwards contained in his Letters and " Addition, upon his *new*. Accordingly, to obviate all misconstruction, I wrote immediately the following letter,* of which " an account has been previously given (p. 26, note).

EDINBURGH, 30*th March*, 1847.

" Your paper read to the Society I have cursorily perused ; " but though opposed to many of its doctrines, I admire the " ingenuity which charac-[328]terises it throughout. On one " point, I find we coincide, in principle, at least, against logicians " in general. They have referred the quantifying predesignations *plurimi*, and the like, to the most opposite heads ; some " making them universal,—some, particular,—and some between " both ; (for you are not correct in saying, (p. 6), that logicians " are unanimous in regarding them as particular, [though most

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tive numbers of the two premises (not both negative) is greater than the *whole number of cases* of the *middle term.*

And the excess is the number of cases involved in the inference, of all which are mentioned in the conclusion-term (or terms) of the positive premiss (or premises).

For instance, let b be the whole number of Ys in existence : I ask whether we can infer anything from

$X_t \quad Y_u$ effective number t
$Z_w \quad Y_v$ v

Answer, if t+v be greater than b, we can infer

$$X_{t+v-b} : Z_w$$

Or, if each of t Xs be one or other of u Ys, and no one of w Zs be any one of v Ys, then if t and v together are more in number than there are Ys, we may infer that no one of t+v—b Xs is any one of the w Zs just spoken of."

[*] This letter (the first paragraph of which is omitted, as not relevant to *this* appendix), was addressed to me, and was sent open to my friend Dr. Sharpey, to be delivered to me. Dr. Sharpey refused to deliver (and, as it happened, I was as much prepared to refuse to receive) any thing on the literary subject matter of the controversy which did not contain a retraction of Sir W. Hamilton's then subsisting charge against me. Accordingly, I never saw it till it appeared in print.

" do]). This confliction attracted my attention ; and a little
" consideration showed me, that besides the quantification of
" the pure quantities, *universal* and *particular*, (which I call
" *definite* and *indefinite*,) there are two others of these, mixed
" and half developed, which ought to be taken into account by
" the logician, as affording valid inference ; but which, without
" scientific error, cannot be referred either to *universal*, (definite,)
" or to *particular*, (indefinite) quantity, far less left to vacillate
" ambiguously between these. I accordingly introduced them
" into my modification, in English doggerel, of ' *Asserit A*,' *&c.*,
" which [in the original cast] I formerly said was at your service ;
" and as it affords a brief view of my doctrine on this point, I
" may now quote it.

" A, it affirms *of this, that, all*,[*]
Whilst E denies of *any* ;
I, it affirms, whilst O denies,
Of *some* (or few or many).

" Thus A affirms, as E denies,
And definitely either ;
Thus I affirms, as O denies,
And definitely neither.

" A *half*, left semi-definite,
Is worthy of its score ;
U, then, affirms, as Y denies,
This, neither less nor more.

" Indefinito-definites,
To UI, YO, last we come ;
And that affirms, and this denies,
Of *more, most*, (half plus some).

" ' The rule of the logicians, that the middle term should be
" ' once at least distributed [or indistributable,] (*i.e.* taken
" ' universally or singularly, =definitely,) is untrue. For it is
" ' sufficient, if, in both the premises together, its quantification
" ' be more than its quantity as a definite whole. (Ultratotal) '
" ' - - - - - - - - ' It is enough for a valid syllogism, that the two
" ' extreme notions should (or should not), of necessity, partially
" ' coincide in the third or middle notion ; and this is necessarily
" ' shown to be the case, if the one extreme coincide [329] with
" ' the middle, to the extent of a half, (dimidiate quantification) ;
" ' and the other, to the extent of aught more than a half, (ultra-
" ' dimidiate quantification). - - - - -

" ' The first and highest quantification of the middle term (. .)
" ' is sufficient not only in combination with itself, but with any
" ' of all the three inferior. The second (. ,) suffices, in com-

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[*] Better : " A, it affirms of *this, these, all*."

" ' bination with the highest, with itself, and with the third, but " ' not with the lowest. The third (.) suffices, in combination " ' with either of the higher, but not with itself, far less with the " ' lowest. The fourth and lowest (,) suffices only in combination with the highest. [1. Definite ; 2. Indefinito-definite ; " ' 3. Semi-definite ; 4. Indefinite.] ' "

" Of the effect of this new system of quantification in amplifying the syllogistic moods, (which in all the figures remain the " same,) I say nothing. It should be noted, however, that the " letters A, E, &c. do not mark the quantification [and qualification] of *propositions*, (as of old) but of *propositional terms*. " The sentences within inverted commas are taken from notes " for the ' Essay towards,' &c.

" Before concluding, I ought to apologise, in the circumstances, " for the details, that have insensibly lengthened out, of a part " of my doctrine, which I have found, to a certain extent, coincident with what appears in your paper. I was anxious, " however, that you and others should have no grounds for surmising, that I borrowed any thing from my predecessors without due acknowledgment.—On second thoughts, however, I " deem it more proper to make this communication through a " third party."

The discussion between Sir William Hamilton and myself called a very able third party into the field, who addressed the following letter to the editor of the *Athenæum*, in which journal it was published, June 19, 1847.

" Sir,—As two great logical innovations—the one due to Sir " William Hamilton, the other due to Mr. De Morgan—used in " conjunction, have led me to the simplest and most general " formulæ of syllogism that ever have been given (formulæ which " correct a serious mistake into which both Sir William Hamilton " and Mr. De Morgan have fallen), I think it will gratify those " interested in logical science if you would give them publicity " through your columns.

" n^{I}, n^{II}, n^{III}, &c. are any numbers. When placed before a " term, as $n^{\mathrm{II}}xs$, n^{II} marks the total number of the class x; placed " before a proposition, it marks the number of things of which " we mean to speak. Thus, n^{I}, of $n^{\mathrm{II}}xs$ are of $n^{\mathrm{III}}ys$, means " that a number of things n^{I} are alleged to have both the characteristics x and y; and are to the whole class of xs as n^{I} to n^{II}, " and to the whole class of ys as n^{I} to n^{III}: similarly with the " negative proposition n^{I} of $n^{\mathrm{II}}xs$ are not of $n^{\mathrm{III}}ys$, n^{I} things " being here said to have the characteristic x, and to want the " characteristic y. It is clear, from the nature of a proposition,

“ that in affirmatives, n^{I} can never be greater than the least
“ extensive of the terms, and in negatives never greater than
“ the number of the class whose characteristic it is said to have.
“ But within these limits the pro-[330]portion n^{I} to n^{II} may be
“ wholly undetermined ; we then mark it with the word *some*,
“ —we call this, with Sir William Hamilton, indefinite quantity.
“ It may be perfectly determined ; as of equality when we mark
“ it with *all*, *every*, or, following Mr. De Morgan, any other arith-
“ metical proportion—as a half. (Sir William Hamilton has
“ erred in calling a half, semi-definite ; it is thoroughly definite).
“ All this we call definite quantity. Lastly, the indefinitude
“ may be reduced within limits—indefinito-definite, as *most*, &c.

“ The first formula contains all syllogisms with an affirmative
“ conclusion, without any exception.

“ I. n^{I} of $n^{\mathrm{II}}xs$ are of $n^{\mathrm{III}}ys$
“ n^{IV} of $n^{\mathrm{V}}zs$ are of $n^{\mathrm{III}}ys$
“ $(n^{\mathrm{I}}+n^{\mathrm{IV}}-n^{\mathrm{III}})$ of $n^{\mathrm{II}}xs$ are of $n^{\mathrm{V}}zs$

“ As Sir William Hamilton's principle takes away all distinc-
“ tion of subject and predicate in affirmative propositions, it
“ will be seen that, by varying the proportions of the symbols,
“ n^{I}, &c., every possible affirmative logical inference, in whatever
“ mood or figure, emerges.

“ The syllogisms with negative questions or conclusions, are
“ not so simple. They fall into two divisions, according as, in
“ the negative premiss, the things spoken of have the character-
“ istic of the extreme, or of the middle ; and from each of these,
“ *two* conclusions, not *one*, are drawn, according as the things
“ to be spoken of in the conclusion have the characteristic of the
“ extreme in the affirmative premiss, or of that in the negative
“ premiss.

“ II. n^{I} of n^{II} xs are of $n^{\mathrm{III}}ys$
“ n^{IV} of n^{V} zs are not of $n^{\mathrm{III}}ys$ concludes ;
“ doubly 1°$(n^{\mathrm{I}}+n^{\mathrm{IV}}-n^{\mathrm{V}})$ of $n^{\mathrm{II}}xs$ are not of $n^{\mathrm{V}}zs$
“ 2°$(n^{\mathrm{I}}+n^{\mathrm{IV}}-n^{\mathrm{II}})$ of $n^{\mathrm{V}}zs$ are not of $n^{\mathrm{II}}xs$.

“ It is to this formula I referred as correcting a serious error
“ into which Sir William Hamilton and Mr. De Morgan have
“ fallen—of holding, as a general principle of all inference, that
“ the two quantifications of the middle term should exceed it
“ as a whole ; for this syllogism proceeds wholly irrespective
“ of the total quantity of the middle, which is excluded from our
“ symbolic conclusion.

“ III. n^{I} of $n^{II}xs$ are of $n^{III}ys$
“ n^{IV} of $n^{III}ys$ are not of $n^{V}zs$ concludes ; also,
“ doubly 1° $(n^{IV}+n^{I}-n^{III})$ of $n^{II}xs$ are not of $n^{V}zs$
“ 2° $(n^{IV}+n^{I}+n^{V}-n^{III}-n^{II})$ of $n^{V}zs$ are not of $n^{II}xs$.

“ Such are the three symbolical formulæ of every possible
“ logical inference. I have the demonstrations that these are
“ in all their extent valid, and are the only possible forms ;
“ but it is sufficient to give here the results.

“ It will surprise no one who considers that the negative pro-
“ position is not converted in the same sense as the affirmative,
“ that the negative syllogistic formulæ are not reducible to one.
“ For the rule of negative [331] conversion changes the things
“ spoken of, and is as follows : n^{I} of $n^{II}xs$ are not of $n^{III}ys$;
“ converts $(n^{III}+n^{I}-n^{II})$ of $n^{III}ys$ are not of $n^{II}xs$. The conse-
“ quence of a form universally true, $(n^{III}-n^{II})$ of $n^{III}ys$ are not
“ of $n^{II}xs$. As to the two conclusions, they are but the converse
“ of each other.

“ It will not be difficult to interpret these, by $n^{I}=n^{II}$ as *every*
“ or $n^{I} : n^{II}$ indefinite=*some*, &c. The usual Aristotelic forms
“ will be seen to be derived from them. Thus the mood Cesare,
“ and the corresponding indirect mood (or, if you will, the mood
“ of the fourth figure, call it at another time Celantes or Cadere
“ at will, but let it be Celantes for the nonce), come forth from
“ the third formula.

“ $n^{IV}=n^{III}$ gives no y is z ... $n^{IV}:n^{V}$ indefinite
“ $n^{I}=n^{II}$ every z is y ... $n^{I}:n^{III}$ indefinite.

“ Hence in Cesare, no x is z from our first,
“ and in Celantes, no z is x from our second conclusion, and so
“ of all the others.

“ I owe it to Sir William Hamilton and Mr. De Morgan to say
“ that without their improvements I could not have advanced
“ one step. Mr. De Morgan has even attempted a like reduction
“ to general formulæ, and has failed, chiefly through a misappre-
“ hension of Sir William Hamilton's principle of quantified
“ predicate. He has introduced a superfluous quantity,—one
“ logically useless, or worse than useless, as the result has shown.
“ This confusion explains his errors. Had it not been for this
“ circumstance, I should not have had the honour of presenting
“ these formulæ to logicians.

“ Permit me to add what I think also of some value. I am
“ not of those who think with Sir William Hamilton that the
“ syllogism always proceeds in the two counter wholes of inten-

" sion and extension—that it must always be an involution or " evolution in respect of classification. This is, no doubt, true " in the most important reasonings of science ; but it is not " scientifically accurate to assert this universally.

" Quality, which is the comprehensive element, is of three " kinds—not two, as heretofore affirmed ; for since Kant, the " division of affirmatives into analytic and synthetic, or (as " Sir William Hamilton wishes) explicative and ampliative, has " been established. James Bernouilli has puzzled himself to " reduce these two to the same form, but without success ; for " that contains an immediate relation of part to whole, and only " a remote one of part to part, while this contains an immediate " relation of part to part, and remote of part to whole. These, " as distinct kinds of quality, are erroneously elided in language. " As the words *ampliative* and *restrictive* are generally opposed " in logic, perhaps we might replace the old division of proposi- " tions, according to quality, into affirmative and negative— " by one into *Explicative, Ampliative,* and *Restrictive.*

" Where, then, both premises are ampliative, the syllogism " proceeds purely by force of extension. There is neither involu- " tion nor evolution—neither induction nor deduction—but a " passage or transition from one mark to another, or from class " to class. Of this kind are all singular, or, as Ramus calls " them, proper syllogisms. Let us call this new [332] class of " syllogisms traductive, to contrast it with the inductive and " deductive.

" The use of these in philosophy as independent modes of " inference will easily appear. When we collect the scattered " fragments of our knowledge into unity of science, we use " *induction and inductive syllogism ;* when we apply the prin- " ciples of science to special events of things, we use *deduction* " *and deductive syllogism ;* but when, abandoning one scheme of " classification, we transfer our knowledge *directly* to another, " we use *traduction and traductive syllogism.* Thus, in political " science, what has been predicated by historians of men classed " geographically is transferred to men classed according to con- " stitutions of government by traduction. This last escapes " Sir William Hamilton's rule, and never concludes through a " comprehensive containing and contained.

" I shall not add, at present, any attempt to prove *à priori* the " exclusive validity of syllogistic inference.

" I admit that I ought not, without good ground, to dissent " from a matured opinion of Sir William Hamilton in any part " of philosophy, still more in logic ; but I obey the force of

" demonstration,—and, as Ludovicus Vives said in respect to
" Aristotle, *Verecundè dissentio.*

"' Yours, &c.
" James Broun.

" *Temple, June* 9, 1847."

My reply to this consisted in forwarding, on the same 19th of June, to the editor of the *Athenæum*, a summary of the results of chapter VIII, then written. This summary appeared on the 26th: I do not insert it, because the chapter in question is a better answer; and though the publication saved my rights, the republication is unnecessary. Mr. Broun's three forms are the first (without the contranominal), the ninth, and the eleventh, of page 161. Mr. Broun was wrong in deducing from the two latter forms that the principle of the middle term was erroneous: for in these very forms the two quantifications exceed the whole: being the whole (in premise one) plus some (in the other). As to the superfluous quantity, it only becomes superfluous when such quantifications are introduced as distinguish spurious from admissible propositions: see pages 145, 146, in which it is shown that the forms are correct.

Nothing but close comparison, and that after practice, would detect the accordance of the two symbolic modes of expression in pages 145 and 161. I am not therefore surprised that Mr. Broun should, having obtained cases of that in page 161, pronounce that in page 145 erroneous.

In the answer which I made, I promised to state distinctly how much of the chapter was written before Mr. Broun's letter appeared. This I now do. With the exception of pages 145, 146, the matter of which is mostly from my Cambridge Memoir, the whole of it was then written, excepting such verbal alterations and occasional introduction of sentences, as take place at the press, or at the last reading of the manuscript. I had [333] thought that there would be no necessity to introduce those pages, except slightly, and in answer to certain objections which seemed likely to occur. The examination which the assertion that they are erroneous made me give my previous forms, pointed out the desirableness of introducing them as they now stand.

September 17, 1847. I had finished the preceding appendix, when I became aware of the existence of the " Commentationes Philosophicæ Selectiores " of Godfrey Ploucquet, of Tubingen, Utrecht, 1781, quarto. The last title (p. 561) is " De Arte Characteristica. Subjicitur Methodus calculandi in logicis, ab auctore inventa. 1763." I find by a catalogue * that this *methodus calculandi* had been previously published in 1773, at Tubingen, at the end of a work entitled "Principia de Substantiis et Phænomenis:" also that the "Methodus demonstrandi directé omnes syllogismorum species" of the same author (which is probably the thing I am going to describe) was published at Tubingen in 1763. From the title of a work which, I am informed, exists, namely, "Sammlung der Schriften welche von logischen Calcul des Prof. Ploucquet betreffen," Tubingen, 1773, one would suppose that this system had obtained great local currency. I give a short account of it: premising that Ploucquet appears to have been a well informed mathematician, much given to pure speculation on mental subjects.

The *calculus* (a term which Ploucquet uses in as wide a sense as I do when I call the contents of Chapter V. a part of the calculus of inference) consists in the invention of a simple notation, and the mechanical substitution, in one premise, of an identical equivalent to the middle term therein contained, taken from the other premise (this last being one in which the middle term is universal). There is neither use of contraries, nor numerical definition: but there is every variety of quantity of the predicate which can be produced by simple conversion of the ordinary forms. A term used universally is denoted by the capital letter; particularly, by the small letter: affirmation by juxtaposition; negation, by interposing ⊳. Thus X)Y is Xy; X.Y is X⊳Y; XY is xy; X:Y is x⊳Y. The following is a complete specimen:

Sint præ- Pm
missæ s⊳M
———
Calculo: s⊳mP quoddam s non est P
Omnis ducatus est aureus
Quædam moneta non est aurea.
———
Da
m⊳A

Calc. m⊳aD. feu m⊳D, quædam moneta non est ducatus.

As Ploucquet seems to think that this actual application of the *calculus* to concrete instances, by aid of their initial letters, is a material part of [334] his system, I have inserted the case entire. The rationale of the system consists in that substitution of identicals for each other, which I understand Sir William Hamilton (with perfect truth) to employ in every case. Thus we have in the above " Some of the Ss are not any Ms, are not those Ms which make up all the Ps, are not therefore any Ps." This demand for identical substitutes requires both kinds of quantity for every predicate, and Ploucquet uses them accordingly, as far as wanted to establish the Aristotelian syllogisms. Sir W. Hamilton goes further, and invents syllogisms for all the kinds of quantity. Thus Ploucquet uses mP or " some Ms are all the Ps " and P⊳ m or " all Ps are not some of the Ms ; " but not MP or p⊳ m.

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* The *second edition* of Mr. Blakey's "Essay on Logic" recently published, contains a catalogue of upwards of a thousand works on logic, briefly titled.

At the same time with the knowledge of Ploucquet I obtained that of the work of a follower and extender, M. W. Drobitsch, author of " Neue Darstellung der Logik . . . Nebst einen logisch-mathematischen Anhange," Leipzig, 1836, octavo. As far as the symbolic part is concerned, Mr. Drobitsch begins by a convention which would reconcile any one to the sound, not merely of *Barbara* and *Celarent*, but even of *Baroko* and *Fresison*. He makes S and P the subject and predicate of the conclusion and M the middle term ; and puts the Aristotelian vowel between them : thus S)P is SAP, and P:S is POS. Hence his premises may be *map sam* or *mop sam ;* and one of his syllogisms is *mepsamsep*. In the *algebraical* part, he uses large and small letters for the universal and particular, or for the whole and part extent of a term. He also introduces the signs = and ⊲ to signify identity and (what I call) subidentity. This use of the mathematical signs involves an extension, which is made by all those who signify the identity of X and Y by X=Y. The mathematician thinks of extent as quantity only : the logician includes both quantity and position. Thus when the former says that five feet are less than seven feet, he means any five feet, be they part of the seven feet or not : the latter, when he says that X is a name of less extent than Y, means not only that the former *can be* contained in the latter, but *that it is*. To make negative propositions, Mr. Drobitsch takes a limited universe (call it U, as I have done) an extent greater than the utmost extent of all the names, otherwise indefinite. And here he falls into some confusion : X and Y being the names, he says U must be of greater extent than X+Y: now if we had X)Y, U need only be of greater extent than Y. If from the genus Y be taken all the species X, the remainder is denoted by Y—X. Accordingly, the contrary of X is U—X.

Mr. Drobitsch then lays down eight forms of predication, of which, however, he only uses the ordinary ones. And I cannot find out that the limited universe, or the contrary, has any use except to furnish means of notation. The eight forms are;—first, $X=y$, or my $X)Y$; secondly, $X=Y$, or $X)Y+Y)X$; thirdly, $x=y$, or XY; fourthly, $u=Y$, or $Y)X$; fifthly, $X \lhd U-Y$, which tells us that X is all contained in what is left of the universe after Y is removed, or is $X.Y$; sixthly, $X=z \lhd Z \lhd U-Y$, a very roundabout way of saying that X is *sub*contrary of Y, or $X.Y+xy$; seventhly, $x=U-Y$ or $X{:}Y$; eighthly, [335] $x=X-Y$, which tells us that Y is a subidentical of X, or $Y)X+X{:}Y$.

This is in fact a mixture of two systems, both in principle and notation. The forms are $A_{\prime}$, A', $O_{\prime}$ (and O'), $E_{\prime}$, $I_{\prime}$, D, $D_{\prime}$ (and D'), and $C_{\prime}$. Also C is virtually given: but E', I', C', do not appear. The ordinary rules under which the mathematicians use $=$ and $\lhd$, remain true in this logical use of them: and thus there is an elegant mode of exhibiting the inference in syllogisms. For instance, in *Camestres* we have $P=m$, $S \lhd U-M \therefore \lhd U-m \therefore \lhd U-P$; or $S \lhd U-P$.

It would have been more-consistent to have made $=$, $\lhd$, and $\rhd$, (introducing this last) serve all purposes. But it has happened very often that a system of notation, already exhibited, has been extended by a better one, and mended only, instead of being reconstructed. Ploucquet had used the large and small letters, and $\rhd$ for denial: the latter symbol a strange one, if mathematical analogy were intended. Mr. Drobitsch has ingeniously contrived that $\lhd$ should represent denial, and has been led to what might have usefully amended all he had to begin with. Taking little x to represent a part of the extent of X, &c. and U for the extent of the universe, the following notation might have been adopted:

First when $\lhd$ and $\rhd$ both include their limit, $=$. We should have

$A_{\prime}$	$X \lhd Y$	or $Y \rhd X$		A'	$Y \lhd X$	or $X \rhd Y$
$O_{\prime}$	$x \lhd U-Y$	or $U-Y \rhd x$		O'	$y \lhd U-X$	or $U-X \rhd y$
$E_{\prime}$	$X \lhd U-Y$	or $U-Y \rhd X$		E'	$X \lhd U-Y$	or $U-Y \lhd X$
$I_{\prime}$	$x \lhd Y$	or $Y \rhd x$		I'	is inexpressible.	

To express I', we must invent a symbol for a part of $U-X$.

Next, when $\lhd$ and $\rhd$ do not include their limits, we have

$D_{\prime}$	$X \lhd Y$	or $Y \rhd X$		$C_{\prime}$	$X \lhd U-Y$	or $U-Y \rhd X$
D	$X=Y$	or $Y=X$		C	$X=U-Y$	or $U-Y=X$
D'	$X \rhd Y$	or $Y \lhd X$		C'	$X \rhd U-Y$	or $U-Y \lhd X$

P is inexpressible.

I am inclined to think that the representation of quantity and location both under one symbol is objectionable, if that symbol be one already appropriated in mathematics to quantity only. I would on no account accustom myself to read A $\prec$B as A is less than (because a part of) B. Mr. Drobitsch is much more complete than his predecessors in his enumeration of the various kinds of sorites.

October 29, 1847. While this sheet was passing through the press, I became acquainted with " A syllabus of logic, in which the views of Kant are generally adopted, and the laws of syllogism symbolically expressed. By Thomas Solly, Esq." Cambridge, 1839, 8vo. The symbolical expression here given is of a peculiar character: the algebraic signs are adopted in a sense which preserves the rules of sign, while the symbols represent the terms of the syllogism, or else the notions of particular and universal. Thus, if p stand for particular, u for universal, and m for one of the terms of a syllogism, $m=u$ or $m-u=0$ implies [336] that m is a universal term, and $(m-u)(n-p)=0$ implies the alternative that either m is universal, or n is particular. By means of such alternative relations, the conditions of validity of the various figures are expressed. Mr. Solly contends for six forms in each figure, by introducing all forms which have weakened conclusions, and proves *à priori*, from his equations, that six and no more are possible in each figure. If I had admitted weakened forms, there would have been sixteen more syllogisms, which might be deduced, either from the eight universals, or from the sixteen particulars.

THE END.

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